Exam preparation
Specialist Mathematics 2015

I recently had the opportunity of visiting Berlin and in particular, the Humboldt University (29 Nobel Laureates). The main building, a very grand one at that, is on the elegant central thoroughfare leading from the famous Brandenburg Gates, Unter den Linden. Established as the University of Berlin in 1810, it was renamed Humboldt University in honour of Wilhelm and Alexander von Humboldt. Wilhelm was a linguist and educational reformer who was closely involved in the establishment of the University, and his brother Alexander was an extraordinary scientist who climbed mountains, lowered himself into volcanoes, and took with him on his explorations equipment so he could take accurate measurements. Alexander (1769-1859) was a colleague of the great mathematician Carl Gauss (1777-1855) and when the examinations are over, I recommend to you an excellent book telling of the lives of these two geniuses *Measuring the World* by Daniel Kehlmann.

But for now, we need to get the measure of the Specialist Mathematics course content and prepare for the examinations. By now you will have basically completed the course and with your teacher, will be embarking on a revision program. Your teacher knows you well and make sure that you heed their advice and work closely with them. But I add a few points:

1. Read the study design carefully. Make sure that you understand every word, definition and know the key knowledge and be able to perform the key skills. Tick off the dot points for each Area of study when you are confident with your understanding of the content, and connect each Key skill dot point with problems from your text and/or examination papers.
2. Read the examination reports. Each year, the examiners comment on areas of concern and there is a common thread. Whenever you are answering question, set them out neatly and accurately. Do not take short cuts when practising as this can lead to bad habits. For example, make sure that vectors are always distinguished from scalars by the addition of the tilda ie $\underset{\sim}{a}.\underset{\sim}{b}$ is different from $a.b$
3. Complete as many practise examinations as possible, and at least a number of them under examination conditions. Learn how to use the reading time productively – read each question carefully and decide on the order in which you will answer the questions, knowing that some questions will be more difficult than others. Always commence with the question that you find the most straight forward.
4. Note in the examiners report the questions which were answered well, and most importantly, the questions which were answered badly. Note the comments. Non-transferal of Mathematical Methods skills and basically poor algebraic techniques let students down. This will not happen if skills are solid because of consistent practice.

Consider the following questions from the 2012 examinations.

Question 2, Examination 1

Find all real solutions of the equation $2\cos(x) = \sqrt{3}\cot(x)$

This question was worth 3 marks, 40% were awarded 0 marks, and only 11% were awarded full marks. So this is a question worth examining

Possible solution: $2\cos(x) = \sqrt{3}\cot(x)$

Step 1: Change cot(*x*) to $\frac{\cos(x)}{\sin(x)}$ so the equation becomes $2\cos(x) = \sqrt{3}\frac{\cos(x)}{\sin(x)}$

Step 2: There are several ways you could progress now but subtracting $\sqrt{3}\frac{\cos(x)}{\sin(x)}$ from each side of the equation is one approach giving

$$2\cos(x) - \sqrt{3}\frac{\cos(x)}{\sin(x)} = 0$$

Step 3: Multiply both sides by sin(*x*)

$$2\cos(x)\sin(x) - \sqrt{3}\cos(x) = 0$$

Step 4: Factor: $\cos(x)(2\sin(x) - \sqrt{3}) = 0$

Step 5: Use the null factor law to get solutions.

$\cos(x) = 0$

$$x = \frac{(2n+1)\pi}{2}, n \in Z$$

.

$$\sin(x) = \frac{\sqrt{3}}{2}$$

Reference angle: $\frac{\pi}{3}$ Quadrants I & 2

$$x = \frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi, n \in Z$$

So the solution is : $x = \frac{(2n+1)\pi}{2}, \frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi, n \in Z$

This is a basic general solution of trigonometric equation and everyone should know that:

- cos(x) = 0 **has** solutions $\ldots -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$ and is expressed in the form $x = \frac{(2n+1)\pi}{2}, n \in Z$
- In the solutions, *n* is an integer ie $n \in Z$ not *R*

In **Examination 2**, several multiple choice questions were poorly answered.

Question 12

The volume of the solid of revolution formed by rotating the graph of $y = \sqrt{9-(x-1)^2}$ about the *x*-axis is given by

A $4\pi(3)^2$

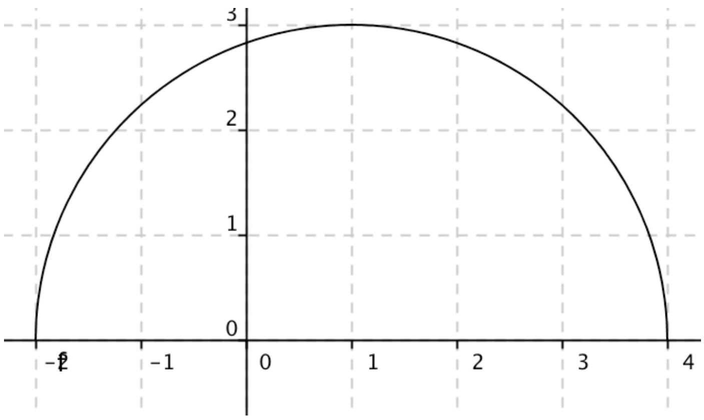
B $\pi\int_{-3}^{3}\left(9-(x-1)^2\right)dx$

C $\pi\int_{-2}^{4}\sqrt{9-(x-1)^2}\,dx$

D $\pi\int_{-2}^{4}\left(9-(x-1)^2\right)^2 dx$

E $\pi\int_{-4}^{2}\left(9-(x-1)^2\right)dx$

The graph is helpful! You will note that this is the graph of a semi-circle centre (1, 0) with radius 3, so the volume will just be the volume of a sphere with radius 3



Hence the answer is **A.** If you used the revolution formula you would have $\pi\int_{-2}^{4}\left(9-(x-1)^2\right)dx$. This is not listed as an option in this form but evaluation (using your CAS if required) gives 36π, option A. You can see that with the sketch, and just using common sense, this question can be readily answered, yet only 48% of students answered correctly! Remember that the options given may be given in an 'expected' form.

Question 19

A body is moving in a straight line and, after *t* seconds, it is *x* metres from the origin and travelling at *v* ms^{-1}. Given that *v* = *x* and that *t* = 3 where *x* = −1, the equation for *x* in terms of *t* is

A $x = e^{t-3}$

B $x = -e^{3-t}$

C $x = \sqrt{2t-5}$

D $x = -\sqrt{2t-5}$

E $x = -e^{t-3}$

Again a reasonably straight forward question which only 44% answered correctly. As stated, *v* = *x* hence $\frac{dx}{dt} = x, \frac{dt}{dx} = \frac{1}{x}, \ln|x| = t + c$ .Substituting *t* = 3 where *x* = −1 gives $\ln|x| = t - 3, \pm x = e^{t-3}$ hence **A** or **E** appear to be the options. Option **E** gives *x* = −1 when *t* = 3 so that is the answer.

These few examples illustrate the need for care and methodical thinking. To achieve this in examinations, you must be very well prepared so I stress the need for consistent, rigorous practice. And read the questions very carefully! The distractors in the Multiple Choice questions are just that: the two examples we have just looked at presented more of a challenge than they might have otherwise!

Work hard, work with your teacher and enjoy the mathematics.

Elizabeth Burns

MAV Councillor