

2024 Trial Examination

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STUDENT Letter

NUMBER

SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

• Question and answer book of 13 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions Answer all questions in pencil on the answer sheet provided. Unless otherwise specified, an **exact** answer is required to a question. In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are not drawn to scale. Take the acceleration due to gravity to have magnitude $g \, ms^{-2}$, where g = 9.8.

Question 1 (4 marks)	
Prove by mathematical induction that $3^n + 3$, $n \in N$ is divisible by 6.	
	4 marks

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Question 2 (5 marks) The quadratic equation: $z^2 + bz + c = 0$, with real coefficients, has a root of $z_1 = 2cis\left(\frac{\pi}{6}\right)$ **a.** Write down, in polar form, z_2 , the second root to the quadratic equation. 1 mark **b.** Find the values of coefficients b and c.

2 marks

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c.	The quadratic $z^2 + 2iz - 4 = 0$, has one of the roots from the equation in part a. and a second root z_3 . Show that the product of the roots of $z^2 + 2iz - 4 = 0$ is a real number.
	······································
	2 marks
	1 + 2 + 2 = 5 marks

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Question 3 (4 marks)
Find the equation of the normal to the curve $3x^2y - 2xy^2 + y = 1$ when $y = 1$ and $x > 0$.
4 mai

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The following approximations may be useful for this question.

Let Z be an observation from a standard normal distribution.

$$Pr(-1 \le Z \le 1) \approx 0.68, Pr(-2 \le Z \le 2) \approx 0.95, Pr(-3 \le Z \le 3) \approx 0.997$$

GREENAFIELD produce strips of turf with an advertised length of $1m$. One of the machines cutting the strips is suspected of producing strips that are undersize. A sample of n strips are measured. The results are normally distributed with a mean length of $99cm$ and a standard deviation of $3cm$. It is correctly concluded that there is a 2.5% chance that the actual mean length produced by the machine is above $100cm$. Find n , the number of strips of turf analysed in the sample.

3 marks

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Question	5	(4	marks)
Question	•	ι.	11141 115)

Clarry and Tammy are best friends.

Clarry claims that: "The graph of a function can never cross one of its asymptotes."

Tammy disagrees. To support her opinion, Tammy graphs the function $y = \frac{x^3}{x^2-9}$

	Find the equations of all asymptotes for $y =$	x^3
a.	Find the equations of all asymptotes for $y =$	$x^{2}-9$

2 marks

b.	Find any intercepts for $y = \frac{x^3}{x^2 - 9}$	

1 mark

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c.	What conclusion can be drawn regarding Clarry and Tammy's disagreement?
	1 mark
	2 + 1 + 1 = 4 marks

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Question 6 (4 marks)		
A population of insects (P thousands), is growing according to the differential equation:		
$\frac{dP}{dt} = \frac{1}{8}P(4-P)$ where t is in months. Initially, (t=0), 2000 insects are present.		

a.	Solve the differential equation, writing P as a function of t .	
		3 mark
h	Hance find the possible range of values for D	
υ.	Hence find the possible range of values for <i>P</i> .	
		1 marl

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3 + 1 = 4 marks

Qυ	testion 7 (8 marks)
Co	nsider the function: $f:[0,1] \to R$ where $f(x) = \tan^{-1} x$
a.	Complete: For a suitable domain, $y = \tan^{-1} x$ and $x = $ are equivalent.
	1 mark
b.	Let $a = f(0)$ and $b = f(1)$. Evaluate a and b .
	1 mark
с.	By first considering the area bound by $x = \tan y$, the y axis and the line $y = b$, find the area bound by $f(x) = \tan^{-1} x$, the x axis and the line $x = 1$.
	3 marks

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d.	Find the volume formed when the area bound by $f(x) = \tan^{-1} x$, the y axis and the line $y = b$ is rotated around the y axis.
	y = D is found the y axis.
	3 marks
Qu	1 + 1 + 3 + 3 = 8 marks estion 8 (8 marks)
Coı	nsider the points in three dimensional space: $A(1,1,-2)$, $B(0,2,-1)$, $C(2,-1,-5)$.
a.	Find \overrightarrow{AB} and \overrightarrow{AC} .
	2 marks
	2 marks

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b.	Hence find $\overrightarrow{AB} \times \overrightarrow{AC}$, the cross product of \overrightarrow{AB} and \overrightarrow{AC} .	
		2 marks
c.	Hence find a unit vector normal to the plane containg points A, B, C .	
		1 mark

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d.	Find the Cartesian equation of the plane containing the points <i>A</i> , <i>B</i> , <i>C</i> .
	3 marks

END OF QUESTION AND ANSWER BOOK

2 + 2 + 1 + 3 = 8 marks

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