Neap

VCE Specialist Mathematics Units 3&4

Question and Answer Booklet

2024 Trial Examination 1

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Materials supplied

- Question and Answer Booklet of 11 pages
- Formula Sheet

Instructions

- Write your responses in English.
- Write **your name** and your **teacher's name** in the spaces above on this page.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents pages

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2024 VCE Specialist Mathematics Units 3&4 Examination.

Instructions

- Answer **all** questions in the spaces provided.
- Unless otherwise specified, an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, diagrams in this booklet are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$.

Question 1 (4 marks)

Find the equation of the line that is perpendicular to the graph of $x\tan^{-1}(y) + 2y = x^2 + 2$ at the point (0, 1).

Question 2 (3 marks)

Evaluate $\int_0^{\pi} x\cos(x)\,dx$.

Question 3 (5 marks)

a. Find $\left(2-\frac{2}{\sqrt{3}}i\right)^4$. Express your answer in the form $x + iy$, where $x, y \in R$. 4 marks

b. Find the integer values of n for which $\left(2-\frac{2}{\sqrt{3}}i\right)^n$ is real. 1 mark

Question 4 (4 marks)
Consider the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - 4\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = -\underset{\sim}{i} + 3\underset{\sim}{j} - 5\underset{\sim}{k}$.

a. Find the vector resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$. 2 marks

b. The vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ lie in a plane that passes through the point (3, 1, –1).
Find the Cartesian equation of the plane. 2 marks

Question 5 (3 marks)

A factory produces bags of garden soil that vary in weight. The bags have a mean weight of 25 kg and a standard deviation of 1.8 kg. The factory sells the bags in pallets, which contain 36 bags.

Determine the maximum mean weight of the bags in a randomly selected pallet, such that 84% of the pallets have a mean weight that is less than this value. Give your answer in kilograms, correct to one decimal place.

Question 6 (4 marks)

Consider the graph of $f(x) = x^4e^{2x}$.

State all values of x where the shape of the graph is concave up.

Question 7 (4 marks)

A curve with the equation $y = x^3$, where $x \in [0, 1]$, is rotated about the x-axis to form a solid of revolution.

Find the surface area of the solid of revolution.

Question 8 (6 marks)

A particle's position at time t is given by the vector $\underset{\sim}{r} = -e^t \underset{\sim}{i} + \left(e^{2t} - 2e^t - 1\right) \underset{\sim}{j}$, where $t \geq 0$.

a. Find the initial speed of the particle. 2 marks

b. Find the Cartesian equation of the particle's path and state its domain. 2 marks

c. The particle is closest to the origin when $x = a$.

Show that $2a^3 + 6a^2 + 3a - 2 = 0$. 2 marks

Question 9 (4 marks)

Find $\int \frac{-2x^2 + 13x - 1}{(x+5)(x^2+4)} dx$.

Question 10 (3 marks)

Use proof by contradiction to prove that $\sin(x) + \cos(x) \geq 1$ for all $x \in \left[0, \frac{\pi}{2}\right]$.

End of examination questions