2024
VCE
Specialist Mathematics
Year 12
Trial Examination 2

Kilbaha Education

Quality educational content

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Victorian Certificate of Education
2024

STUDENT NUMBER

									Letter
Figures									
Words									

SPECIALIST MATHEMATICS

Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 31 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Write your **name** and **student number** on your answer sheet for multiple-choice questions and sign your name in the space provided.
- All written responses must be in English.
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Consider the following two statements

(1) If the heating is on, then the room will not be cold.
(2) If the room is cold, then the heating is not on.

Then

A. statement (2) is the negation of statement (1).

B. statement (2) is the converse of statement (1).

C. statement (2) is the contrapositive of statement (1).

D. statement (2) is the inverse of statement (1).

Question 2

Consider the function with the rule $f(x) = \dfrac{x+b}{x^2 - a^2}$, where a and b are real non-zero constants.

Alan stated that the graph of the function has three straight line asymptotes if $b \neq a$.

Ben stated that the graph of the function has two straight line asymptotes and a point of discontinuity if $|b| = \sqrt{a^2}$.

Colin stated that the graph crosses the x-axis at $x = -b$ and crosses the y-axis at $y = -\dfrac{b}{a^2}$ and has a maximal domain of $R \setminus \{\pm a\}$ Then

A. Only Alan is correct.

B. Only Ben is correct.

C. Only Colin is correct.

D. All of Alan, Ben and Colin are correct.

Question 3

Which of the following statements concerning complex numbers is necessarily true?

A. If $z = x + yi$ and $x \neq 0$ then $\text{Arg}(z) = \tan^{-1}\left(\frac{y}{x}\right)$.

B. If $z = x + yi$ and $x > 0$ then $\text{Arg}(z) = \tan^{-1}\left(\frac{y}{x}\right)$.

C. If $z_1 = r_1\text{cis}(\theta_1)$ and $z_2 = r_2\text{cis}(\theta_2)$ then $\text{Arg}(z_1 z_2) = \theta_1 + \theta_2$.

D. If $z_1 = r_1\text{cis}(\theta_1)$ and $z_2 = r_2\text{cis}(\theta_2)$ and $z_1 = z_2$ then then $r_1 = r_2$ and $\theta_1 = \theta_2$.

Question 4

Consider the two lines $\underset{\sim}{r}(t) = (3+2t)\underset{\sim}{i} - (2+t)\underset{\sim}{j} + (4+t)\underset{\sim}{k}$ and $\frac{x+3}{-2} = y - 1 = 1 - z$.

Then

A. The two lines are parallel and do not intersect.

B. The two lines are the same line.

C. The two lines are not parallel and they intersect.

D. The two lines are not parallel and they do not intersect.

Question 5

To prove $\binom{2n}{n} < 2^{2n-2}$ for $n \geq 5$ using mathematical induction, in the inductive step it is necessary to assume that $\binom{2k}{k} < 2^{2k-2}$ and show that

A. $\binom{2k+2}{k+1} < 2^{2k}$

B. $\binom{2k+2}{k+1} < 2^{2k-1}$

C. $\binom{2k+1}{k+1} < 2^{2k}$

D. $\binom{2k+1}{k+1} < 2^{2k-1}$

Question 6

Let $I_n = \int x^n e^{-kx}\,dx$ for $n \in N$, then

A. $I_n = \dfrac{x^n e^{-kx}}{k} + \dfrac{n}{k} I_{n-1}$

B. $I_n = \dfrac{x^n e^{-kx}}{k} - \dfrac{n}{k} I_{n-1}$

C. $I_n = \dfrac{-x^n e^{-kx}}{k} - \dfrac{n}{k} I_{n-1}$

D. $I_n = \dfrac{x^{n+1} e^{-kx}}{n+1} + \dfrac{k}{n+1} I_{n+1}$

Question 7

Given $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$ and $\underset{\sim}{n} = 4\underset{\sim}{i} + 2\underset{\sim}{j} - 4\underset{\sim}{k}$, the closest distance between the planes $\underset{\sim}{r}.\underset{\sim}{n} = 4$ and $-2x - y + 2z = 1$ is

A. $\dfrac{1}{3}$

B. $\dfrac{2}{3}$

C. 1

D. $\dfrac{4}{3}$

Question 8

A particle is travelling on a circle. If $\dfrac{dt}{dx} = \dfrac{1}{y}$ then

A. $\dfrac{dy}{dt} = x$ and the particle moves clockwise around the circle.

B. $\dfrac{dy}{dt} = -x$ and the particle moves clockwise around the circle.

C. $\dfrac{dy}{dt} = x$ and the particle moves anticlockwise around the circle.

D. $\dfrac{dy}{dt} = -x$ and the particle moves anticlockwise around the circle.

Question 9

The scalar resolute of $\underset{\sim}{a}$ in the direction $\underset{\sim}{b}$, is p.

The scalar resolute of $m\underset{\sim}{a}$ in the direction $n\underset{\sim}{b}$ where p,n,m are all non-zero real numbers is

A. mp

B. np

C. mnp

D. $\dfrac{np}{m}$

Question 10

If θ is the angle between the two non-zero vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, then which of the following is **false**?

A. $\dfrac{\left|\underset{\sim}{a}\times\underset{\sim}{b}\right|}{\underset{\sim}{a}.\underset{\sim}{b}}=\tan(\theta)$

B. $\underset{\sim}{\hat{a}}.\underset{\sim}{\hat{b}}=\cos(\theta)$

C. If $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$, then $\underset{\sim}{a}\times\underset{\sim}{b}=\underset{\sim}{0}$ and if $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$, then $\underset{\sim}{a}.\underset{\sim}{b}=0$

D. $\underset{\sim}{\hat{a}}\times\underset{\sim}{\hat{b}}$ is a unit vector perpendicular to both $\underset{\sim}{a}$ and $\underset{\sim}{b}$

Question 11

If $x=\ln(kt)$ and $y=\cos(kt)$ where k is a non-zero real constant, then $\dfrac{d^2y}{dx^2}$ is equal to

A. $-k^2t^2\cos(kt)$

B. $k\,t^2\cos(kt)$

C. $-kt\left(kt\cos(kt)+\sin(kt)\right)$

D. $-k\left(kt\cos(kt)+\sin(kt)\right)$

Question 12

Let z be a complex number, in the Argand plane, $|z-a|=2|z-ai|$ represents

A. a straight line

B. a circle

C. an ellipse

D. an hyperbola

Question 13

Consider the function $f : R \to R,\ f(x) = \left(x^2 + bx + 2\right)e^{-2x}$ where $b \in R$.

Then which of the following is **false**?

A. If $|b| < \sqrt{6}$ then graph of the function f has no turning points and no points of inflexion.

B. If $|b| > \sqrt{7}$ then graph of the function f has two turning points and two points of inflexion.

C. If $|b| = \sqrt{7}$ then graph of the function f has one turning point and two points of inflexion.

D. If $\sqrt{6} < |b| < \sqrt{7}$ then graph of the function f has no turning points and no points of inflexion.

Question 14

The number of penguins $n = n(t)$ in an enclosure at a time t years, satisfies the equation

$\log_e(n) - \log_e(100 - n) = \frac{t}{5} - \log_e(9)$, which of the following is **false**?

A. The number of penguins satisfies the differential equation $\frac{dn}{dt} = \frac{n}{9}\left(1 - \frac{n}{100}\right)$

B. The initial number of penguins was 10 and the number of penguins cannot exceed 100.

C. $n(t) = \dfrac{100}{1 + 9e^{-\frac{t}{5}}}$

D. The penguin growth rate is increasing most rapidly when $t = 5\log_e(9)$

Question 15

When Euler's method, with a step size of $\frac{\pi}{18}$, is used to solve the differential equation

$\frac{dy}{dx} = \sin^2(3x)$ with $x_0 = 0$ and $y_0 = 3$, the value of y_3 is equal to

A. $3 + \frac{\pi}{18}$

B. $3 + \frac{\pi}{18} - \frac{\sqrt{3}}{24}$

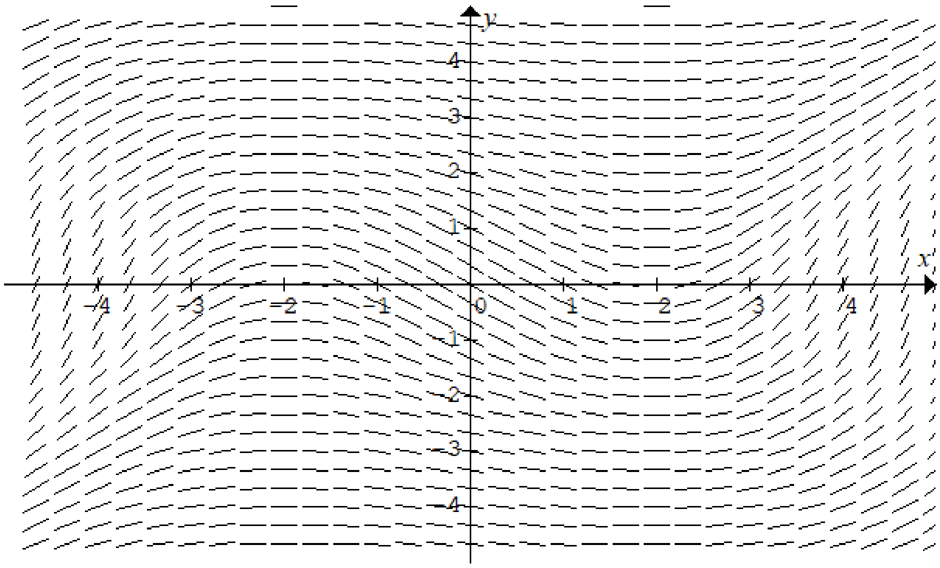
C. $3 + \frac{\pi}{72}$

D. $3 + \frac{\pi}{12}$

Question 16

The direction (slope) field for a certain differential equation is shown below.

The differential equation could be



A. $\dfrac{dy}{dx} = \dfrac{x^2+4}{y^2+4}$

B. $\dfrac{dy}{dx} = \dfrac{x^2+4}{y^2-4}$

C. $\dfrac{dy}{dx} = \dfrac{x^2-4}{y^2+4}$

D. $\dfrac{dy}{dx} = \dfrac{x^2-4}{y^2-4}$

Question 17

A hot air balloon is accelerating upwards with an acceleration of $1\,\text{ms}^{-2}$. At a particular instant it is 100 metres above ground level and rising upwards with a speed of $2\,\text{ms}^{-1}$, a small stone falls from the balloon to the ground. Assuming air resistance is negligible, the time taken, for the stone to hit the ground in seconds, is closest to

A. 4.55

B. 4.68

C. 4.73

D. 5.00

Question 18

A 95% confidence interval for a population mean is given by (a,b).

The 99% confidence interval for the population mean is closest to

A. $(1.157a-0.157b\ ,\ -0.157a+1.157b)$

B. $(-1.157a+0.157b\ ,\ 0.157a-1.157b)$

C. $(1.314a\ ,\ 1.314b)$

D. $(0.157a\ ,\ 1.157b)$

Question 19

The masses of packages of butter produced by a company are assumed to be normally distributed with a known mass of μ grams with a standard deviation of 6 grams. The containers of butter are labelled to contain 250 grams. Given there is a 20% probability that the mean mass of 25 randomly selected containers will be less than the labelled amount, then the value of μ is closest to

A. 255.05

B. 251.01

C. 248.99

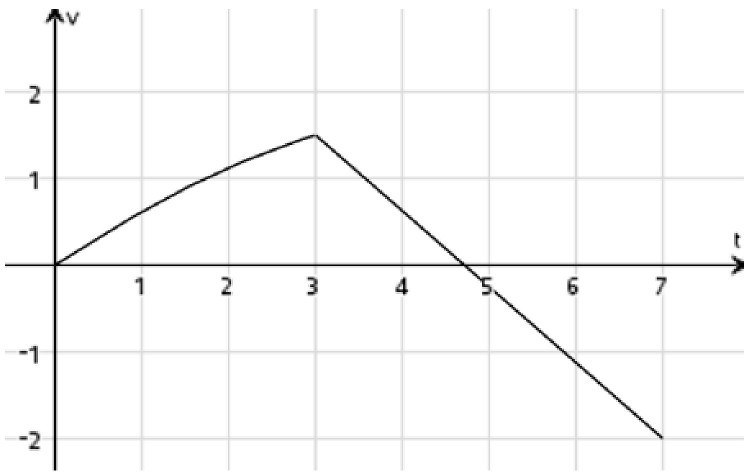
D. 244.95

Question 20

A toy car moves in a straight line, its velocity v ms^{-1} at a time t seconds is given by

$$v(t)=\begin{cases}\dfrac{6}{\pi}\tan^{-1}\left(\dfrac{t}{3}\right) & 0\le t<3\\ a+bt & 3\le t\le 7\end{cases}$$ and the velocity time graph is shown below.

Over the first seven seconds, which of the following is **false**?



A. $a=\dfrac{33}{8}$ and $b=-\dfrac{7}{8}$

B. The toy car moves a total distance of $\dfrac{6}{\pi}\displaystyle\int_0^3 \tan^{-1}\left(\dfrac{t}{3}\right)dt+\dfrac{25}{7}$ metres.

C. The toy car has a displacement of $\dfrac{6}{\pi}\displaystyle\int_0^3 \tan^{-1}\left(\dfrac{t}{3}\right)dt\ -1$ metres.

D. The average speed of the toy car in ms^{-1} is $\dfrac{1}{7}\displaystyle\int_0^7 v(t)\,dt$.

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (10 marks)

Let $f(x) = \dfrac{x^2 - 2x - 3}{x^2 - x}$

a. State the equations of the asymptotes on the graph of f.

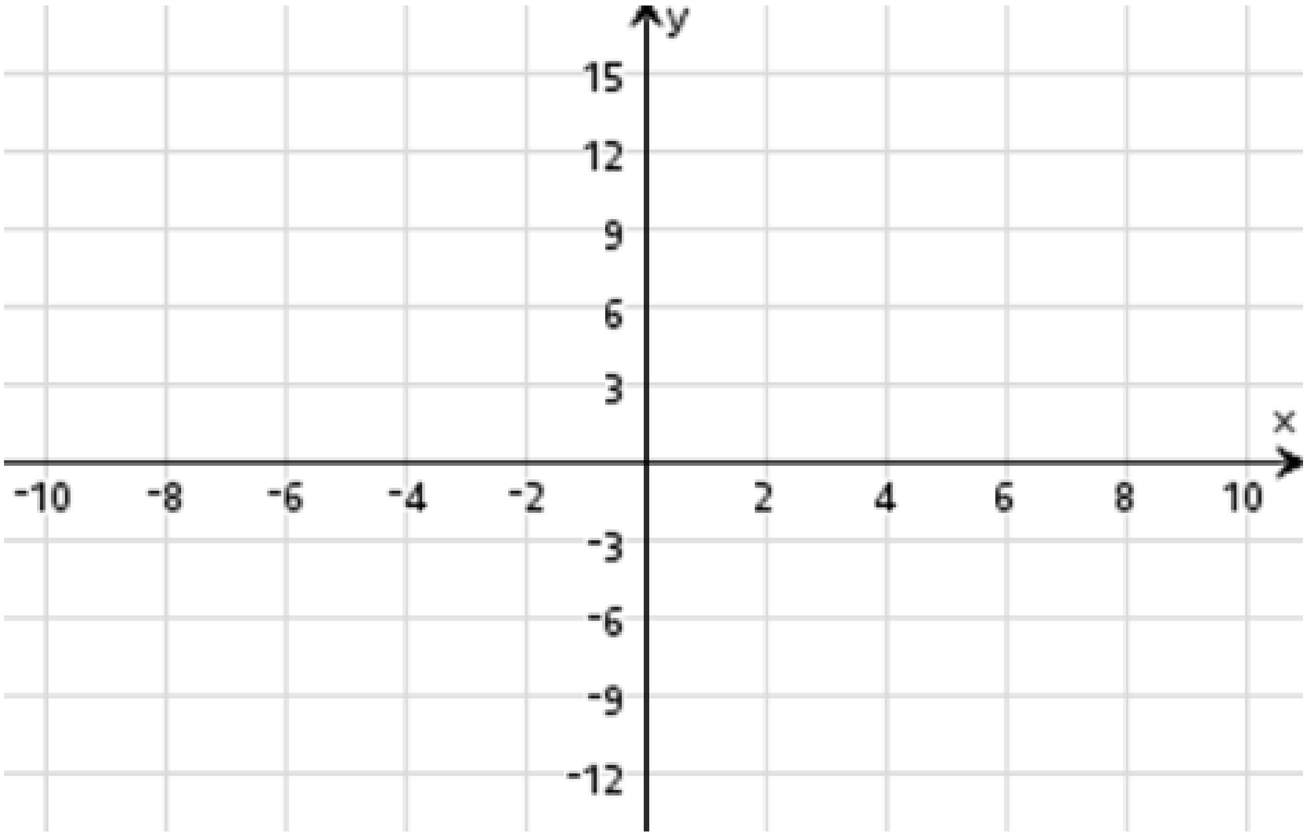
1 mark

b. Find the coordinates of the turning points on the graph of f.

1 mark

c. Sketch the graph of f on the set of axes below. Label the asymptotes with their equations, and label the turning points and the point of inflexion with their coordinates, correct to two decimal places. Label the axial intercepts with the coordinates.

3 marks



d. Consider now the graph of $f_k(x)=\dfrac{x^2-kx-3}{x^2-x}$, where k is a real constant.

i. For what values of k will the graph of f_k have a point of discontinuity.

1 mark

ii. For what values of k will the graph of f_k cross the horizontal asymptote.

1 mark

iii. Complete the table below.

3 marks

values of k	the graph of f_k has
	two turning points
	one turning point
	no turning points
	no points of inflection
	one point of inflection

Question 2 (10 marks)

a. Let $C_1 = \{z : |z-2| = 2, z \in C\}$ and $C_2 = \{z : |z-4i| = 4, z \in C\}$ be two circles in the complex plane.

i. Express both C_1 and C_2 in Cartesian form.

2 marks

ii. The non-zero point of intersection between C_1 and C_2 can be expressed as the complex number $u = a + bi$, find the values of *a* and *b*, where $a, b \in R$.

1 mark

iii. A ray *S*, which passes through the complex number *u* and the centre of the circle C_2 can be expressed as $S = \{z : \text{Arg}(z-d) = \pi + \tan^{-1}(p), z \in C\}$

Determine the real numbers *d* and *p*.

2 marks

b. Sketch the circles C_1, C_2 and the ray S on the Argand diagram below.

2 marks

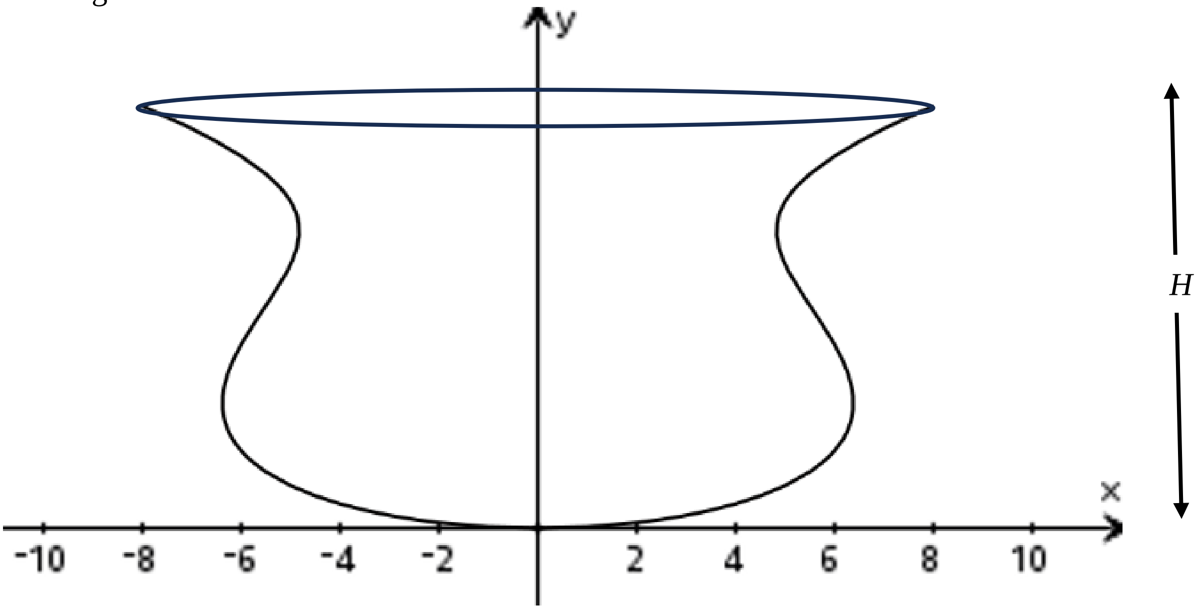
c. Determine and shade the area corresponding to

$\{z : |z-2| \le 2, z \in C\} \cap \{z : |z-4i| \le 4, z \in C\}$.

3 marks

Question 3 (10 marks)

The diagram shows a jug, with an open top, which is formed by rotating the curve $x = \sqrt{y\left(y^2 - 12y + 40\right)}$ for $0 \le x \le 8$ and $0 \le y \le H$ about the y-axis, the jug has a height of H.
All lengths are measured in centimetres.



a. **i.** Find the value of H and determine the total capacity of the jug in cm^3 when filled to a height of H.

2 marks

ii. When the jug is filled with water to a height of h centimetres, where $0 \le h \le H$ show that the volume V in cm^3 is given by $V = \dfrac{\pi h^2}{4}\left(h^2 - 16h + 80\right)$

1 mark

b. **i.** The total curved surface area S cm^2 of the jug can be expressed as the definite integral $S = \pi \int_0^b \sqrt{py^4 + qy^3 + ry^2 + sy + t}\, dy$, state the values of b,p,q,r,s and t.

3 marks

ii. Determine the total curved surface area of the jug, in cm^2 giving your answer correct to two decimal places.

1 mark

c. Unfortunately the jug has a small hole in the base and the water leaks out at a rate equal to $k\sqrt{h}$ cm^3/s where h is the height of the remaining water in the jug. When filled the jug takes two minutes to empty, determine the value of the constant k.

3 marks

Question 4 (9 marks)

a. A particle P moves such that its position vector in metres after a time t seconds, is given by $\underset{\sim}{r}_P(t) = 2t\,\underset{\sim}{i} + 3\sqrt{2}\,t\,\underset{\sim}{j} + 6t\,\underset{\sim}{k}$ where $t \geq 0$. Another particle Q initially at the point $A\left(0, 12\sqrt{2}, 32\right)$ moves at some later time, and travels towards the point $B\left(6, 15\sqrt{2}, 34\right)$, such that its speed is double the speed of particle P. Given that the two particles collide, determine the time between the release of the two particles, and the point at which they collide.

Assume all positions are given in metres.

5 marks

b. Let $\underset{\sim}{c} = \overrightarrow{OC} = c\left(\cos(\alpha)\underset{\sim}{i} + \sin(\alpha)\underset{\sim}{j}\right)$ and $\underset{\sim}{d} = \overrightarrow{OD} = d\left(\cos(\beta)\underset{\sim}{i} + \sin(\beta)\underset{\sim}{j}\right)$

where c and d are positive real numbers and $0 < \alpha < \beta < \frac{\pi}{2}$.

Prove that $\left|\underset{\sim}{c}\right| + \left|\underset{\sim}{d}\right| > \left|\underset{\sim}{d} - \underset{\sim}{c}\right|$

4 marks

Question 5 (11 marks)

a. Given the point $A(5,-1,2)$ and the plane $-2x+5y-3z=17$, find the coordinates of the point on the plane which is closest to the point *A*, and hence find the shortest distance between the point and the plane.

3 marks

b. Given the point $B(3,4,-1)$ and the line $\underset{\sim}{r}(t)=(-13+6t)\underset{\sim}{i}+(-10+3t)\underset{\sim}{j}+(-1+t)\underset{\sim}{k}$,

find the coordinates of the point on the line which are closest to the point *B*, and hence find the shortest distance between the point and the line.

4 marks

c. Given the line $\underset{\sim}{r}(t) = (-13+6t)\underset{\sim}{i} + (-10+3t)\underset{\sim}{j} + (-1+t)\underset{\sim}{k}$, and the plane $-2x+5y-3z=17$. Show that the line and the plane do not intersect and hence find the shortest distance between the line and the plane. Explain your results.

2 marks

d. Consider now the different line $\frac{x+13}{6} = \frac{y+10}{b} = z+1$ and the plane $-2x+5y-3z=d$. Determine the values of b and d for which this line and this plane intersect in

i. a unique point.

ii. an infinite number of points.

2 marks

Question 6 (10 marks)

a. A watermelon is in the shape of an ellipsoid, and has a total length of 40 centimetres and a total height of 20 centimetres.

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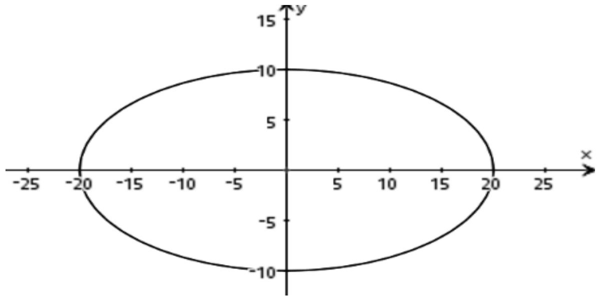

i. Determine the total surface area of the watermelon, giving your answer in square centimeters correct to two decimal places.

1 mark

ii. Determine the volume of the watermelon, giving your answer in cubic centimetres.

1 mark

The weights of watermelons grown on an orchard are normally distributed with a mean mass of 3 kilograms with a standard deviation of 400 grams. The orchard also grows cantaloupes, the masses of the cantaloupes are normally distributed with a mean mass of 1.5 kilograms with a standard deviation of 200 grams.

b. Find the probability the total mass of three independent watermelons and two independent cantaloupes have a mass exceeding 11 kilograms. Give your answer correct to four decimal places.

2 marks

c. A random sample of n cantaloupes was calculated and a 95% confidence interval was found to be $(1.45, 1.55)$ kilograms. Determine the value of n.

1 mark

The farmer who grows the watermelons on the orchard, suspects that his watermelons have not fully grown and it is desired to have the mean masses of the watermelons to be 3 kilograms. The farmer tests 36 watermelons and finds their mean mass to be 2.9 kilograms.

d. Write down the null and alternative hypothesis for the test.

1 mark

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____

The farmer decides to apply a one-tailed test at the 5% level of significance and assumes that the weights of watermelons grown on the farm are normally distributed with a mean mass of 3 kilograms with a known standard deviation of 400 grams

e. Find the p value for the test correct to four decimal places and hence determine a conclusion about the alternative hypothesis from this test.

2 marks

____

____

____

f. What is the critical sample mean in this test for the masses of the random sample of 36 watermelons, for the null hypothesis not to be rejected. Give your answer correct to four decimal places.

1 mark

____

____

____

g. Suppose that the true mean mass of the watermelons is 2.8 kg and the standard deviation is still 400 grams. What is the probability of making a type II error in this statistical test.

Give your answer as a percentage correct to one decimal place.

1 mark

END OF SECTION B

End of question and answer book for the
2024 Kilbaha VCE Specialist Mathematics Trial Examination 2

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SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics formulas

Mensuration

area of a circle segment	$\frac{r^2}{2}\left(\theta-\sin(\theta)\right)$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z=x+yi=r\left(\cos(\theta)+i\sin(\theta)\right)=r\operatorname{cis}(\theta)$	$\lvert z\rvert=\sqrt{x^2+y^2}=r$	
$-\pi<\operatorname{Arg}(z)\le\pi$	$z_1z_2=r_1r_2\operatorname{cis}(\theta_1+\theta_2)$	
$\frac{z_1}{z_2}=\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$	de Moivre's theorem	$z^n=r^n\operatorname{cis}(n\theta)$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\sin(2x)=2\sin(x)\cos(x)$	
$\cos(2x)=\cos^2(x)-\sin^2(x)$ $=2\cos^2(x)-1=1-2\sin^2(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$
$\sin^2(ax)=\frac{1}{2}\left(1-\cos(2ax)\right)$	$\cos^2(ax)=\frac{1}{2}\left(1+\cos(2ax)\right)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2 .... X_n$	$E(aX_1+b)=aE(X_1)+b$ $E(a_1X_1+a_2X_2+...+a_nX_n)$ $=a_1E(X_1)+a_2E(X_2)+...+a_nE(X_n)$
	$\text{Var}(aX_1+b)=a^2\text{Var}(X_1)$ $\text{Var}(a_1X_1+a_2X_2+...a_nX_n)$ $=a_1^2\text{Var}(X_1)+a_2^2\text{Var}(X_2)+...+a_n^2\text{Var}(X_n)$
for independent identically distributed variables $X_1, X_2 .... X_n$	$E(X_1+X_2+...+X_n)=n\mu$
	$\text{Var}(X_1+X_2+...+X_n)=n\sigma^2$
approximate confidence interval for μ	$\left(\bar{x}-z\dfrac{s}{\sqrt{n}}\ ,\ \bar{x}+z\dfrac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean: $E(\bar{X})=\mu$
	variance: $\text{Var}(\bar{X})=\dfrac{\sigma^2}{n}$

Vectors in two and three dimensions

$\underset{\sim}{r}(t)=x(t)\underset{\sim}{i}+y(t)\underset{\sim}{j}+z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert=\sqrt{x(t)^2+y(t)^2+z(t)^2}$
	$\dot{\underset{\sim}{r}}(t)=\dfrac{d\underset{\sim}{r}}{dt}=\dfrac{dx}{dt}\underset{\sim}{i}+\dfrac{dy}{dt}\underset{\sim}{j}+\dfrac{dz}{dt}\underset{\sim}{k}$
for $\underset{\sim}{r_1}=x_1\underset{\sim}{i}+y_1\underset{\sim}{j}+z_1\underset{\sim}{k}$ and $\underset{\sim}{r_2}=x_2\underset{\sim}{i}+y_2\underset{\sim}{j}+z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r_1}\cdot\underset{\sim}{r_2}=\lvert r_1\rvert\lvert r_2\rvert\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$
	vector cross product $\underset{\sim}{r_1}\times\underset{\sim}{r_2}=\begin{vmatrix}\underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k}\\ x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\end{vmatrix}=(y_1z_2-y_2z_1)\underset{\sim}{i}+(x_2z_1-x_1z_2)\underset{\sim}{j}+(x_1y_2-x_2y_1)\underset{\sim}{k}$
vector equation of a line	$\underset{\sim}{r}(t)=\underset{\sim}{r_1}+t\underset{\sim}{r_2}=(x_1+x_2t)\underset{\sim}{i}+(y_1+y_2t)\underset{\sim}{j}+(z_1+z_2t)\underset{\sim}{k}$
parametric equation of line	$x(t)=x_1+x_2t \quad y(t)=y_1+y_2t \quad z(t)=z_1+z_2t$
vector equation of a plane	$\underset{\sim}{r}(s,t)=\underset{\sim}{r_0}+s\underset{\sim}{r_1}+t\underset{\sim}{r_2}$ $=(x_0+x_1s+x_2t)\underset{\sim}{i}+(y_0+y_1s+y_2t)\underset{\sim}{j}+(z_0+z_1s+z_2t)\underset{\sim}{k}$
parametric equation of a plane	$x(s,t)=x_0+x_1s+x_2t \quad y(s,t)=y_0+y_1s+y_2t \quad z(s,t)=z_0+z_1s+z_2t$
Cartesian equation of a plane	$ax+by+cz=d$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e\left(x\right)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\left(\lvert x\rvert\right)+c$
$\frac{d}{dx}\left(\sin\left(ax\right)\right)=a\cos\left(ax\right)$	$\int \sin\left(ax\right)dx=-\frac{1}{a}\cos\left(ax\right)+c$
$\frac{d}{dx}\left(\cos\left(ax\right)\right)=-a\sin\left(ax\right)$	$\int \cos\left(ax\right)dx=\frac{1}{a}\sin\left(ax\right)+c$
$\frac{d}{dx}\left(\tan\left(ax\right)\right)=a\sec^2\left(ax\right)$	$\int \sec^2\left(ax\right)dx=\frac{1}{a}\tan\left(ax\right)+c$
$\frac{d}{dx}\left(\cot\left(ax\right)\right)=-a\,\text{cosec}^2\left(ax\right)$	$\int \text{cosec}^2\left(ax\right)dx=-\frac{1}{a}\cot\left(ax\right)+c$
$\frac{d}{dx}\left(\sec\left(ax\right)\right)=a\sec\left(ax\right)\tan\left(ax\right)$	$\int \sec\left(ax\right)\tan\left(ax\right)dx=\frac{1}{a}\sec\left(ax\right)+c$
$\frac{d}{dx}\left(\text{cosec}\left(ax\right)\right)=-a\,\text{cosec}\left(ax\right)\cot\left(ax\right)$	$\int \text{cosec}\left(ax\right)\cot\left(ax\right)dx=-\frac{1}{a}\text{cosec}\left(ax\right)+c$
$\frac{d}{dx}\left(\sin^{-1}\left(ax\right)\right)=\frac{1}{\sqrt{1-\left(ax\right)^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}\left(ax\right)\right)=\frac{-1}{\sqrt{1-\left(ax\right)^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}\left(ax\right)\right)=\frac{a}{1+\left(ax\right)^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int \left(ax+b\right)^n dx=\frac{1}{a\left(n+1\right)}\left(ax+b\right)^{n+1}+c,\ n\neq -1$
	$\int \left(ax+b\right)^{-1} dx=\frac{1}{a}\log_e\left(\lvert ax+b\rvert\right)+c$

Calculus- continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n, y_n)$
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about the x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about the y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about the x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about the y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}t^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u + v)t$

END OF FORMULA SHEET

ANSWER SHEET

STUDENT NUMBER

											Letter
Figures											
Words											

SIGNATURE ___________________________________

SECTION A

1		**A**		**B**		**C**		**D**
2		**A**		**B**		**C**		**D**
3		**A**		**B**		**C**		**D**
4		**A**		**B**		**C**		**D**
5		**A**		**B**		**C**		**D**
6		**A**		**B**		**C**		**D**
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15		**A**		**B**		**C**		**D**
16		**A**		**B**		**C**		**D**
17		**A**		**B**		**C**		**D**
18		**A**		**B**		**C**		**D**
19		**A**		**B**		**C**		**D**
20		**A**		**B**		**C**		**D**

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