The Mathematical Association of Victoria

Trial Exam 2024

SPECIALIST MATHEMATICS

WRITTEN EXAMINATION 1

STUDENT NAME ____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room any technology (calculators or software) or notes of any kind. blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 13 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale .

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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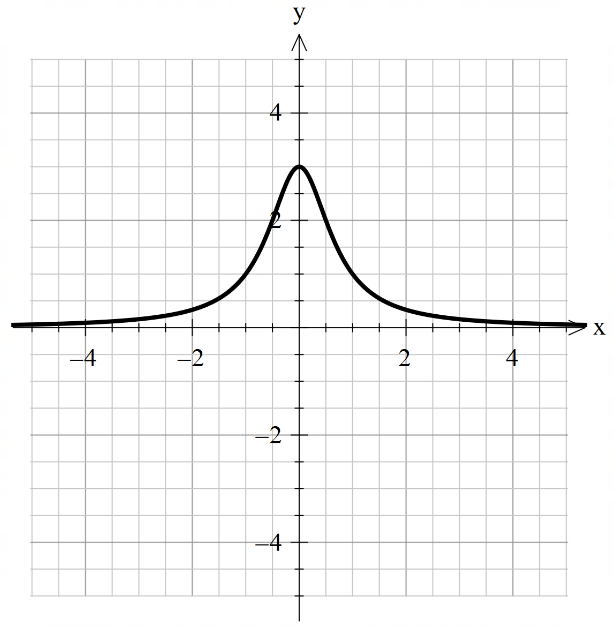
Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

Part of the graph of $g(x)=\dfrac{3}{2x^2+1}$ is shown below.



a. Show that points of inflection occur at $x=\pm\dfrac{\sqrt{6}}{6}$. 2 marks

b. Find the area under the curve $y = g(x)$ from $x = -\frac{\sqrt{6}}{6}$ to $x = \frac{\sqrt{6}}{6}$. 2 marks

Question 2 (4 marks)

Let $z_1 = 2 - 2\sqrt{3}i$ and $z_2 = 5\,\text{cis}\left(-\frac{5\pi}{6}\right)$.

a. Find, in Polar Form, $z_1 z_2$. 2 marks

b. Find $z_1 + \overline{z_2}$, expressing your answer in the Cartesian Form $a + bi$, where $a, b \in R$.

2 marks

Question 3 (3 marks)

Evaluate the gradient of the normal to the curve $-2xy^2 + ky = 3x$ at the point $(1,1)$ where k is a real constant.

Question 4 (3 marks)

Evaluate $\int_1^2 \left(\frac{2x}{2x^2 - 5x - 3} \right) dx$. Express your answer in the form $\frac{\log_e(a)}{b}$ where $a, b \in Q$.

Question 5 (3 marks)

Solve the differential equation $2y\frac{dy}{dx}=\frac{1}{\sqrt{12-x^2-4x}}$ where $y(0)=2$.

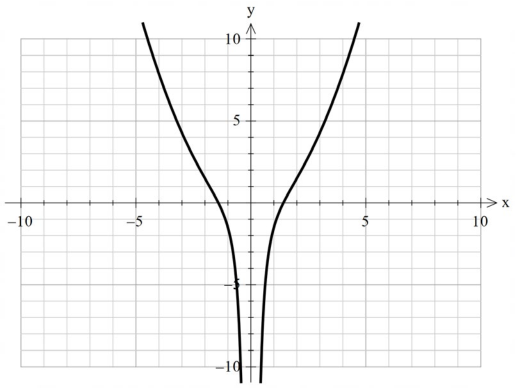
Write your answer as y in terms of x.

Question 6 (2 marks)

Find $\int x\sin(x+1)dx$.

Question 7 (5 marks)

Part of the graph of f with rule $f(x)=\frac{x^2}{2}-\frac{2}{x^2}$ is shown below where $x\in R\setminus\{0\}$.



a. Find the coordinates of the x -intercepts of the graph of $f(x)$. 1 mark

b. Show there are no stationary points on the graph of $f(x)$. 1 mark

c. The graph of $f(x)$ where $x > 0$ is rotated around the x-axis from $x = \sqrt{2}$ to $x = 2\sqrt{2}$. Find the volume of the solid of revolution formed.
Write your answer in the form of $\pi a\sqrt{b}$ where $a, b \in Q$. 3 marks

Question 8 (4 marks)

Mond Patisserie packs three croissants, two traditional and one almond, into a box that is sold to customers. It is known that the masses of traditional croissants are normally distributed with a mean of 65 grams and a standard deviation of 5 grams.
Almond croissants have masses that are normally distributed with a mean of 125 grams and a standard deviation of 10 grams.

Assume that the mass of the box packaging is negligible and the masses of the traditional croissants are independent of the masses of the almond croissants.

a. Let M be the masses of the box with two traditional and one almond croissant.
Find

i. $\mathrm{E}(M)$ 1 mark

ii. $\mathrm{sd}(M)$ 1 mark

b. Find the value of k, given that the probability of the two traditional croissants are lighter than the almond croissant is equal to $\Pr(Z < k)$ where $Z \sim N(0,1)$. 2 marks

Question 9 (3 marks)

Let $M = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}$.

Use mathematical induction to prove that $M^n = M$ where $n \geq 2$, n is an integer.

Question 10 (5 marks)

Let $\underset{\sim}{u} = -\underset{\sim}{i} + 2\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{v} = 4\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$.

a. Find the cosine of the angle θ between $\underset{\sim}{u}$ and $\underset{\sim}{v}$. 1 mark

b. Expand $\underset{\sim}{w} \cdot \left(\underset{\sim}{u} \times \underset{\sim}{v}\right) = -85$ where $\underset{\sim}{w} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$, expressing your answer in the form $ax + by + cz = -85$ 2 marks

c. Hence, find the non-zero vector $\underset{\sim}{w}$ that is perpendicular to both $\underset{\sim}{u}$ and $\underset{\sim}{v}$ such that $\underset{\sim}{w} \cdot \left(\underset{\sim}{u} \times \underset{\sim}{v}\right) = -85$.

2 marks

Question 11 (4 marks)
The position vector of a particle at time t seconds, $t \geq 0$ is given by

$$\underset{\sim}{r}(t) = \left(\sin^2(t)\right)\underset{\sim}{i} + \left(\cos(t) - \cos^3(t)\right)\underset{\sim}{j}$$

a. Find $\underset{\sim}{r}\left(\frac{\pi}{4}\right)$ and $\dot{\underset{\sim}{r}}\left(\frac{\pi}{4}\right)$. 2 marks

b. Verify that the path of $\underset{\sim}{r}(t)$ is a subset of the Cartesian curve 2 marks

$$C = \left\{\left(x, y\right) \in R^2 \mid x^3 + y^2 = x^2\right\}$$

END OF QUESTION AND ANSWER BOOK