Multiple-choice questions

7 The coordinates of the x-axis intercepts of the graph of the ellipse with the equation $\frac{x^2}{9} + \frac{y^2}{25} = 1$ are:

A (–9, 0) and (9, 0)
B (–5, 0) and (5, 0)
C (0, –3) and (0, 3)
D (–3, 0) and (3, 0)
E (3, 0) and (5, 0)

8 The solutions of $\cos 3x = \frac{1}{2}$ for $x \in [0, 2\pi]$ are:

A $\frac{\pi}{3}, \frac{5\pi}{3}$ only
B $\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}$ only
C $\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}$ only
D $\frac{\pi}{9}, \frac{7\pi}{9}, \frac{13\pi}{9}$ only
E $\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}$ only

9 The equations for the asymptotes of the hyperbola with equation $\frac{(y+1)^2}{25} - \frac{(x-2)^2}{16} = 1$ are:

A $y = \frac{5}{4}x + \frac{8}{3}$ and $y = \frac{-3}{4}x + \frac{2}{3}$
B $y = \frac{4}{5}x + \frac{10}{5}$ and $y = \frac{-4}{5}x + \frac{7}{5}$
C $y = \frac{5}{4}x + \frac{10}{3}$ and $y = \frac{-5}{4}x + \frac{2}{3}$
D $y = \frac{4}{5}x + \frac{10}{5}$ and $y = \frac{-4}{5}x + \frac{7}{3}$
E $y = \frac{5}{4}x - \frac{7}{2}$ and $y = \frac{-5}{4}x + \frac{3}{2}$

11 Let $\sec x = 3$, $\frac{3\pi}{2} < x \le 2\pi$. The exact value of $\cot x$ is:

A $-2\sqrt{2}$
B $2\sqrt{2}$
C $\frac{-1}{\sqrt{10}}$
D $\frac{\sqrt{2}}{4}$
E $\frac{-\sqrt{2}}{4}$

12 The equations of the asymptotes of $y = 2\tan^{-1}x + \pi$ are:

A $y = -\pi$ and $y = 3\pi$
B $y = 0$ and $y = 2\pi$
C $y = \frac{\pi}{2}$ and $y = \frac{3\pi}{2}$
D $y = -2 + \pi$ and $y = 2 + \pi$
E $y = -\pi + 2$ and $y = 3\pi + 2$

14 $\frac{1}{2}\sin^{-1} x + \frac{1}{2}\cos^{-1} x = a$ where a is a constant. The value of a is:

A 0
B 1
C π
D $\frac{\pi}{2}$
E $\frac{\pi}{4}$

15 The family of equations of the vertical asymptotes of the function with rule $f(\theta) = \frac{1}{1 + \cos\theta}$ is:

A $\theta = \frac{3\pi}{2}k$ where $k \in Z\backslash\{0\}$
B $\theta = \frac{\pi}{2}(3 - 2k)$ where $k \in Z$
C $\theta = \frac{\pi}{2}(3 + 2k)$ where $k \in Z\backslash\{0\}$
D $\theta = \frac{\pi}{2}(2 + 4k)$ where $k \in Z$
E $\theta = \frac{\pi}{2}(3 + 4k)$ where $k \in Z$

17 The gradient of the tangent to the ellipse with equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$ at the point $\left(1, \frac{4\sqrt{2}}{3}\right)$ is:

A $\frac{-\sqrt{2}}{6}$
B $\frac{1}{3\sqrt{2}}$
C $\frac{-2}{3}$
D $\frac{4}{9}$
E $\frac{-4}{9}$

18 Using an appropriate substitution, $\int_0^1 x\sqrt{2x+1}\,dx$ is equal to:

A $\frac{1}{4}\int_1^3 (u-1)\sqrt{u}\,du$

B $\int_1^3 (u-1)\sqrt{u}\,du$

C $\frac{1}{4}\int_0^1 (u-1)\sqrt{u}\,du$

D $\int_0^1 (u-1)\sqrt{u}\,du$

E $\frac{1}{2}\int_1^3 (u-1)\sqrt{u}\,du$

19 An antiderivative of $\frac{9}{x^2-9x}$ is:

A $\log_e|x^2-9x|$

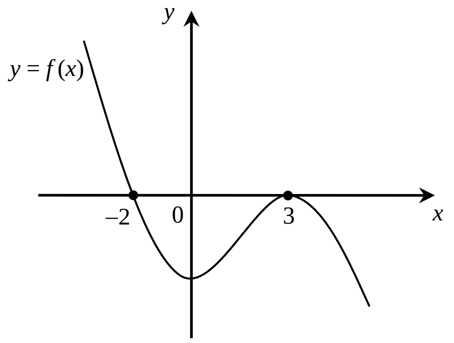
B $(2x-9)\log_e|x^2-9x|$

C $\frac{-9}{x} - \log_e|x|$

D $\log_e|x| - \log_e\left|\frac{9}{x}\right|$

E $\log_e\frac{|x-9|}{|x|}$

20 The graph of $y = f(x)$ is shown below.



If $F(x)$ is an antiderivative of $f(x)$, the stationary points of the graph of $y = F(x)$ are:

A a local minimum at $x = 0$, a local maximum at $x = 3$

B stationary points of inflexion at $x = 0$ and $x = 3$, a local maximum at $x = -2$

C a stationary point of inflexion at $x = 3$, a local maximum at $x = -2$

D a stationary point of inflexion at $x = 0$, a local maximum at $x = -2$

E a stationary point of inflexion at $x = 3$, a local minimum at $x = -2$

21 If $\frac{dy}{dx} = 4 + y^2$ and $y = 0$ when $x = 0$, then y is equal to:

A $\frac{1}{3}x^2 + 4x$

B $\frac{1}{2}\tan(2x)$

C $2\tan\left(\frac{1}{2}x\right)$

D $2\tan x$

E $2\tan(2x)$

22 A particle moves in a straight line so that its position x cm from a fixed point O at time t seconds ($t \geq 0$) is given by $x = t^3 - 9t^2 + 24t - 1$. The position of the particle (in cm) the second time it is instantaneously at rest is:

A 4
B 2
C 10
D 14
E 15

23 A body is projected up from the ground with a velocity of 30 m/s. Its acceleration due to gravity is -10 m/s^2. The body's velocity is equal to zero:

A after 0 seconds
B after 1 second
C after 2 seconds
D after 3 seconds
E never

24 A car accelerating uniformly from rest reaches a speed of 50 km/h in 5 seconds. In that time the car will have travelled:

A $\frac{625}{18}$ metres

B 125 metres

C $\frac{625}{9}$ metres

D 1. 25 kilometres

E 34.72 kilometres

25 A particle is moving along Ox so that, at time t, $x = 5\sin(2t)$. The acceleration of the particle when $t = \frac{\pi}{4}$ is:

A -20
B -10
C 0
D 10
E 20

26 A particle moves in a straight line. At time t, $t \geq 0$, its displacement x to the right of a fixed point O on the line is given by $x = 9t^2 - t^3$. The interval of time for which the particle is moving to the right is:

A (0, 6)
B $(6, \infty)$
C $(-\infty, 0)$
D $(-\infty, 6)$
E (0, 9)

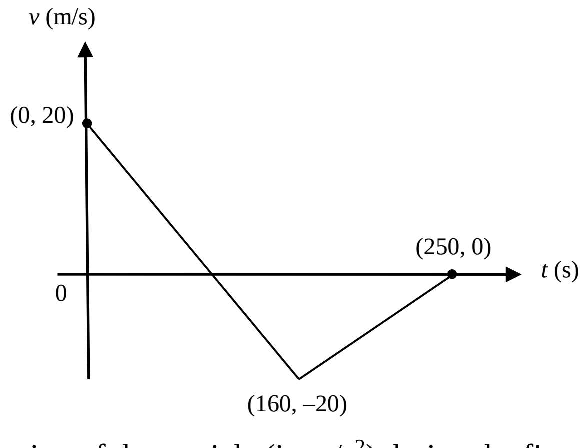
27 A particle moves along a straight line such that at time t seconds its position in metres relative to a fixed point O on the line is given by $x(t) = 5t^2 - 4$. The velocity (in m/s) when $t = 2$ is:

A 8
B 10
C 20
D 6
E −10

28 The displacement x from the origin of a particle travelling in a straight line is given by $x = 2t^3 - 10t^2 - 44t + 112$. The average speed (in m/s) of the particle during the first 4 seconds is:

A –76
B –24
C 4
D 52
E 76

29 The velocity–time graph shown describes the motion of a particle.



The acceleration of the particle (in m/s^2) during the first 160 seconds is:

A –40
B –0.25
C 0
D 0.25
E 40

30 The displacement x metres from the origin of a particle travelling in a straight line is given by $x = 2 - 2\cos\left(\frac{3\pi}{4}t - \frac{\pi}{2}\right)$. The maximum displacement of the particle (in m) is:

A –4
B –2
C 0
D 2
E 4

31 A particle moves in a straight line so that its position x cm from a fixed point O at time t seconds ($t \geq 0$) is given by $x = t^3 - 9t^2 + 24t - 1$. The particle's initial position (in cm) is:

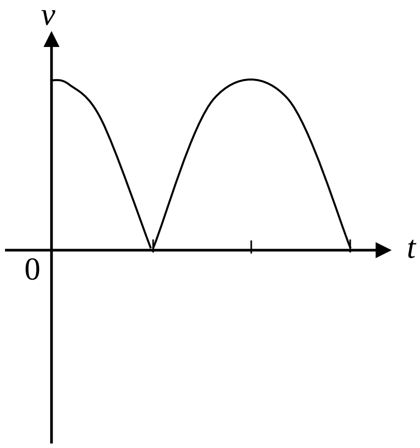
A 0
B 5
C 1
D −1
E 2

32 A particle moves in a straight line so that its position x cm from a fixed point O at time t seconds ($t \geq 0$) is given by $x = t^3 - 9t^2 + 24t - 1$. The particle's initial velocity (in cm/s) is:

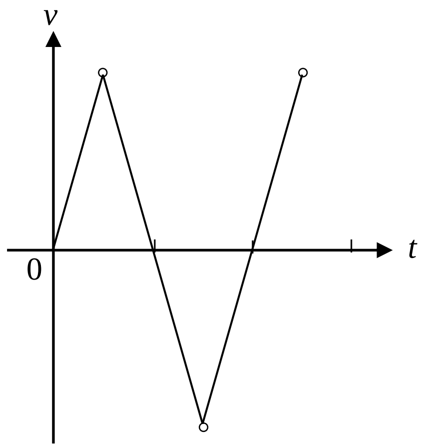
A 0
B 24
C 1
D −1
E 9

33 A ball is dropped vertically, hits the ground and bounces vertically upwards to its original height. It continues bouncing, returning to its original height after each bounce. The velocity–time graph that best represents the ball's motion from when it is dropped until it hits the ground for the second time is:

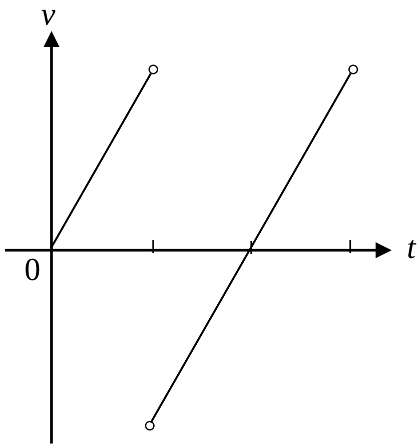
A



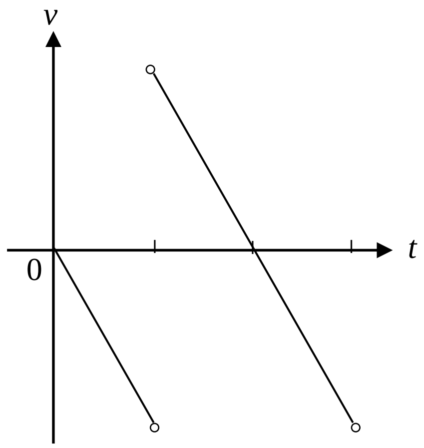
B



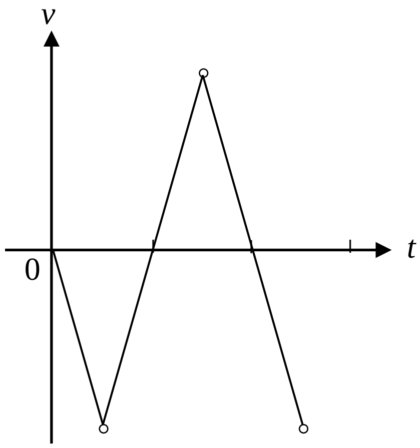
C



D



E



34 A body is projected up from the ground with a velocity of 30 m/s. Its acceleration due to gravity is -10 m/s^2. The maximum height (in m) reached by the body is:

A 90
B 30
C 6
D 45
E 3

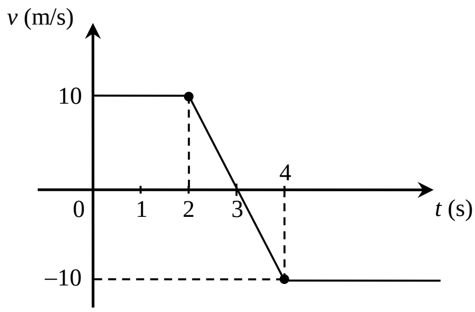
35 A particle moves in a straight line. At time t seconds its displacement from a fixed origin is x metres and its velocity is v m/s. Given that $v = \sqrt{16x - 2x^2}$, the acceleration of the particle in m/s^2 when $x = 2$ is:

A 0
B 2
C 4
D 6
E 8

36 The displacement x from the origin of a particle travelling in a straight line is given by $x = 2t^3 - 10t^2 - 44t + 112$. The acceleration (in m/s^2) at time $t = 3$ seconds is:

A –24
B –16
C 0
D 16
E 24

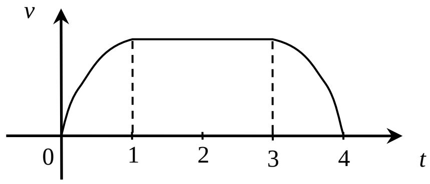
37 A particle moves with velocity v m/s as indicated in the velocity–time graph.



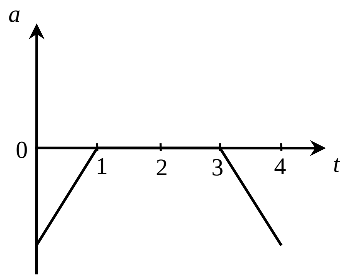
The distance, in metres, travelled by the particle in the first 4 seconds is:

A 10
B 15
C 20
D 25
E 30

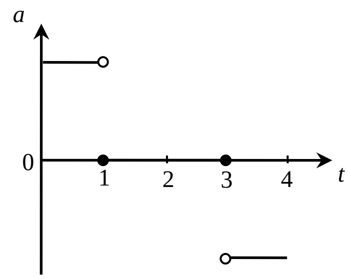
38 The following is the velocity–time graph of a racing car over a short course.

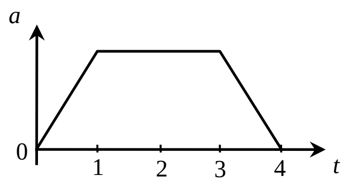


Which one of the following could be the acceleration–time graph of the car's motion?

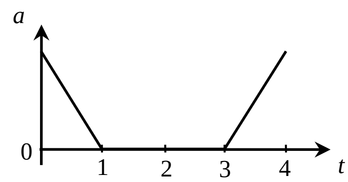


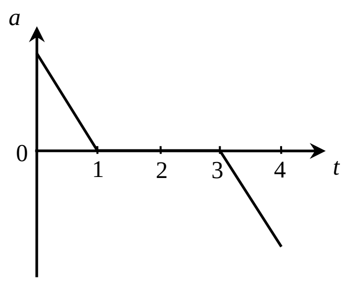
B



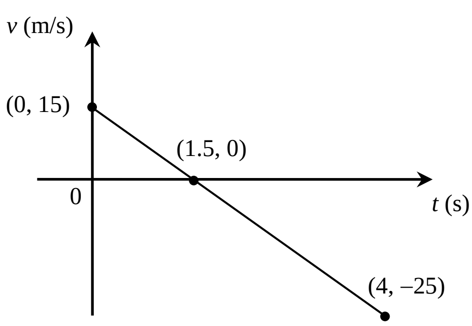


D





39 This velocity–time graph represents the motion of a ball that is thrown vertically upwards from a high balcony and then falls to the ground below. The air resistance is negligible.



The height in metres of the balcony above the ground is:

A 11.25
B 15
C 20
D 25
E 31.25

Short-answer questions (technology-free)

2 Solve for x the equation $2 \sin (2x) \cos (2x) = \cos (2x)$ for $x \in [0, \pi]$.

3
- **a** On the same set of axes, sketch the graphs of $f: [0, 2\pi] \to R$, $f(x) = \sin x$ and $g: [0, 2\pi] \to R$, $g(x) = \cos x$.
- **b** Find the coordinates of the points of intersection of the graphs of $y = f(x)$ and $y = g(x)$.
- **c** Hence find $\{x: \sin x < \cos x, 0 \le x \le 2\pi\}$.

7 Give a vector of magnitude 4 in the direction of vector $\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k}$.

9 Are the vectors $\boldsymbol{a} = 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$, $\boldsymbol{b} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{c} = -2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ linearly independent? Prove your result.

10 OAB is an isosceles triangle with $OA = OB$. M is the midpoint of AB. Let $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$. Use a vector proof to show that OM is perpendicular to AB.

11 Let A be the point (1, 2, 1) and let B be the point (4, 2, –1).
- **a** Find the point on OB which is closest to A.
- **b** What is the shortest distance between A and OB?

12 For vectors $\boldsymbol{a} = \boldsymbol{i} + 3\boldsymbol{j}$ and $\boldsymbol{b} = 2\boldsymbol{i} - \boldsymbol{j}$ describe, through a cartesian equation, the set of points with position vector $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j}$ such that:
- **a** $|\boldsymbol{r} - \boldsymbol{a}| = |\boldsymbol{r} - \boldsymbol{b}|$
- **b** $\boldsymbol{r} \cdot (\boldsymbol{r} - \boldsymbol{a}) = 0$

13 Solve the equation $\operatorname{cosec}(2x) = -\sqrt{2}$, for $x \in [0, 2\pi]$.

14 Find the exact value of:
- **a** $\sin\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$
- **b** $\cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right)$.

15 Sketch the graph of $y = \sec\left(2x - \frac{\pi}{3}\right)$, for $x \in [-\pi, \pi]$.

16 Sketch the graph of $y = \cos^{-1}(x + 4)$.

17
- **a** Given that $\sin(\theta + \alpha) = \lambda \sin(\theta - \alpha)$ show that $\tan\theta = \frac{(\lambda + 1)\tan\alpha}{\lambda - 1}$.
- **b** If $\lambda = 2$ and $\alpha = \frac{\pi}{3}$, solve the equation $\sin(\theta + \alpha) = \lambda \sin(\theta - \alpha)$ for θ where $-2\pi \le \theta \le 2\pi$.

18 If $\sin A = \frac{12}{13}$, $\frac{\pi}{2} < A < \pi$, and $\cos B = \frac{-4}{5}$, $\pi < B < \frac{3\pi}{2}$, find the exact value of $\cos(A - B)$.

19 For each of the following, find $\text{Arg}(z_1 z_2)$ and $\text{Arg}(z_1) + \text{Arg}(z_2)$.

a $z_1 = \text{cis}\left(\frac{\pi}{4}\right)$ and $z_2 = \text{cis}\left(\frac{\pi}{3}\right)$

b $z_1 = \text{cis}\left(\frac{-2\pi}{3}\right)$ and $z_2 = \text{cis}\left(\frac{-3\pi}{4}\right)$

c $z_1 = \text{cis}\left(\frac{2\pi}{3}\right)$ and $z_2 = \text{cis}\left(\frac{\pi}{2}\right)$

20 For the transformation $z \rightarrow z + 2$, sketch the image of each of the following sets of points on an Argand diagram.

a $|z| = 3$

b $\text{Arg}(z) = \frac{\pi}{3}$

c $\frac{-\pi}{3} \le \text{Arg}(z) \le \frac{\pi}{3}$

d $|z - (1 + i)| = |z - 2|$

21 **a** If $0 < \text{Arg}(z) < \frac{\pi}{2}$, show that $\text{Arg}(1 - z) = -\pi + \text{Arg}(z - 1)$.

b If $\frac{-\pi}{2} < \text{Arg}(z) < 0$, show that $\text{Arg}(1 - z) = \text{Arg}(z - 1) + \pi$.

22 Find the locus defined by $\arg(z + i) - \arg(z + 1) = \frac{\pi}{2}$.

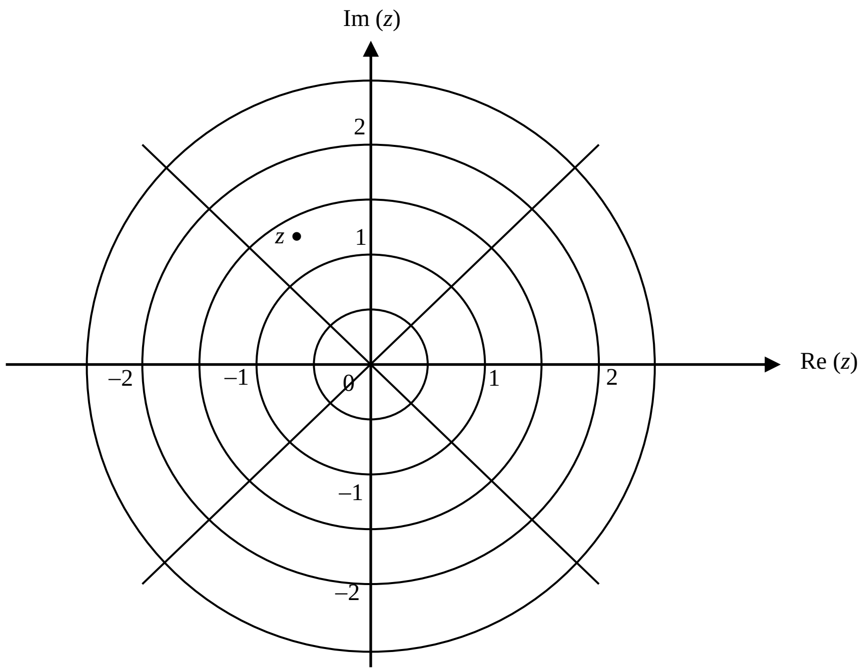
23 Shade the region of the complex plane defined by $\{z: |z - 1 + i| \ge 4\}$.

24 For the equation $P(z) = z^3 + (3 - 2i)z^2 + z + 3 - 2i$:

a show that $-3 + 2i$ is a solution of the equation $P(z) = 0$

b find all the solutions of the equation $P(z) = 0$.

25 The complex number $z = \sqrt{2}\,\text{cis}\,\theta$ is shown on the Argand diagram below. Plot and label the complex numbers u, v and w on the same diagram, where $u = z^2$, $v = \frac{1}{z}$ and $w = z^2 + \frac{1}{z}$.



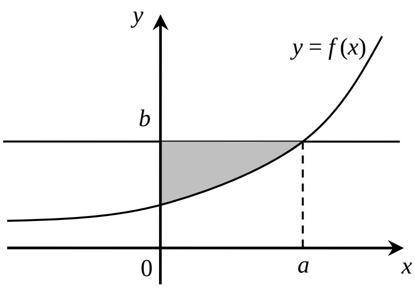
26 Shade the region of the complex plane defined by $\{z: iz - i\bar{z} < 3\}$.

27 Find the equation of the tangent to the ellipse with equation $\frac{x^2}{4} + y^2 = 1$ at the point(s) at which:

a $x = 2$
b $x = 0$
c $x = 1$.

28 If $f(x) = \log_e(\sin x)$, find $f''(x)$.

29 The shaded region is rotated around the x-axis to form a solid of revolution. Find an expression for the volume of the resultant solid.



30 **a** Show that $\frac{d}{dx}(\sin^{-1}(\sqrt{2}x)) = \frac{\sqrt{2}}{2\sqrt{x - 2x^2}}$.

b Hence find the exact value of $\int_{0.25}^{0.5} \frac{1}{\sqrt{x - 2x^2}}\,dx$.

31 Find the area of the region bounded by the two curves $y = 4x^2 + 2x$ and $y = -2x^2 + x + 1$.

32 Verify that $y = ae^{kx^2}$ is a solution to the differential equation $x\dfrac{d^2y}{dx^2} - (2kx^2 + 1)\dfrac{dy}{dx} = 0$.

33 Solve the differential equation $(x + 1)^2\dfrac{dy}{dx} = 1$ where $y = 2$ when $x = 0$.

34 Find the values of a and b if $y = a\cos(2x) + b\sin(2x)$ satisfies the differential equation $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} + 2y = \cos(2x) + \sin(2x)$.

35 Solve the differential equation $f'(x) = \dfrac{3x}{\sqrt{x^2 + 1}}$, given that $f(0) = 2$.

36 A tank contains 50 litres of a salt solution which contains 40 grams of dissolved salt. Water runs into the tank at the rate of 1.5 litres/minute and the mixture is kept uniform by stirring. The mixture then runs out at the same rate as the water runs in. If m grams of salt remain after t minutes, express:

a $\dfrac{dm}{dt}$ in terms of m

b m in terms of t.

37 Solve the differential equation $\dfrac{dy}{dx} = \dfrac{2}{1 - x^2}$ given that $y = 0$ when $x = 0$.

38 Sand is poured into a conical heap so that the radius length r cm is always $\dfrac{3}{4}$ of the height h cm. The volume of sand in the heap is V cm^3 at time t minutes.

a Express V in terms of h.

b If the height is increasing at the rate of 2 cm/min, express $\dfrac{dV}{dt}$ in terms of h.

39 The velocity, v m/s, of a particle moving in a line is given by $v = e^{\frac{-x}{2}} + 4$, $t \geq 0$, where x metres is the position of the particle at time t seconds.

a Find the acceleration of the particle in terms of x.

b Find, correct to one decimal place, the time it takes for the particle to travel 20 metres.

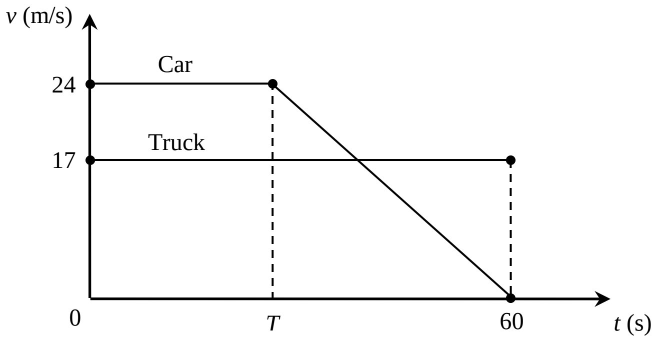
40 An object is thrown vertically upwards from the top of a building, 50 metres above ground level. The object takes 10 seconds to reach the ground.

a Find the initial speed.

b Find the maximum height reached, correct to one decimal place.

41 A particle moves in a line. At time t seconds, $t \geq 0$, its displacement from a fixed origin O is x metres and its acceleration, a m/s^2, is given by $a = 2t - \cos t$. If the particle starts at the point where $x = 3$, with a velocity of 2 m/s towards O, express x in terms of t.

42 A car travelling at 24 m/s overtakes a truck travelling at a constant speed of 17 m/s along a straight road. T seconds later, the car decelerates uniformly to rest. The truck continues at constant speed and it passes the car at the instant the car comes to a stop. This is exactly 60 seconds after the car had passed the truck.



The velocity–time graph representing this situation is shown above. Find T.

43 A particle is moving in a line so that its displacement, x m, from a fixed origin O, at time, t seconds, is given by $x = \cos(2t) + 4\cos t$, $0 \le t \le 2\pi$. If v m/s is the velocity and a m/s^2 is the acceleration at time t, find at what time(s) the particle:

a is at rest

b has zero acceleration.

44 Find the cartesian equation for the graph represented by the vector equation $\boldsymbol{r}(t) = \sec(t)\,\boldsymbol{i} + (1 + \tan(t))\boldsymbol{j}$, $t \in \left[0, \frac{\pi}{2}\right)$.

45 The following vector equations each represent the position of a particle at time t, $t \ge 0$. For each equation:

i find the corresponding cartesian equation stating domain and range

ii sketch the path of the particle indicating the initial position and the initial direction of motion.

a $\boldsymbol{r}(t) = \cos\left(t + \frac{\pi}{4}\right)\boldsymbol{i} + \sin\left(t + \frac{\pi}{4}\right)\boldsymbol{j}$

b $\boldsymbol{r}(t) = (3 - t)\boldsymbol{i} + (t^2 + 2t)\boldsymbol{j}$

c $\boldsymbol{r}(t) = \tan(t)\,\boldsymbol{i} + \sec(t)\,\boldsymbol{j}$, $t \in \left[0, \frac{\pi}{2}\right)$

46 For each of the following vector equations:

i find the corresponding cartesian equation stating domain and range

ii sketch the relation.

a $\boldsymbol{r}(t) = (3 - t)\boldsymbol{i} + 4(t + 1)\boldsymbol{j}$, $t \in R$

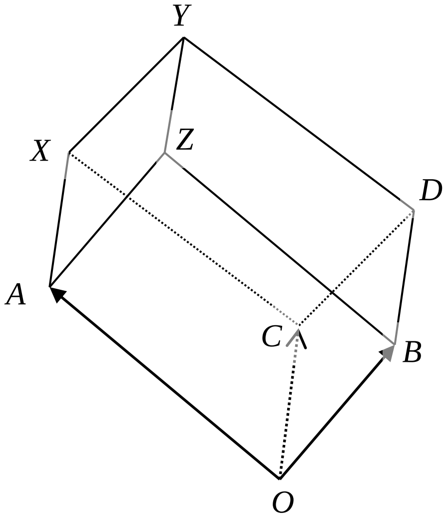
b $\boldsymbol{r}(t) = \cos(t)\,\boldsymbol{i} + (1 - \sin(t))\,\boldsymbol{j}$, $t \in R$

c $\boldsymbol{r}(t) = \sin^2\left(\frac{\pi t}{2}\right)\boldsymbol{i} + 2\cos^2\left(\frac{\pi t}{2}\right)\boldsymbol{j}$, $t \in [0, \infty)$

Extended-response questions

3 A cuboid is positioned on level ground so that it rests on one of its vertices, O.

$\overrightarrow{OA} = 3\boldsymbol{i} - 4\boldsymbol{j} - \boldsymbol{k}$, $\overrightarrow{OB} = \boldsymbol{i} + 2w\boldsymbol{j} - 5\boldsymbol{k}$, $\overrightarrow{OC} = x\boldsymbol{i} + y\boldsymbol{j} + 5\boldsymbol{k}$.



a **i** Find $\overrightarrow{OA} \cdot \overrightarrow{OB}$ in terms of w.

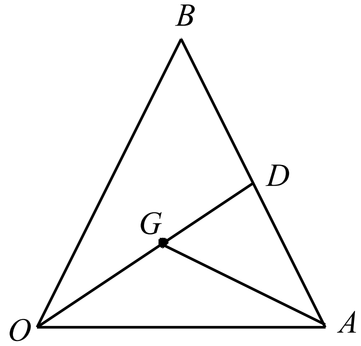
ii Hence find the value of w.

b **i** Use the fact that OA is perpendicular to OC to write an equation relating x and y.

ii Find another equation relating x and y and hence find the values of x and y.

c Hence find the exact volume of this cuboid.

4 In the figure OAB is a triangle with D the midpoint of AB. Let $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$. G is a point on OD such that $\overrightarrow{OG} = \frac{2}{3}\overrightarrow{OD}$.



a Find $\overrightarrow{OG}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

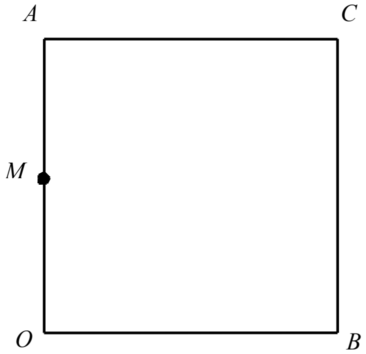
b Find $\overrightarrow{GA}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

c Find $\overrightarrow{GA} \cdot \overrightarrow{OG}$.

d **i** If GA is perpendicular to OG show that angle BOA has magnitude θ° where $\cos\theta = \dfrac{|\boldsymbol{b}|^2 - 2|\boldsymbol{a}|^2}{|\boldsymbol{a}|\,|\boldsymbol{b}|}$

ii If $|\boldsymbol{b}| = \sqrt{3}\,|\boldsymbol{a}|$, give the magnitude of angle *BOA* correct to two decimal places.

5 *OACB* is a square with $\overrightarrow{OA} = a\boldsymbol{j}$ and $\overrightarrow{OB} = a\boldsymbol{i}$. *M* is the midpoint of *OA*.



a Find, in terms of *a*:

i $\overrightarrow{OM}$

ii $\overrightarrow{MC}$.

b *P* is a point on *MC* such that $\overrightarrow{MP} = \lambda\overrightarrow{MC}$. Find $\overrightarrow{MP}$, $\overrightarrow{BP}$ and $\overrightarrow{OP}$ in terms of λ and *a*.

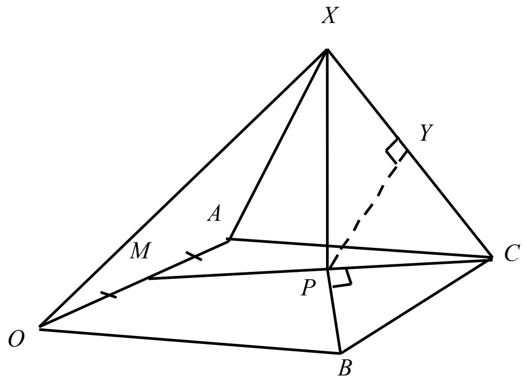
c If *BP* is perpendicular to *MC*:

i find the value of λ and also find $|\overrightarrow{BP}|$, $|\overrightarrow{OP}|$ and $|\overrightarrow{OB}|$. Comment.

ii if $\theta = \angle PBO$, evaluate cos θ.

d If $|\overrightarrow{OP}| = |\overrightarrow{OB}|$ find the possible values of λ and illustrate these two cases carefully.

In the diagram, $\overrightarrow{OA} = a\boldsymbol{j}$, $\overrightarrow{OB} = a\boldsymbol{i}$ and *BP* is perpendicular to *MC* where *M* is the midpoint of *OA*. $\overrightarrow{PX} = a\boldsymbol{k}$. *Y* is a point on *XC* such that *PY* is perpendicular to *XC*.



e Find $\overrightarrow{OY}$.

6 Let $P(z) = -z^3 - z^2 + 2z - 12$, $z \in C$.

a **i** Find $P(u)$, where $u = 1 - \sqrt{3}\, i$.

ii What can be deduced about u?

b **i** Find all the roots of the equation $P(z) = 0$, expressing your answers in cartesian form.

ii Plot the roots on an Argand diagram.

c Express u in polar form, and hence find Arg (iu).

7 Let $u = -4\sqrt{2} - 4\sqrt{2}\, i$ and $v = 2 \text{ cis}\left(\frac{-\pi}{4}\right)$.

a Express u in exact polar form.

b Show that one of the cube roots of u is v.

c Find the remaining two cube roots of u in exact polar form.

d Express v in exact cartesian form.

e Plot the three cube roots of u on an Argand diagram.

f Show that the equation $z^3 - 3\sqrt{2}\, z^2 i - 6z = -4\sqrt{2} - 6\sqrt{2}\, i$ can be expressed in the form $(z - w)^3 = -4\sqrt{2} - 4\sqrt{2}\, i$ where $w \in C$.

g Hence find one root of the equation $z^3 - 3\sqrt{2}\, z^2 i - 6z = -4\sqrt{2} - 6\sqrt{2}\, i$ in exact cartesian form.

8 **a** Plot the complex numbers $u = 8 - 6i$ and $v = -1 - 7i$ on an Argand diagram.

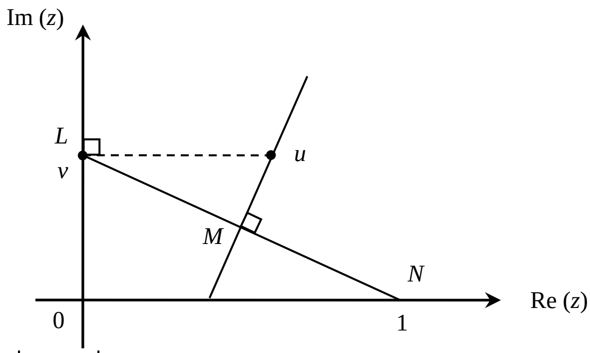
b Verify that u is a member of the subset S, where $S = \{z: |z| = 10, z \in C\}$.

c Sketch S on the Argand diagram in part **a**.

d Let w be such that $w + i\bar{v} = \bar{v}$. Find w in cartesian form.

e Sketch $T = \{z: |z| \le 10\} \cap \{z: |z - w| = |z - u|\}$ on the Argand diagram in part **a**.

9 In the Argand diagram shown, M is the midpoint of LN.



a If $|z - 1| = |z - v|$, show that the locus of z is given by the relation $2vy = 2x + v^2 - 1$ where $z = x + iy$.

b Show that $u = \frac{v^2 + 1}{2} + vi$.

c As v moves along the positive Im (z)-axis, u moves along a curve. Find the cartesian equation of this curve.

10 Let $u = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$.

a Express u in polar form, where $-\pi < \text{Arg } u \leq \pi$.

b Using an Argand diagram, show that $\text{Arg}(u + 1) = \frac{\pi}{8}$.

c **i** Express $u + 1$ in cartesian form, and in polar form.

ii Hence show that the exact value of $\sin\left(\frac{\pi}{8}\right) = \frac{\sqrt{2-\sqrt{2}}}{2}$.

d Using an appropriate compound angle formula, verify your exact value for $\sin\left(\frac{\pi}{8}\right)$.

11 **a** For $g(x) = \frac{1}{f(x)}$, find the rule for $g''(x)$ in terms of $f'(x)$, $f''(x)$ and $f(x)$.

b Show that if there is a point of inflexion at $(a, g(a))$ then $[f'(a)]^2 = \frac{1}{2} f''(a) f(a)$.

Consider $f(x) = x^2 - 2bx + 16$.

c Given that Δ is the discriminant of $f(x)$, find the values of b for which:

i $\Delta = 0$

ii $\Delta < 0$

iii $\Delta > 0$.

d Find the coordinates of stationary points and points of inflexion for the graph of $y = \frac{1}{f(x)}$ in terms of b when the discriminant of $f(x) < 0$.

e Sketch the graph of $y = \frac{1}{f(x)}$ when the discriminant of $f(x) = 0$.

f Find an antiderivative for $\frac{1}{f(x)}$ in each of the cases discussed in part **c**.

12 **a** For $y = \tan x + \frac{1}{3}\tan^3 x$, find $\frac{dy}{dx}$.

b Solve the equation $\tan x + \frac{1}{3}\tan^3 x = 0$ for $x \in \left[0, \frac{\pi}{2}\right]$.

c Find the equation of the tangent to the curve with equation $y = \tan x + \frac{1}{3}\tan^3 x$ at the point where $x = \frac{\pi}{4}$.

d Find the area between the curve with equation $y = \tan x + \frac{1}{3}\tan^3 x$, the axes and the line $x = \frac{\pi}{4}$.

e The region contained between the axes, the line $x = \frac{\pi}{4}$ and the curve $y = \sec^2 x$ is rotated around the x-axis to form a solid of revolution. Find the volume of this solid.

13 The volume, v litres, of oil in an irregularly shaped tank, when the oil depth is h metres, is given by $v = 8000h \tan^{-1} h$.

a **i** Find the exact volume of oil in the tank, in litres, when the oil depth is 1 metre.

ii Find the oil depth, correct to the nearest centimetre, when the volume is 10 000 litres.

The tank is initially empty. Oil is then poured into the tank at a constant rate of 2000 litres per minute.

b Find, in terms of h, an expression for the rate at which the oil depth is increasing, in metres per minute, when the depth is h metres.

c **i** Write a definite integral, the value of which gives the time it takes in minutes for the oil depth in the tank to reach $\sqrt{3}$ metres.

ii Show that the exact time taken for the oil depth to reach $\sqrt{3}$ metres is $\frac{4\pi}{\sqrt{3}}$ minutes.

14 In a small town of population 1000, the rate of infection of a type of influenza is modelled by the differential equation $\frac{dN}{dt} = kN(1000 - N)$ where N is the number of people infected after t days and k is an unknown constant.

a If the rate of infection is 100 people per day when the number already infected is 500, show that the differential equation can be expressed as $\frac{dt}{dN} = \frac{2500}{N(1000 - N)}$.

b Express t in terms of N, given that initially 10 people are infected.

c By the start of which day will the number of people infected first exceed 750?

15 A helicopter is hovering 25 m above the ground and drops a package of food to people below. The acceleration a m/s^2 of this package is given by $a = 9.8 - 0.05v^2$, where v m/s is the vertical speed at time t s. If x metres is the distance fallen at time t s, find:

a the terminal velocity of the package

b the speed of the package when it hits the ground, in m/s, correct to one decimal place

c the time it takes the package to reach the ground, in seconds, correct to one decimal place.

16 After brakes are applied in a car, under the influence of ABS (anti-lock brakes), the car comes to rest, and two different models have been conjectured. In a controlled experiment the brakes are applied when the car is moving at 25 m/s.

a In the first model, the acceleration, in m/s^2, is given by $a = \frac{-52}{(t+1)^3}$, $t \geq 0$, where t seconds is the time since the brakes were applied.

- **i** Express v in terms of t, where v m/s is the speed at time t.
- **ii** Find the time taken for the car to come to rest using this model.

b In the second model, $a = \frac{-(900 + v^2)}{60}$.

- **i** Express t in terms of v for this situation.
- **ii** Find the time taken for the car to come to rest using this second model.

c What model takes longer, and by how much?

17 A dragster racing car accelerates uniformly over a straight line course and completes a ‘standing’ (that is, starting from rest) 400 metres in 8 seconds.

a **i** Find the acceleration, in m/s^2, of the dragster over 400 metres.

- **ii** Show that the dragster reaches a speed of 100 m/s at the end of the 400 metre course.

At the 400 metre mark, the dragster stops accelerating. At this instant, the dragster’s brakes are applied and, in addition, a small parachute opens at the rear to slow the car down. Due to these factors the deceleration of the dragster during this stage of the motion is $\frac{5000 + 0.5v^2}{400}$ m/s^2.

b **i** Show that the differential equation relating v to x, where v m/s is the velocity of the dragster x metres beyond the 400 metre mark, is $\frac{dv}{dx} = \frac{-(10^4 + v^2)}{800v}$.

- **ii** Hence find, to the nearest metre, the distance the dragster takes to stop from the instant the brakes are applied.

c Use calculus to find the time, in seconds, taken to bring the dragster to rest from the 400 metre mark.

Answers to Revision exercises 1

Answers to multiple-choice questions

1	D
2	C
3	D
4	B
5	C
6	D
7	D
8	E
9	E
10	B
11	E
12	B
13	B
14	E
15	D
16	D
17	A
18	A
19	E
20	C
21	E
22	E
23	D
24	A
25	A
26	A
27	C
28	D
29	B
30	E
31	D
32	B
33	C
34	D
35	C
36	D
37	E
38	E
39	C
40	C
41	D
42	E
43	D

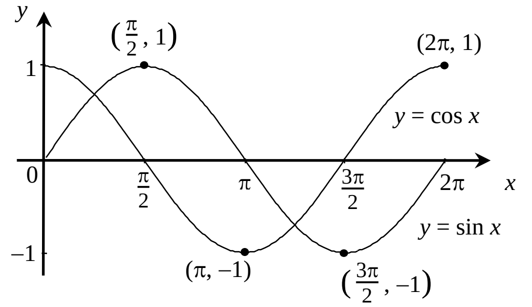
Answers to short-answer (technology-free) questions

1 **a** $\dfrac{211n}{5} - 179, \dfrac{211n^2}{10} - \dfrac{1579n}{10}$

b $3^{n-5} \times 2^{10-n}, \dfrac{1024}{81}\left(\left(\dfrac{3}{2}\right)^n - 1\right)$

2 $\dfrac{\pi}{12}, \dfrac{\pi}{4}, \dfrac{5\pi}{12}, \dfrac{3\pi}{4}$

3 **a**



b $\left(\dfrac{\pi}{4}, \dfrac{\sqrt{2}}{2}\right), \left(\dfrac{5\pi}{4}, \dfrac{-\sqrt{2}}{2}\right)$

c $\left\{x\colon 0 \le x < \dfrac{\pi}{4}\right\} \cup \left\{x\colon \dfrac{5\pi}{4} < x \le 2\pi\right\}$

5 $\dfrac{8}{9}$

7 $\dfrac{4}{\sqrt{30}}(\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k})$

8 $10\sqrt{\dfrac{13}{29}}$

9 Yes

11 **a** $\left(\dfrac{4}{3}, \dfrac{2}{3}, \dfrac{-1}{3}\right)$

b $\dfrac{\sqrt{33}}{3}$

12 **a** $y = \dfrac{1}{4}x + \dfrac{5}{8}$

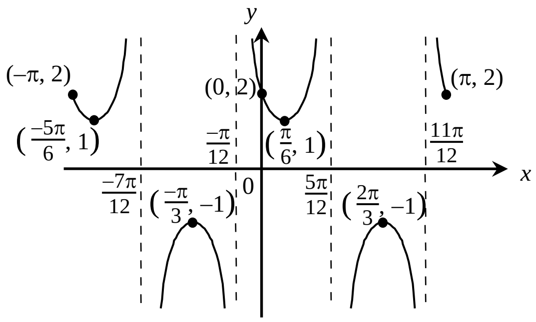
b $\left(x - \dfrac{1}{2}\right)^2 + \left(y - \dfrac{3}{2}\right)^2 = \dfrac{5}{2}$

13 $x = \dfrac{5\pi}{8}, \dfrac{7\pi}{8}, \dfrac{13\pi}{8}, \dfrac{15\pi}{8}$

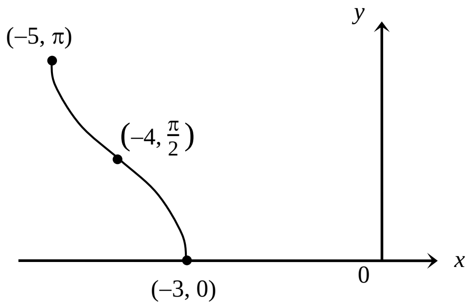
14 **a** $\dfrac{3}{5}$

b $\dfrac{12}{13}$

15



16



17 **b** $\theta = -2\pi + \tan^{-1}(3\sqrt{3}), -\pi + \tan^{-1}(3\sqrt{3}), \tan^{-1}(3\sqrt{3}), \pi + \tan^{-1}(3\sqrt{3})$

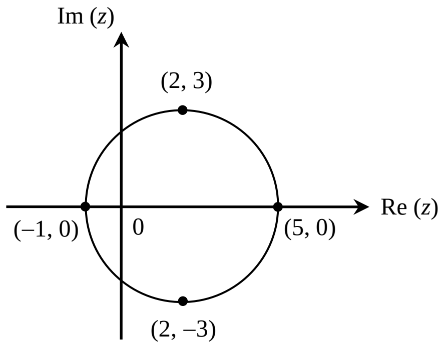
18 $\frac{-16}{65}$

19 **a** $\frac{7\pi}{12}, \frac{7\pi}{12}$

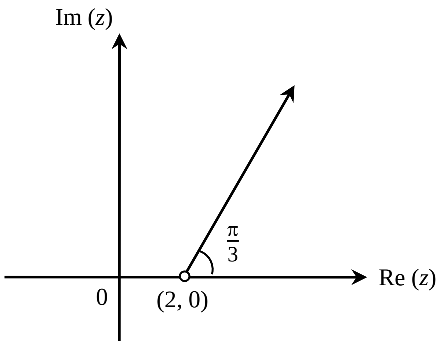
b $\frac{7\pi}{12}, \frac{-17\pi}{12}$

c $\frac{-5\pi}{6}, \frac{7\pi}{6}$

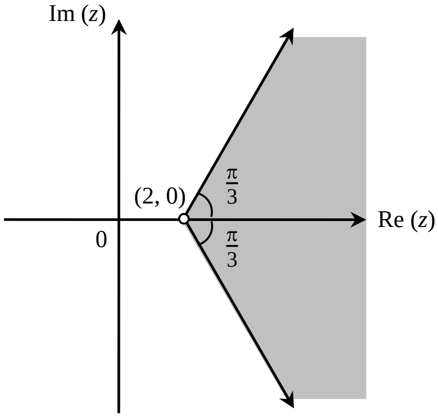
20 **a**



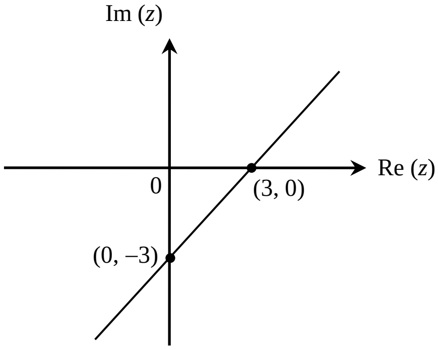
b



c

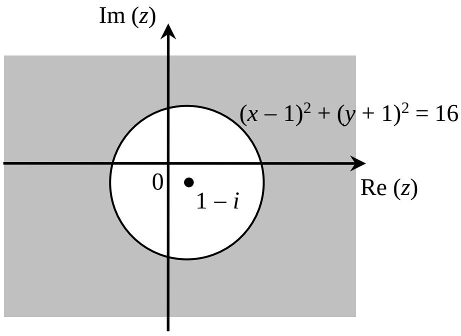


d



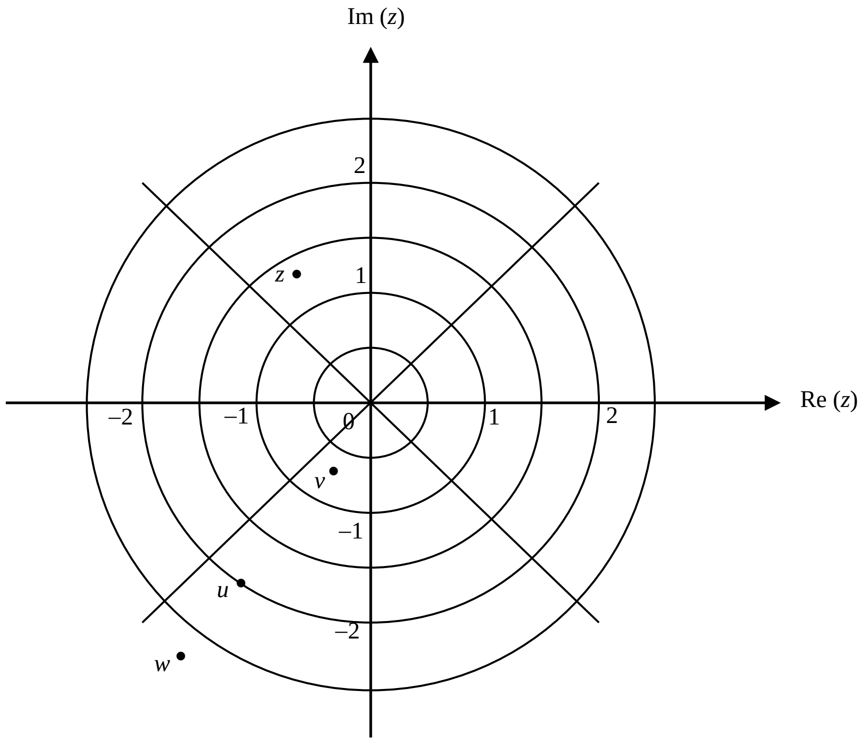
22 $z \in \left\{x + iy: \left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{2}, x > 0\right\} \cup \left\{x + iy: \left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{2}, y > 0\right\}$

23

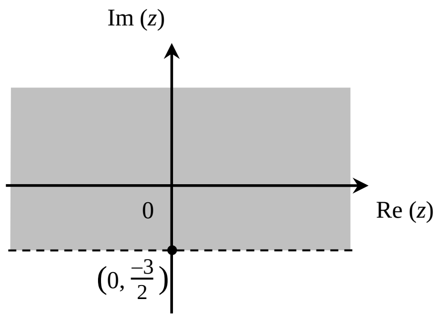


24 **b** $-3 + 2i, \pm i$

25



26



27 **a** $x = 2$

b $y = \pm 1$

c $y = \pm\frac{\sqrt{3}}{6}(x - 4)$

28 $-\text{cosec}^2\, x$

29 $\pi b^2 a - \pi\int_0^a (f(x))^2\, dx$

30 **b** $\frac{\pi\sqrt{2}}{4}$

31 **a** $\frac{125}{216}$ square units

33 $y = \frac{3x + 2}{x + 1}$

34 $a = 0,\ b = \frac{-1}{2}$

35 $f(x) = 3\sqrt{x^2 + 1} - 1$

36 **a** $\frac{dm}{dt} = \frac{-3m}{100}$

b $m = 40\,e^{\frac{-3t}{100}}$

37 $y = \log_e\left(\frac{1 + x}{1 - x}\right)$

38 **a** $V = \frac{3\pi}{16}h^3$

b $\frac{dV}{dt} = \frac{9\pi}{8}h^2$

39 **a** $a = \frac{-1}{2}e^{-x} - 2\,e^{\frac{-x}{2}}$

b 4.9 seconds

40 **a** 44 m/s

b 148.8 m

41 $x = \frac{1}{3}t^3 - 2t + 2 + \cos t$

42 $T = 25$

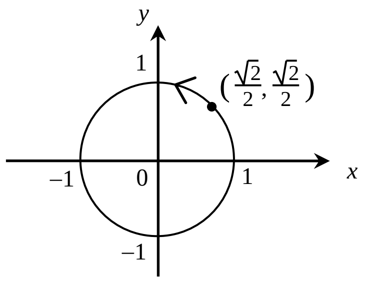
43 **a** $t = 0, \pi, 2\pi$

b $t = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

44 $x^2 - (y - 1)^2 = 1, x, y \geq 1$

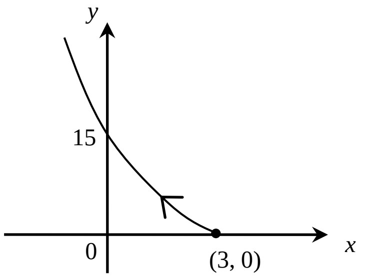
45 **a** **i** $x^2 + y^2 = 1$, dom: $[-1, 1]$, ran: $[-1, 1]$

ii



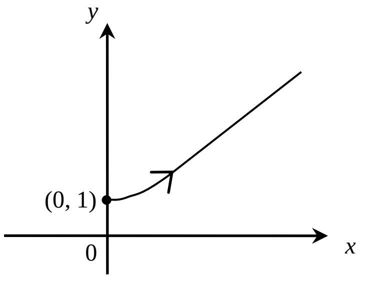
b **i** $y = x^2 - 8x + 15$, dom: $(-\infty, 3]$, ran: $[0, \infty)$

ii



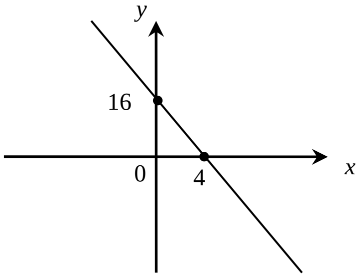
c **i** $y^2 - x^2 = 1$, dom: $[0, \infty)$, ran: $[1, \infty)$

ii



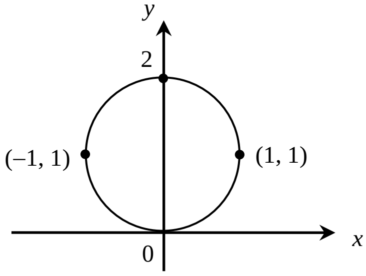
46 **a** **i** $y = 16 - 4x$, dom: R, ran: R

ii



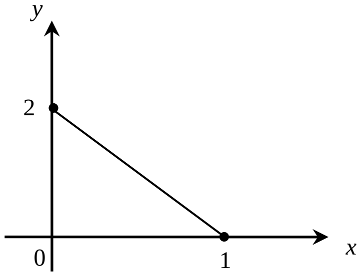
b **i** $x^2 + (1 - y)^2 = 1$, dom: $[-1, 1]$, ran: $[0, 2]$

ii



c **i** $y = -2x + 2$, dom: $[0, 1]$, ran: $[0, 2]$

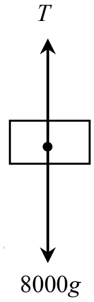
ii



47 **a** $a = \frac{17}{2}, b = \frac{5\sqrt{3}}{2}$

b 9.54 N

48 **a**



b 72 000 N

49 $-7\boldsymbol{i} + 4\boldsymbol{j} - 5\boldsymbol{k}$

50 **b** 1.02 m/s^2

Answers to extended-response questions

1 **a** **i** 2 hours 14 minutes
ii 1 hour 47 minutes
iii 2 hours 0 minutes
iv 1 hour 23 minutes
v 1 hour 52 minutes

b **i** $t = \sqrt{5 - 4\sin\theta°} + \frac{\pi\,\theta}{360}$

ii $\theta = 74.5°$, shortest possible time is 1 hour 43 minutes

2 **a** $AB = \sqrt{10}$, $AC = \sqrt{20}$, $BC = \sqrt{10}$

b $y = \frac{1}{2}x + \frac{5}{2}$

c $y = 3x$

d (1, 3)

e $(x-1)^2 + (y-3)^2 = 5$

3 **a** **i** $8 - 8w$
ii $w = 1$

b **i** $3x - 4y - 5 = 0$
ii $x + 2y - 25 = 0$, $x = 11$, $y = 7$

c 390 units^3

4 **a** $\frac{1}{3}(\boldsymbol{a} + \boldsymbol{b})$

b $\frac{2}{3}\boldsymbol{a} - \frac{1}{3}\boldsymbol{b}$

c $\frac{1}{9}(2|\boldsymbol{a}|^2 - |\boldsymbol{b}|^2 + \boldsymbol{a}\cdot\boldsymbol{b})$

d **ii** $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 54.74°$

5 **a** **i** $\frac{a}{2}\boldsymbol{j}$

ii $a\,\boldsymbol{i} + \frac{a}{2}\boldsymbol{j}$

b $\lambda\left(a\,\boldsymbol{i} + \frac{a}{2}\boldsymbol{j}\right)$, $a(\lambda - 1)\,\boldsymbol{i} + \frac{a}{2}(1 + \lambda)\boldsymbol{j}$, $\lambda a\,\boldsymbol{i} + \frac{a}{2}(1 + \lambda)\boldsymbol{j}$

c **i** $\lambda = \frac{3}{5}$, $|\overrightarrow{BP}| = \frac{2}{\sqrt{5}}a$, $|\overrightarrow{OP}| = a$, $|\overrightarrow{OB}| = a$, isosceles triangle

ii $\frac{1}{\sqrt{5}}$

d $\lambda = \frac{3}{5}, -1$

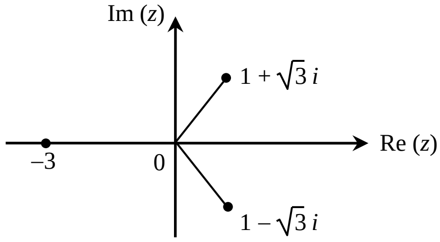
e $\frac{a}{30}(28\boldsymbol{i} + 29\boldsymbol{j} + 5\boldsymbol{k})$

6 **a** **i** 0

ii u is a root of $P(z)$

b **i** $1 \pm \sqrt{3}\,i, -3$

i



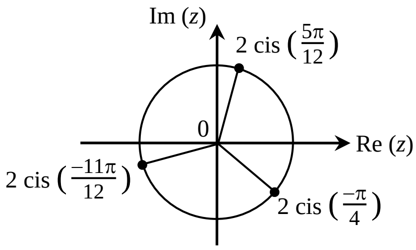
c $u = 2 \text{ cis}\left(\frac{-\pi}{3}\right)$, $\text{Arg}\,(iu) = \frac{\pi}{6}$

7 **a** $8 \text{ cis}\left(\frac{-3\pi}{4}\right)$

c $2 \text{ cis}\,\frac{5\pi}{12}, 2 \text{ cis}\left(\frac{-11\pi}{12}\right)$

d $\sqrt{2} - \sqrt{2}\,i$

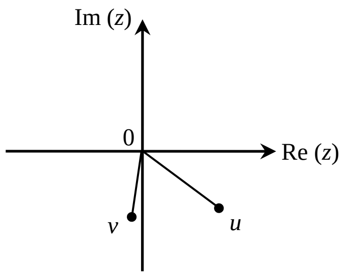
e



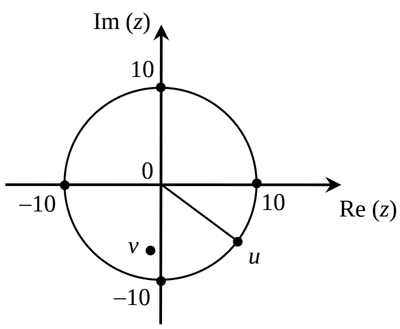
f $w = \sqrt{2}\,i$

g $\sqrt{2}$

8 **a**

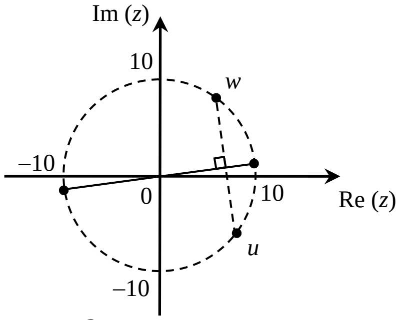


c



d $6 + 8i$

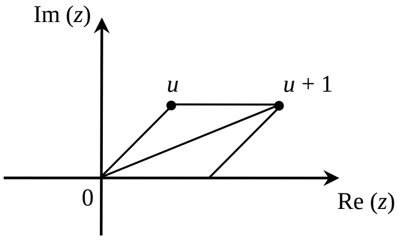
e



9 **c** $2x = y^2 + 1$

10 **a** $u = \text{cis}\,\frac{\pi}{4}$

b



c **i** $\left(1 + \frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\,i$, $\sqrt{2 + \sqrt{2}}\ \text{cis}\,\frac{\pi}{8}$

11 **a** $g''(x) = \frac{2}{(f(x))^3}(f'(x))^2 - \frac{f''(x)}{(f(x))^2}$

c **i** $b = \pm 4$

ii $-4 < b < 4$

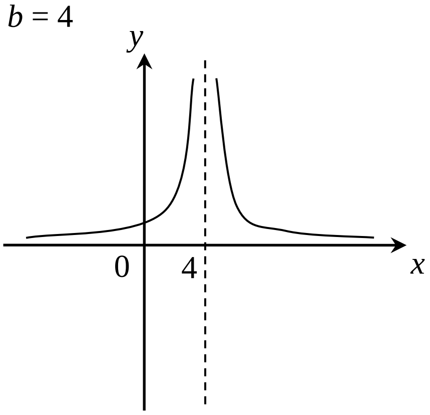
iii $b > 4$ or $b < -4$

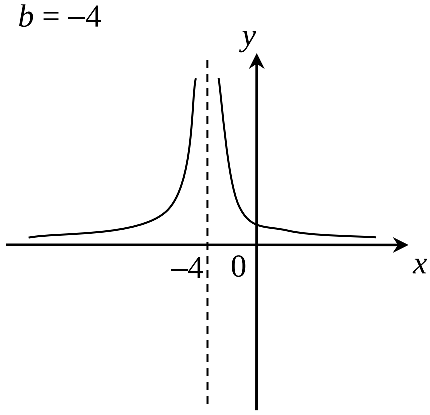
d Stationary point $\left(b, \frac{1}{16 - b^2}\right)$;

points of inflection $\left(\frac{\sqrt{48 - 3b^2} + 3b}{3}, \frac{3}{64 - 4b^2}\right)$ and

$\left(\frac{-\sqrt{48 - 3b^2} + 3b}{3}, \frac{3}{64 - 4b^2}\right)$

e $b = 4$ $b = -4$





Value of *b*	**Antiderivative**
$b = 4$	$\frac{1}{4-x}$
$b = -4$	$\frac{-1}{4+x}$
$b > 4$ or $b < -4$	$\frac{1}{2\sqrt{b^2-16}}\log_e\left(\frac{x-\sqrt{b^2-16}-b}{x+\sqrt{b^2-16}-b}\right)$
$-4 < b < 4$	$\frac{1}{\sqrt{16-b^2}}\tan^{-1}\left(\frac{x-b}{\sqrt{16-b^2}}\right)$

12 **a** $\sec^4 x$
b $x = 0$
c $y = 4x - \pi + \frac{4}{3}$
d $\frac{1}{6}(1 + 2\log_e 2)$
e $\frac{4\pi}{3}$

13 **a** **i** 2000π litres
ii 134 cm
b $\frac{dh}{dt} = \frac{1+h^2}{4(h + (1+h^2)\tan^{-1}h)}$
c **i** $\int_0^{\sqrt{3}} \frac{4(h + (1+h^2)\tan^{-1}h)}{1+h^2}\, dh$

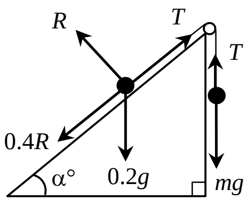
14 **b** $t = 2.5\log_e\left(\frac{99N}{1000-N}\right)$
c The start of the 15th day

15 **a** 14 m/s
b 13.4 m/s
c 2.7 s

16 **a** **i** $v = \frac{26}{(t+1)^2} - 1$
ii $\sqrt{26} - 1$ seconds
b **i** $t = 20\tan^{-1}\left(\frac{5}{6}\right) - 20\tan^{-1}\left(\frac{v}{30}\right)$
ii $20\tan^{-1}\left(\frac{5}{6}\right)$ seconds
c Model 2 takes longer by 9.8 s

17 **a** **i** 12.5 m/s^2
b **ii** 277 metres
c 2π seconds

18 **a** **i**



b $m = 0.112$

c **ii** $\dfrac{\sqrt{21g}}{15}$ m/s

d $\sqrt{\dfrac{37g}{75}}$ m/s