2024
VCE
Specialist Mathematics
Year 12
Trial Examination 2

Detailed Answers

Kilbaha Education (Est. 1978) (ABN 47 065 111 373)	**PayID: 47065111373**
PO Box 3229	**Email: kilbaha@gmail.com**
Cotham Vic 3101	**Tel: (03) 9018 5376**
Australia	**Web: https://kilbaha.com.au**

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SECTION A

ANSWERS

1		A		B		**C**		D
2		A		B		C		**D**
3		A		**B**		C		D
4		A		**B**		C		D
5		**A**		B		C		D
6		A		B		C		**D**
7		A		B		**C**		D
8		A		**B**		C		D
9		**A**		B		C		D
10		A		B		C		**D**
11		A		B		**C**		D
12		A		**B**		C		D
13		A		B		C		**D**
14		**A**		B		C		D
15		**A**		B		C		D
16		A		B		**C**		D
17		A		B		**C**		D
18		**A**		B		C		D
19		A		**B**		C		D
20		A		B		C		**D**

SECTION A

Question 1 **Answer C**

h: the heating is on
$\neg r$: the room is not cold
If the heating is on then the room is not cold $h \to \neg r$
the contrapositive is $r \to \neg h$ if the room is cold, then the heating is not on.

Question 2 **Answer D**

$f(x) = \dfrac{x+b}{x^2-a^2}$ has $y=0$ as a horizontal asymptote and $x = \pm a$ a vertical asymptotes,
so three straight line asymptotes provided $b \neq a$, Alan is correct
If $|b| = \sqrt{a^2}$ then $b = \pm\sqrt{a^2}$ consider when $b = \sqrt{a^2} = a$ $f(x) = \dfrac{x+a}{x^2-a^2} = \dfrac{1}{x-a}$ $x \neq -a$, $y=0$ as a horizontal asymptote and $x = a$ a vertical asymptote and $x = -a$ is a point of discontinuity,
or consider when $b = -\sqrt{a^2} = -a$ $f(x) = \dfrac{x-a}{x^2-a^2} = \dfrac{1}{x+a}$ $x \neq a$, $y=0$ as a horizontal asymptote and $x = -a$ a vertical asymptote and $x = a$ is a point of discontinuity, Ben is correct.
$f(0) = -\dfrac{b}{a^2}$ and $f(x) = 0 \Rightarrow x = -b$ so $\left(0, -\dfrac{b}{a^2}\right)$ is the y-intercept and $(-b, 0)$ is the x-intercept,
Colin is correct.

Question 3 **Answer B**

If $z = x + yi$, $\text{Arg}(z) = \tan^{-1}\left(\dfrac{y}{x}\right)$, is true in the first and fourth quadrant only, so only when $x > 0$

Question 4 **Answer B**

Line (1), $\underset{\sim}{r}(t) = (3+2t)\underset{\sim}{i} - (2+t)\underset{\sim}{j} + (4+t)\underset{\sim}{k}$ has direction $2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and

parametric equations $x = 3+2t$, $y = -2-t$, $z = 4+t$.

Line (2), $\dfrac{x+3}{-2} = y-1 = 1-z$ rewrite as $\dfrac{x+3}{-2} = \dfrac{y-1}{1} = \dfrac{z-1}{-1} = s$ has direction $-2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$

so the lines are parallel, and has the parametric form $x = -3-2s$, $y = 1+s$, $z = 1-s$

equate the x components $-3-2s = 3+2t$ so $s = -3-t$ then $y = 1+(-3-t) = -2-t$ and

$z = 1-s = 1-(-3-t) = 4+t$, so the two lines are in fact the same line.

Question 5 **Answer A**

Assume $\dbinom{2k}{k} < 2^{2k-2}$ replace k with $k+1$ and show that $\dbinom{2(k+1)}{k+1} = \dbinom{2k+2}{k+1} < 2^{(k+1)-2} = 2^{2k}$

Question 6 **Answer D**

$I_n = \int x^n e^{-kx}\,dx$ integration by parts, let

$u = x^n \qquad \frac{dv}{dx} = e^{-kx}$

$\frac{du}{dx} = nx^{n-1} \quad v = \int e^{-kx}\,dx = -\frac{1}{k}e^{-kx}$

$I_n = -\frac{x^n e^{-kx}}{k} + \frac{n}{k}\int x^{n-1}e^{-kx}\,dx = -\frac{x^n e^{-kx}}{k} + \frac{n}{k}I_{n-1}$ **A. B.** and **C.** are all incorrect.

$I_n = \int x^n e^{-kx}\,dx$ integration by parts, let

$u = e^{-kx} \qquad \frac{dv}{dx} = x^n$

$\frac{du}{dx} = -ke^{-kx} \quad v = \int x^n\,dx = \frac{x^{n+1}}{n+1}$

$I_n = \frac{x^{n+1}e^{-kx}}{n+1} + \frac{k}{n+1}\int x^{n+1}e^{-kx}\,dx = \frac{x^{n+1}e^{-kx}}{n+1} + \frac{k}{n+1}I_{n+1}$

Question 7 **Answer C**

The distance of the plane $ax+by+cz=d$ from the origin is $\frac{|d|}{\sqrt{a^2+b^2+c^2}}$

Plane (1) $\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$ and $\underset{\sim}{n} = 4\underset{\sim}{i} + 2\underset{\sim}{j} - 4\underset{\sim}{k}$ $\quad \underset{\sim}{r}.\underset{\sim}{n} = 4$ gives $4x+2y-4z=4$ or

$2x+y-2z=2$ the distance from the origin is $\frac{2}{\sqrt{2^2+1^2+(-2)^2}} = \frac{2}{3}$

Plane (2) $-2x-y+2z=1$ the distance from the origin is $\frac{1}{\sqrt{(-2)^2+(-1)^2+2^2}} = \frac{1}{3}$

But the normals of the two planes are in the opposite direction, so the planes are on opposite sides of the origin, so the distance between the planes is $\frac{1}{3}+\frac{2}{3}=1$

Question 8 **Answer B**

A particle is travelling on a circle, so that $x^2+y^2=r^2$ using implicit differentiation

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0, \quad \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = -\frac{x}{y}\times(y) = -x$, so y decreases as x increases

the particle moves clockwise around the circle.

Question 9 **Answer A**

The scalar resolute of $\underset{\sim}{a}$ in the direction $\underset{\sim}{b}$, is $\underset{\sim}{a}.\hat{\underset{\sim}{b}} = \dfrac{\underset{\sim}{a}.\underset{\sim}{b}}{|\underset{\sim}{b}|} = p$.

Let $\underset{\sim}{c} = n\underset{\sim}{b}$, now $|\underset{\sim}{c}| = n|\underset{\sim}{b}|$

The scalar resolute of $m\underset{\sim}{a}$ in the direction $\underset{\sim}{c}$ is $m\underset{\sim}{a}.\hat{\underset{\sim}{c}} = \dfrac{m\underset{\sim}{a}.\underset{\sim}{c}}{|\underset{\sim}{c}|} = \dfrac{m\underset{\sim}{a}.n\underset{\sim}{b}}{n|\underset{\sim}{b}|} = m\dfrac{\underset{\sim}{a}.\underset{\sim}{b}}{|\underset{\sim}{b}|} = mp$

Question 10 **Answer D**

$\underset{\sim}{a}\times\underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\sin\theta\,\hat{\underset{\sim}{n}}$ where $\hat{\underset{\sim}{n}} = \dfrac{\underset{\sim}{a}\times\underset{\sim}{b}}{|\underset{\sim}{a}\times\underset{\sim}{b}|}$ is a unit vector perpendicular to both $\underset{\sim}{a}$ and $\underset{\sim}{b}$, so $|\hat{\underset{\sim}{n}}| = 1$

(1) $|\underset{\sim}{a}\times\underset{\sim}{b}| = |\underset{\sim}{a}||\underset{\sim}{b}|\sin\theta$ and (2) $\underset{\sim}{a}.\underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos(\theta)$ $\dfrac{(1)}{(2)}$ $\dfrac{|\underset{\sim}{a}\times\underset{\sim}{b}|}{\underset{\sim}{a}.\underset{\sim}{b}} = \tan(\theta)$ **A.** is true

$\hat{\underset{\sim}{a}}.\hat{\underset{\sim}{b}} = \dfrac{\underset{\sim}{a}.\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|} = \cos(\theta)$ **B.** is true

If $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$, then $\underset{\sim}{a}\times\underset{\sim}{b} = \underset{\sim}{0}$ and if $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$, then $\underset{\sim}{a}.\underset{\sim}{b} = 0$ **C.** is true

D. is false.

Question 11 **Answer C**

$x = \log_e(kt) \qquad y = \cos(kt)$

$\dfrac{dx}{dt} = \dot{x} = \dfrac{1}{t} \qquad \dfrac{dy}{dt} = \dot{y} = -k\sin(kt)$

$\dfrac{dy}{dx} = \dfrac{dy}{dt}.\dfrac{dt}{dx} = \dfrac{\dot{y}}{\dot{x}} = -kt\sin(kt)$

$\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right).\dfrac{dt}{dx}$

$\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(-kt\sin(kt)\right).t$

$\dfrac{d^2y}{dx^2} = t\left(-k^2t\cos(kt) - k\sin(kt)\right)$

$\dfrac{d^2y}{dx^2} = -kt\left(kt\cos(kt) + \sin(kt)\right)$

$\dfrac{d^2}{dx^2}\left(\cos\left(e^x\right)\right) \qquad -e^{2\cdot x}\cdot\cos\left(e^x\right) - e^x\cdot\sin\left(e^x\right)$

$\dfrac{d^2}{dx^2}\left(\cos\left(e^x\right)\right)|x=\ln(k\cdot t)$

$-k^2\cdot t^2\cdot\cos(k\cdot t) - k\cdot t\cdot\sin(k\cdot t)$

Question 12 **Answer B**

$|z-a| = 2|z-ai|, \quad z = x + yi$

$|(x-a)+yi| = 2|x+(y-a)i|$

$\sqrt{(x-a)^2 + y^2} = 2\sqrt{x^2 + (y-a)^2}$

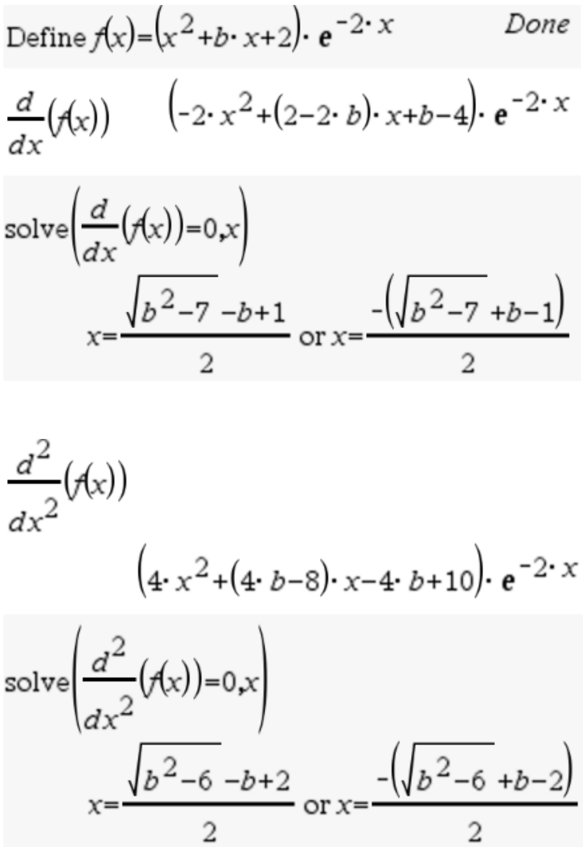
$(x-a)^2 + y^2 = 4\left(x^2 + (y-a)^2\right)$

$x^2 - 2xa + a^2 + y^2 = 4x^2 + 4y^2 - 8ya + 4a^2$

$3x^2 + 2xa + 3y^2 - 8ya = -3a^2$

$\left(x + \frac{a}{3}\right)^2 + \left(y - \frac{4a}{3}\right)^2 = \frac{8a^2}{9}$ a circle

Question 13 **Answer D**



$f(x) = \left(x^2 + bx + 2\right)e^{-2x}$

$f'(x) = \left(-2x^2 + (2-2b)x + (b-4)\right)e^{-2x}$

for turning points

$f'(x) = 0 \Rightarrow -2x^2 + (2-2b)x + (b-4) = 0$

$x = \frac{-b+1 \pm \sqrt{b^2-7}}{2}$

so there are two turning points if $|b| > \sqrt{7}$

one turning point if $|b| = \sqrt{7}$

and no turning points if $|b| < \sqrt{7}$

$f''(x) = \left(4x^2 + (4b-8)x + (10-4b)\right)e^{-2x}$

for inflexion points

$f''(x) = 0 \Rightarrow 4x^2 + (4b-8)x + (10-4b) = 0$

$x = \frac{-b+2 \pm \sqrt{b^2-6}}{2}$

so there are two inflexion points if $|b| > \sqrt{6}$

one inflexion point if $|b| = \sqrt{6}$

and no inflexion points if $|b| < \sqrt{6}$

D. is false

Question 14 **Answer A**

$\log_e(n)-\log_e(100-n)=\frac{t}{5}-\log_e(9)$,

satisfies the differential equation

$$\frac{dn}{dt}=\frac{n(100-n)}{500}=\frac{n}{5}\left(1-\frac{n}{100}\right)=\frac{100n-n^2}{500}$$

A. is false, **B. C.** and **D.** are all true

The initial number of penguins was 10 and

the number of penguins cannot exceed 100.

The solution of the differential equation is

$$n(t)=\frac{100}{1+9e^{-\frac{t}{5}}}$$

The penguin growth rate is increasing most rapidly

when $\frac{d^2n}{dt^2}=\left(\frac{100-2n}{500}\right)\frac{dn}{dt}=0 \quad \Rightarrow n=50$

$$\frac{100}{1+9e^{-\frac{t}{5}}}=50 \quad \Rightarrow t=5\log_e(9)=10\log_e(3)$$

$\text{solve}\left(\ln(n)-\ln(100-n)=\frac{t}{5}-\ln(9),n\right)|t=0$ $\quad n=10$

$\text{impDif}\left(\ln(n)-\ln(100-n)=\frac{t}{5}-\ln(9),t,n\right)$ $\quad \frac{-n\cdot(n-100)}{500}$

$\text{deSolve}\left(n'=\frac{-n\cdot(n-100)}{500} \text{ and } n(0)=10,t,n\right)$ $\quad n=\frac{100\cdot e^{\frac{t}{5}}}{e^{\frac{t}{5}}+9}$

$\text{solve}\left(\frac{100\cdot e^{\frac{t}{5}}}{e^{\frac{t}{5}}+9}=50,t\right)$ $\quad t=10\cdot\ln(3)$

Question 15 **Answer A**

$\frac{dy}{dx}=f(x)=\sin^2(3x)$ using Euler's Method

$h=\frac{\pi}{18}$, $x_0=0$ and $y_0=3$, $x_1=\frac{\pi}{18}$, $x_2=\frac{\pi}{9}$

$$y_1=y_0+h\,f(x_0)=3+\frac{\pi}{18}\sin^2(0)=3$$

$$y_2=y_1+h\,f(x_1)=3+\frac{\pi}{18}\sin^2\left(\frac{\pi}{6}\right)=3+\frac{\pi}{18}\times\frac{1}{4}=3+\frac{\pi}{72}$$

$$y_3=y_2+h\,f(x_2)=3+\frac{\pi}{72}+\frac{\pi}{18}\sin^2\left(\frac{\pi}{3}\right)=3+\frac{\pi}{72}+\frac{\pi}{18}\times\frac{3}{4}=3+\frac{\pi}{18}$$

$\text{euler}\left((\sin(3\cdot x))^2,x,y,\left\{0,\frac{\pi}{6}\right\},3,\frac{\pi}{18}\right)$

$$\begin{bmatrix}0.0000 & 0.1745 & 0.3491 & 0.5236\\ 3.0000 & 3.0000 & 3.0436 & 3.1745\end{bmatrix}$$

$3+\frac{\pi}{18}$ $\quad 3.1745$

Question 16 **Answer C**

when $x=\pm 2$ the slope $m=0$ and when $x=0$ $y=0$, the slope $m=-1$

the slopes are never infinite, only satisfied by $\frac{dy}{dx}=\frac{x^2-4}{y^2+4}$

Question 17 **Answer C**

The stone takes on the initial upwards speed of the balloon, but its acceleration is just due to gravity. Taking upwards as positive and downwards as negative,

$s = -100 \quad u = 2 \quad a = -9.8 \quad t = ? \quad \text{using} \quad s = ut + \frac{1}{2}at^2$

$-100 = 2t - 4.9t^2 \quad \text{solving} \quad \Rightarrow \; t = 4.73$

Question 18 **Answer A**

$(a,b) = \bar{x} \pm z\frac{\sigma}{\sqrt{n}} \quad 95\%, \quad z = 1.96$

$(1) \;\; a = \bar{x} - 1.96\frac{\sigma}{\sqrt{n}} \quad (2) \;\; b = \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}$

$\frac{1}{2}((1)+(2)) \quad \bar{x} = \frac{1}{2}(a+b)$

$\frac{1}{2}((2)-(1)) \quad 1.96\frac{\sigma}{\sqrt{n}} = \frac{1}{2}(b-a), \quad \frac{\sigma}{\sqrt{n}} = \frac{1}{2\times 1.96}(b-a)$

$99\%, \quad z = 2.575$

$$CI: \left(\bar{x} - 2.575\frac{\sigma}{\sqrt{n}}, \bar{x} + 2.575\frac{\sigma}{\sqrt{n}}\right)$$

$$= \left(\frac{1}{2}(a+b) - \frac{2.575}{2\times 1.96}(b-a)\,,\, \frac{1}{2}(a+b) + \frac{2.575}{2\times 1.96}(b-a)\right)$$

$$= \left(\frac{a}{2}\left(1 + \frac{2.575}{1.96}\right) + \frac{b}{2}\left(1 - \frac{2.575}{1.96}\right), \frac{a}{2}\left(1 - \frac{2.575}{1.96}\right) + \frac{b}{2}\left(1 + \frac{2.575}{1.96}\right)\right)$$

$$(1.157a - 0.157b\,,\, -0.157a + 1.157b)$$

Question 19 **Answer B**

$\bar{B} \overset{d}{=} N\left(\mu = ?, \frac{6^2}{25}\right),$

$\Pr(\bar{B} < 250) = 0.2$

$\frac{250-\mu}{\frac{6}{5}} = -0.8416$

$\mu = 251.01$

invNorm(0.2,0,1)	-0.84162
solve$\left(\frac{250-m}{\frac{6}{5}}=-0.8416, m\right)$	m=251.01

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Question 20 **Answer D**

$v(3)=\frac{6}{\pi}\tan^{-1}(1)=\frac{3}{2}, \qquad v(7)=-2$

$b=\frac{\frac{3}{2}+2}{3-7}=-\frac{7}{8}, \qquad a=\frac{3}{2}-3\times\left(-\frac{7}{8}\right)=\frac{33}{8}$ **A.** is true

The area of the triangle when $v>0$ is $A_1=\frac{1}{2}\times\frac{3}{2}\times\left(\frac{33}{7}-3\right)=\frac{9}{7}$,

and when $v<0$ is $A_2=\frac{1}{2}\times 2\times\left(7-\frac{33}{7}\right)=-\frac{16}{7}$

The toy car moves a total distance of

$\int_0^7 |v(t)|\,dt=\frac{6}{\pi}\int_0^3 \tan^{-1}\left(\frac{t}{3}\right)dt+\frac{9}{7}+\frac{16}{7}=\frac{6}{\pi}\int_0^3 \tan^{-1}\left(\frac{t}{3}\right)dt+\frac{25}{7}$ metres. **B.** is true

The toy car has a displacement of

$\int_0^7 v(t)\,dt=\frac{6}{\pi}\int_0^3 \tan^{-1}\left(\frac{t}{3}\right)dt+\frac{9}{7}-\frac{16}{7}=\frac{6}{\pi}\int_0^3 \tan^{-1}\left(\frac{t}{3}\right)dt\ -1$ metres. **C.** is true

$$\int_3^7 \left|\frac{33}{8}-\frac{7\cdot t}{8}\right| dt \qquad \frac{25}{7}$$

$$\int_3^7 \left(\frac{33}{8}-\frac{7\cdot t}{8}\right) dt \qquad -1$$

D. is false the average speed is the total distance travelled over total time.

END OF SECTION A SUGGESTED ANSWERS

SECTION B

Question 1

a. $f(x)=\dfrac{x^2-2x-3}{x^2-x}$

$f(x)=\dfrac{(x-3)(x+1)}{x(x-1)}=1-\dfrac{x+3}{x(x-1)}$

$x=0$ and $x=1$ are vertical asymptotes,

and $y=1$ is a horizontal asymptote.

Note the graph will cross the horizontal asymptote at $x=-3$ A1

Define $f1(x)=\dfrac{x^2-2\cdot x-3}{x^2-x}$	Done
factor$(f1(x))$	$\dfrac{(x-3)\cdot(x+1)}{x\cdot(x-1)}$
propFrac$(f1(x))$	$1-\dfrac{x+3}{x\cdot(x-1)}$

b. $f'(x)=\dfrac{x^2+6x-3}{x^2(x-1)^2}$

for turning points $f'(x)=0$

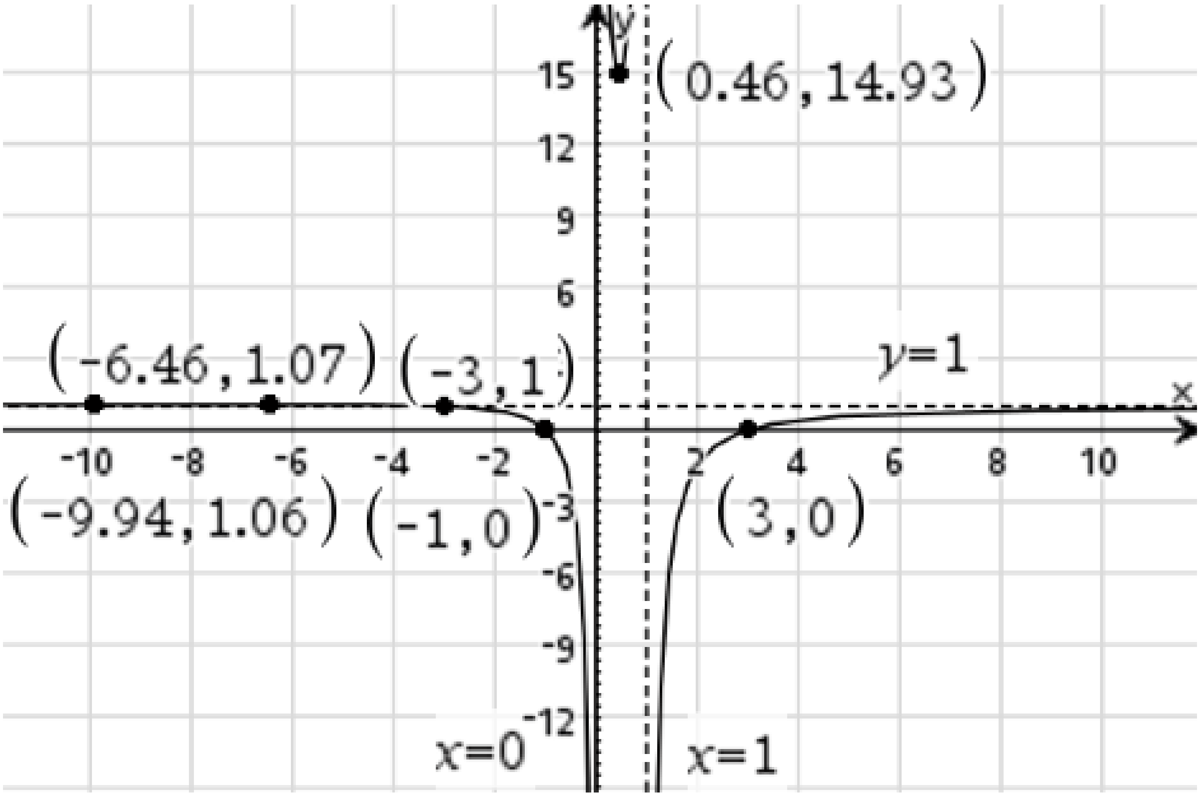
solving $x^2+6x-3=0$

gives $x=-3\pm2\sqrt{3}$

$\dfrac{d}{dx}(f1(x))$	$\dfrac{x^2+6\cdot x-3}{x^2\cdot(x-1)^2}$
zeros$\left(\dfrac{d}{dx}(f1(x)),x\right)\to xtps$	$\{-(2\cdot\sqrt{3}+3),2\cdot\sqrt{3}-3\}$
$f1(xtps)$	$\{-4\cdot(\sqrt{3}-2),4\cdot(\sqrt{3}+2)\}$

$\left(-3+2\sqrt{3},4\sqrt{3}+8\right),\ \left(-3-2\sqrt{3},-4\sqrt{3}+8\right)$ A1

c. The graph crosses the x-axis at $x=3$ and $x=-1$, $(-3,0)$, $(-1,0)$ G3



d.i. when $k=-2$ $f_{-2}(x)=\dfrac{x^2+2x-3}{x^2-x}=\dfrac{(x-1)(x+3)}{x(x-1)}=\dfrac{x+3}{x}=1+\dfrac{3}{x}$ $x\neq 1$

when $k=-2$ the graph has a point of discontinuity and will have no turning points A1

ii. $f_k(x)=\dfrac{x^2-kx-3}{x^2-x}$

$f_k(x)=1-\dfrac{(k-1)x+3}{x^2-x}$

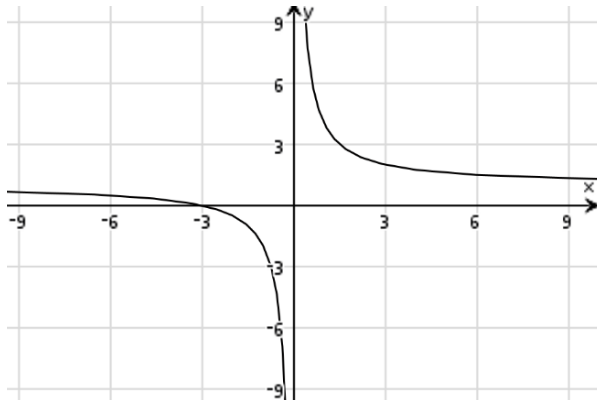
Define $fk(x)=\dfrac{x^2-k\cdot x-3}{x^2-x}$ Done

$\text{propFrac}(fk(x))$ $1-\dfrac{(k-1)\cdot x+3}{x\cdot(x-1)}$

The graph will cross the horizontal asymptote $y=1$ when $k\in R\setminus\{1,-2\}$ A1

Define $fk(x)=\dfrac{x^2-k\cdot x-3}{x^2-x}|k=-2$ Done

$fk(x)$ $\dfrac{x+3}{x}$



iii. $f'(x)=\dfrac{(k-1)x^2+6x-3}{x^2(x-1)^2}$

for turning points $f'(x)=0$

solving $(k-1)x^2+6x-3=0$

$\Delta=36+12(k-1)=12(k+2)$

gives $x=\dfrac{-3\pm\sqrt{3(k+2)}}{k-1}$

investigate the case when $k=1$

$f_1(x)=\dfrac{x^2-x-3}{x^2-x}$

$f_1'(x)=\dfrac{3(2x-1)}{x^2(x-1)^2}$

$\dfrac{d}{dx}(fk(x))$ $\dfrac{(k-1)\cdot x^2+6\cdot x-3}{x^2\cdot(x-1)^2}$

$\text{solve}((k-1)\cdot x^2+6\cdot x-3=0,x)$

$x=\dfrac{-(\sqrt{3\cdot(k+2)}+3)}{k-1}$ or $x=\dfrac{\sqrt{3\cdot(k+2)}-3}{k-1}$

$fk(x)|k=1$ $\dfrac{x^2-x-3}{x\cdot(x-1)}$

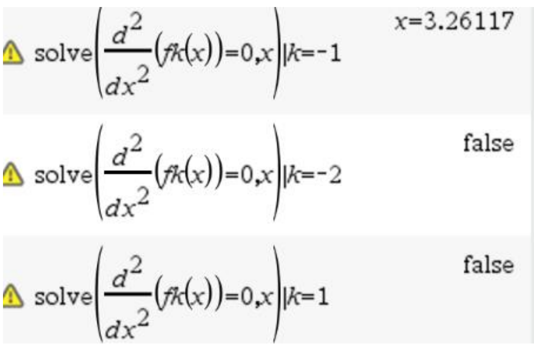
$\dfrac{d}{dx}\left(\dfrac{x^2-x-3}{x\cdot(x-1)}\right)$ $\dfrac{3\cdot(2\cdot x-1)}{x^2\cdot(x-1)^2}$

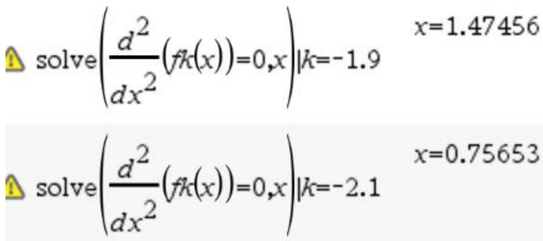
$\text{solve}\left(\dfrac{3\cdot(2\cdot x-1)}{x^2\cdot(x-1)^2}=0,x\right)$ $x=\dfrac{1}{2}$

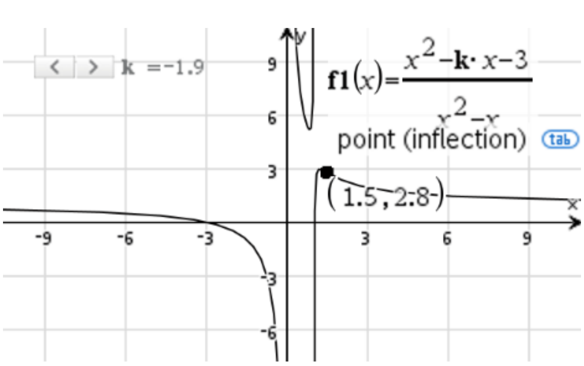
values of k	the graph of f_k has
$k > -2,\ k \neq 1$ $= (-2,1) \cup (1,\infty)$	two turning points
$k = 1$	one turning point
$k \le -2 \quad = (-\infty, -2]$	no turning points
$k = -2$ or $k = 1$	no points of inflection
$k \in R \setminus \{-2,1\}$	one point of inflection

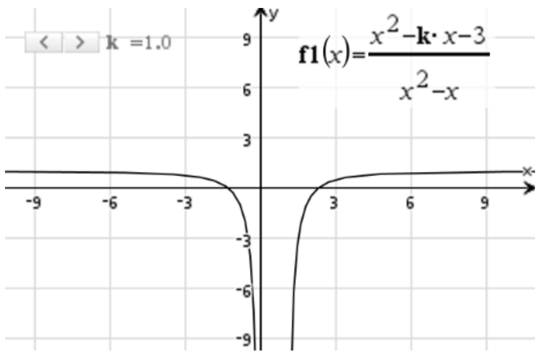
A3

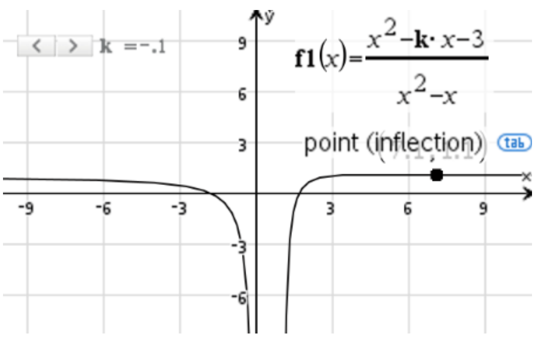
Use a slider on the graphs page for k for $f_k(x)$ and watch the inflection point appear and disappear or use trial and error for the inflection points.

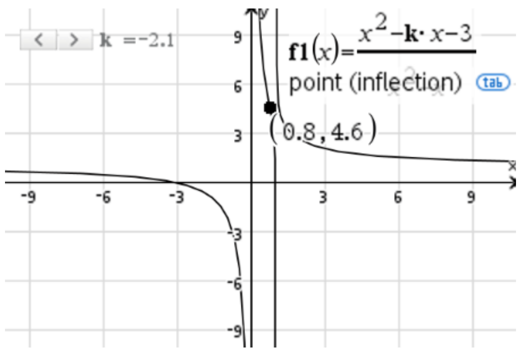












Question 2

a.i. $z = x + yi$

$|z-2| = 2$

$|(x-2)+yi| = 2$

$\sqrt{(x-2)^2 + y^2} = 2$

$(x-2)^2 + y^2 = 4$

C_1 is a circle centre at $(2,0)$ radius 2.

$|z-4i| = 4$

$|x+(y-4)i| = 4$

$\sqrt{x^2 + (y-4)^2} = 4$

$x^2 + (y-4)^2 = 16$

C_2 is a circle centre at $(0,4)$ radius 4. A2

ii. solving $(x-2)^2 + y^2 = 4$ with $x^2 + (y-4)^2 = 16$ gives

$(0,0), \left(\frac{16}{5}, \frac{8}{5}\right)$, $u = \frac{16}{5} + \frac{8}{5}i$ $a = \frac{16}{5}$, $b = \frac{8}{5}$ A1

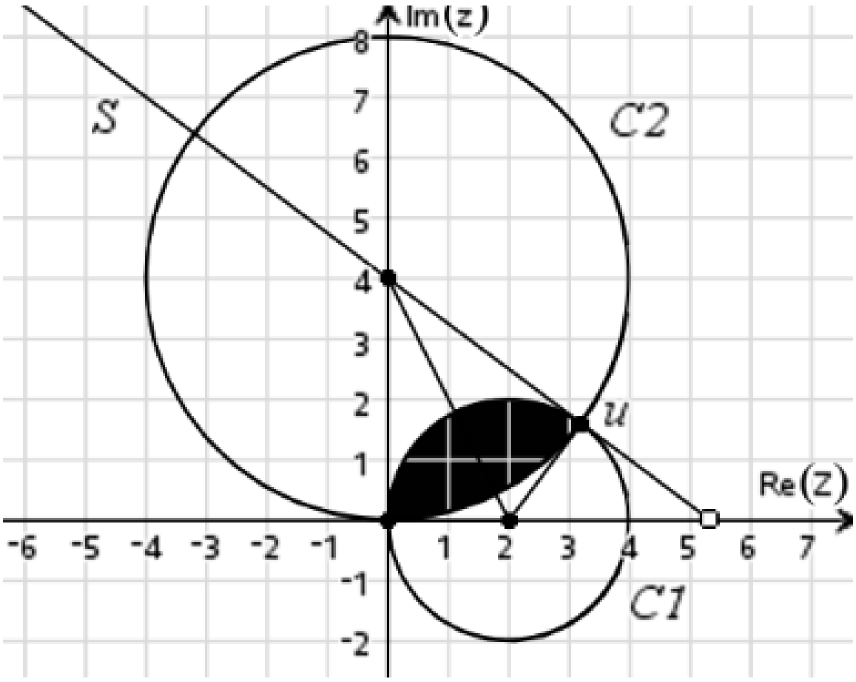
iii. the line through $(0,4), \left(\frac{16}{5}, \frac{8}{5}\right)$ has a gradient $\dfrac{\frac{8}{5}-4}{\frac{16}{5}} = -\frac{3}{4}$

the ray is $y - 4 = -\frac{3x}{4}$, $y = -\frac{3x}{4} + 4$ for $x < \frac{16}{3}$ draw with open circle at $\left(\frac{16}{3}, 0\right)$ A1

now the angle the ray makes with the positive x-axis is $\theta = \pi + \tan^{-1}\left(-\frac{3}{4}\right)$

$\text{Arg}\left(z - \frac{16}{3}\right) = \pi + \tan^{-1}\left(-\frac{3}{4}\right)$, $d = \frac{16}{3}$, $p = -\frac{3}{4}$ A1

b. G2



c. Consider three areas

Area 1: the area of the sector of radius 2, with an angle of 2α where $\alpha = \tan^{-1}(2)$

$A_1 = \frac{1}{2}r^2 2\alpha = \frac{1}{2}\times 2^2 \times 2\tan^{-1}(2) = 4\tan^{-1}(2)$ A1

Area 2: the area of the sector of radius 4, with an angle of 2β but $\alpha + \beta = \frac{\pi}{2}$ so that

$\beta = \tan^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2} - \tan^{-1}(2)$

$A_2 = \frac{1}{2}r^2 2\beta = \frac{1}{2}\times 4^2 \times 2\left(\frac{\pi}{2} - \tan^{-1}(2)\right) = 8\pi - 16\tan^{-1}(2)$ A1

Area 3: the area of two right angle triangles with side lengths 2 and 4, the total area of both triangles is $2\times\frac{1}{2}bh = 2\times\frac{1}{2}\times 2\times 4 = 8$

The required area is

$A = A_1 + A_2 - A_3 = 4\tan^{-1}(2) + \left(8\pi - 16\tan^{-1}(2)\right) - 8$

$A = 8\pi - 12\tan^{-1}(2) - 8$ A1

Alternatively using calculus

$(x-2)^2 + y^2 = 4$ the top half of the circle is $y_1 = \sqrt{4-(x-2)^2}$

$x^2 + (y-4)^2 = 16$ the bottom half of the circle is $y_2 = 4 - \sqrt{16 - x^2}$ A1

The required area is $A = \int_0^a (y_1 - y_2)\,dx$

$A = \int_0^{\frac{16}{5}} \left(\sqrt{4-(x-2)^2} - 4 + \sqrt{16-x^2}\right)dx$ A1

$A = 4\cos^{-1}\left(\frac{\sqrt{5}}{5}\right) + 8\cos^{-1}\left(\frac{3}{5}\right) - 8$ A1

Which is equivalent to $A = 8\pi - 12\tan^{-1}(2) - 8$

Define $f2(x) = 4 - \sqrt{16 - x^2}$	*Done*
Define $f3(x) = \sqrt{4-(x-2)^2}$	*Done*
$\int_0^{\frac{16}{5}} (f3(x) - f2(x))\,dx$	$4\cdot\cos^{-1}\left(\frac{\sqrt{5}}{5}\right) + 8\cdot\cos^{-1}\left(\frac{3}{5}\right) - 8$

$\int_0^{\frac{16}{5}} (f3(x) - f2(x))\,dx$	3.8470
$8\cdot\pi - 12\cdot\tan^{-1}(2) - 8$	3.8470

Question 3

a.i. $x=\sqrt{y\left(y^2-12y+40\right)}$ solving when $x=8,\ y=8,\ H=8$ A1

$x^2=y\left(y^2-12y+40\right)$

$V=\pi\int_0^8 x^2\,dy$

$V=\pi\int_0^8\left(y^3-12y^2+40y\right)dy$

$V=256\pi\ \ \text{cm}^3$ A1

$x^2=y\cdot\left(y^2-12\cdot y+40\right) \quad x^2=y\cdot\left(y^2-12\cdot y+40\right)$

$\text{solve}\left(x^2=y\cdot\left(y^2-12\cdot y+40\right),y\right)|x=8 \quad y=8$

$\pi\cdot\int_0^h\left(y\cdot\left(y^2-12\cdot y+40\right)\right)dy|h=8 \quad 256\cdot\pi$

ii. $V=\pi\int_0^h\left(y^3-12y^2+40y\right)dy$

$V=\pi\left[\frac{y^4}{4}-4y^3+20y^2\right]_0^h=\pi\left(\frac{h^4}{4}-4h^3+20h^2\right)$ A1

$V=\frac{\pi h^2}{4}\left(h^2-16h+80\right),\quad 0\le h\le 8$

b.i. $S=2\pi\int_{y_1}^{y_2}x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$

$x=\sqrt{y\left(y^2-12y+40\right)},\qquad \frac{dx}{dy}=\frac{3y^2-24y+40}{2\sqrt{y\left(y^2-12y+40\right)}}$ A1

$1+\left(\frac{dx}{dy}\right)^2=1+\frac{\left(3y^2-24y+40\right)^2}{4\left(y\left(y^2-12y+40\right)\right)}$

$=\frac{9y^4-140y^3+768y^2-1760y+1600}{4\left(y\left(y^2-12y+40\right)\right)}$

$S=2\pi\int_0^8\sqrt{y\left(y^2-12y+40\right)}\times\frac{\sqrt{9y^4-140y^3+768y^2-1760y+1600}}{2\sqrt{y\left(y^2-12y+40\right)}}\,dy$ M1

$S=\pi\int_0^8\sqrt{9y^4-140y^3+768^2-1760y+1600}\,dy$ A1

$b=8,\ \ p=9,\ q=-140,\ r=768,\ \ s=-1760,\ \ t=1600$

ii. $S=453.24\ \text{cm}^2$ A1

Define $x(y)=\sqrt{y\cdot(y^2-12\cdot y+40)}$ Done

$\text{comDenom}\left(\frac{d}{dy}(x(y))\right)$ $\frac{3\cdot y^2-24\cdot y+40}{2\cdot\sqrt{y\cdot(y^2-12\cdot y+40)}}$

$\text{comDenom}\left(1+\left(\frac{3\cdot y^2-24\cdot y+40}{2\cdot\sqrt{y\cdot(y^2-12\cdot y+40)}}\right)^2\right)$

$\frac{9\cdot y^4-140\cdot y^3+768\cdot y^2-1760\cdot y+1600}{4\cdot y^3-48\cdot y^2+160\cdot y}$

$2\cdot\pi\cdot x(y)\cdot\sqrt{\frac{9\cdot y^4-140\cdot y^3+768\cdot y^2-1760\cdot y\cdot}{4\cdot y^3-48\cdot y^2+160\cdot y}}$

$\pi\cdot\sqrt{9\cdot y^4-140\cdot y^3+768\cdot y^2-1760\cdot y+1600}$

$\int_0^8\left(\pi\cdot\sqrt{9\cdot y^4-140\cdot y^3+768\cdot y^2-1760\cdot y+160}\right.$ 453.2444

c. $V=\frac{\pi h^2}{4}\left(h^2-16h+80\right),\ \frac{dV}{dh}=\pi h\left(h^2-12h+40\right)$ A1

$\frac{dV}{dt}=-k\sqrt{h}$

$\frac{dt}{dh}=\frac{dt}{dV}\cdot\frac{dV}{dh}=\frac{\pi h\left(h^2-12h+40\right)}{-k\sqrt{h}}$ M1

$T=120=\frac{\pi}{-k}\int_8^0\sqrt{h}\left(h^2-12h+40\right)dh$

$T=120=\frac{\pi}{k}\int_0^8\sqrt{h}\left(h^2-12h+40\right)dh$

solving gives $k=\frac{1376\pi\sqrt{2}}{1575}$ A1

Define $v(h)=\frac{h^2\cdot(h^2-16\cdot h+80)\cdot\pi}{4}$ Done

$\frac{d}{dh}(v(h))$ $h\cdot(h^2-12\cdot h+40)\cdot\pi$

$\int_0^8\frac{\pi\cdot h\cdot(h^2-12\cdot h+40)}{k\cdot\sqrt{h}}\,dh$ $\frac{11008\cdot\pi\cdot\sqrt{2}}{105\cdot k}$

$\text{solve}\left(\frac{11008\cdot\pi\cdot\sqrt{2}}{105\cdot k}=120,k\right)$

$k=\frac{1376\cdot\pi\cdot\sqrt{2}}{1575}$

Question 4

a. $\underset{\sim}{r}_P(t) = 2t\,\underset{\sim}{i} + 3\sqrt{2}\,t\,\underset{\sim}{j} + 6t\,\underset{\sim}{k}$

$\underset{\sim}{v}_P(t) = 2\,\underset{\sim}{i} + 3\sqrt{2}\,\underset{\sim}{j} + 6\,\underset{\sim}{k}$, $\left|v_P(t)\right| = \sqrt{4+18+36} = \sqrt{58}$

$A\left(0, 12\sqrt{2}, 32\right)$, $B\left(6, 15\sqrt{2}, 34\right)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 6\,\underset{\sim}{i} + 3\sqrt{2}\,\underset{\sim}{j} + 2\,\underset{\sim}{k}$,

$\left|\overrightarrow{AB}\right| = \left|\underset{\sim}{v}_P(t)\right| = \sqrt{58}$ A1

$\left|\underset{\sim}{v}_Q(t)\right| = 2\left|\underset{\sim}{v}_P(t)\right|$

$\underset{\sim}{v}_Q(t) = 2\overrightarrow{AB} = 12\,\underset{\sim}{i} + 6\sqrt{2}\,\underset{\sim}{j} + 4\,\underset{\sim}{k}$

$\underset{\sim}{r}_Q(t) = \int\left(12\,\underset{\sim}{i} + 6\sqrt{2}\,\underset{\sim}{j} + 4\,\underset{\sim}{k}\right)dt = 12t\,\underset{\sim}{i} + 6\sqrt{2}t\,\underset{\sim}{j} + 4t\,\underset{\sim}{k} + \underset{\sim}{c}$

$\underset{\sim}{r}_Q(0) = \underset{\sim}{c} = \overrightarrow{OA} = 12\sqrt{2}\,\underset{\sim}{j} + 32\,\underset{\sim}{k}$ M1

$\underset{\sim}{r}_Q(t) = 12t\,\underset{\sim}{i} + \left(6\sqrt{2}\,t + 12\sqrt{2}\right)\underset{\sim}{j} + \left(4t + 32\right)\underset{\sim}{k}$

$\underset{\sim}{r}_Q(t) = \underset{\sim}{r}_P(s)$ $\quad \underset{\sim}{r}_P(s) = 2s\,\underset{\sim}{i} + 3\sqrt{2}\,s\,\underset{\sim}{j} + 6s\,\underset{\sim}{k}$

$\underset{\sim}{i} : (1) \quad 12t = 2s, \quad \Rightarrow s = 6t$ M1

$\underset{\sim}{j} : (2) \quad 6\sqrt{2}\,t + 12\sqrt{2} = 3\sqrt{2}s \quad , \quad s = 2t + 4$

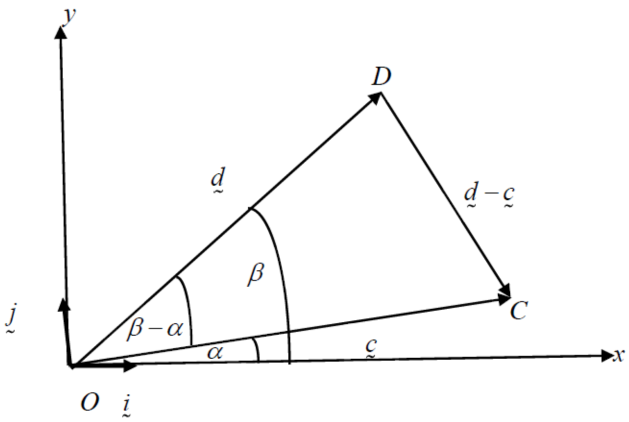
$6t = 2t + 4, \quad t = 1, \quad s = 6$

need to check in $\underset{\sim}{k} : (3)$ $\;4t + 32 = 6s = 36$ yes A1

so the time between release is $6 - 1 = 5$ seconds and the point of collision is

$\left(12, 18\sqrt{2}, 36\right)$ A1

b.



$\underset{\sim}{c}=\overrightarrow{OC}=c\left(\cos(\alpha)\underset{\sim}{i}+\sin(\alpha)\underset{\sim}{j}\right)$ the vector $\underset{\sim}{c}$ has a length of c so $|\underset{\sim}{c}|=c$
and makes an angle of α with the positive x-axis.
$\underset{\sim}{d}=\overrightarrow{OD}=d\left(\cos(\beta)\underset{\sim}{i}+\sin(\beta)\underset{\sim}{j}\right)$ the vector $\underset{\sim}{d}$ has a length of d so $|\underset{\sim}{d}|=d$
and makes an angle of β with the positive x-axis

$$\underset{\sim}{d}-\underset{\sim}{c}=d\left(\cos(\beta)\underset{\sim}{i}+\sin(\beta)\underset{\sim}{j}\right)-c\left(\cos(\alpha)\underset{\sim}{i}+\sin(\alpha)\underset{\sim}{j}\right)$$

$$\underset{\sim}{d}-\underset{\sim}{c}=\left(d\cos(\beta)-c\cos(\alpha)\right)\underset{\sim}{i}+\left(d\sin(\beta)-c\sin(\alpha)\right)\underset{\sim}{j}$$

$$|\underset{\sim}{d}-\underset{\sim}{c}|=\sqrt{\left(d\cos(\beta)-c\cos(\alpha)\right)^2+\left(d\sin(\beta)-c\sin(\alpha)\right)^2}$$

$$=\sqrt{d^2\left(\cos^2(\beta)+\sin^2(\beta)\right)+c^2\left(\cos^2(\alpha)+\sin^2(\alpha)\right)-2dc\left(\cos(\beta)\cos(\alpha)+\sin(\beta)\sin(a)\right)}$$

$$=\sqrt{d^2+c^2-2dc\cos(\beta-\alpha)}$$

Alternatively by the cosine rule, since $\beta-\alpha$ is the angle between the vectors $\underset{\sim}{d}$ and $\underset{\sim}{c}$
so that $|\underset{\sim}{d}-\underset{\sim}{c}|=\sqrt{d^2+c^2-2dc\cos(\beta-\alpha)}$ A1

To prove that $|\underset{\sim}{c}|+|\underset{\sim}{d}|>|\underset{\sim}{d}-\underset{\sim}{c}|$ is equivalent to proving

$c+d>\sqrt{d^2+c^2-2dc\cos(\beta-\alpha)}$ A1

To prove this use a proof by contradiction, assume that

$c+d\le\sqrt{d^2+c^2-2dc\cos(\beta-\alpha)}$ now square both sides A1

$$c^2+2dc+d^2\le d^2+c^2-2dc\cos(\beta-\alpha)$$

$$2dc\left(1+\cos(\beta-a)\right)\le 0$$

but since $c>0$ and $d>0$, $0<\beta-\alpha<\frac{\pi}{2}$, $\cos(\beta-\alpha)>0$ so $2dc\left(1+\cos(\beta-a)\right)>0$

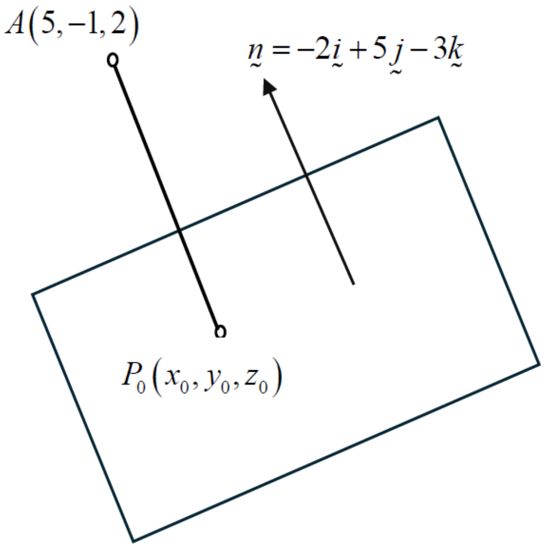
hence we have a contradiction, so the original statement A1

$c+d\le\sqrt{d^2+c^2-2dc\cos(\beta-\alpha)}$ must be false,

hence $c+d>|\underset{\sim}{d}-\underset{\sim}{c}|=\sqrt{d^2+c^2-2dc\cos(\beta-\alpha)}$

Question 5

a.



The plane $-2x+5y-3z=17$ has a normal $\underset{\sim}{n} = -2\underset{\sim}{i} + 5\underset{\sim}{j} - 3\underset{\sim}{k}$ let the point $P_0(x_0, y_0, z_0)$ be the point on the plane closest to the point $A(5,-1,2)$.

The vector $\overrightarrow{AP_0} = (x_0-5)\underset{\sim}{i} + (y_0+1)\underset{\sim}{j} + (z_0-2)\underset{\sim}{k}$ is parallel to the normal to the plane,

so that $\overrightarrow{AP_0} = \lambda \underset{\sim}{n}$, then M1

$\underset{\sim}{i}$ (1) $x_0 - 5 = -2\lambda,$

$\underset{\sim}{j}$ (2) $y_0 + 1 = 5\lambda,$

$\underset{\sim}{k}$ (3) $z_0 - 2 = -3\lambda$

rearranging $x_0 = 5 - 2\lambda, \quad y_0 = -1 + 5\lambda, \quad z_0 = 2 - 3\lambda$

but since $P_0(x_0, y_0, z_0)$ lies on the plane $-2x_0 + 5y_0 - 3z_0 = 17$ M1

$-2(5-2\lambda) + 5(-1+5\lambda) - 3(2-3\lambda) = 17$

$-10 + 4\lambda - 5 + 25\lambda - 6 + 9\lambda = 17$

$38\lambda = 38$

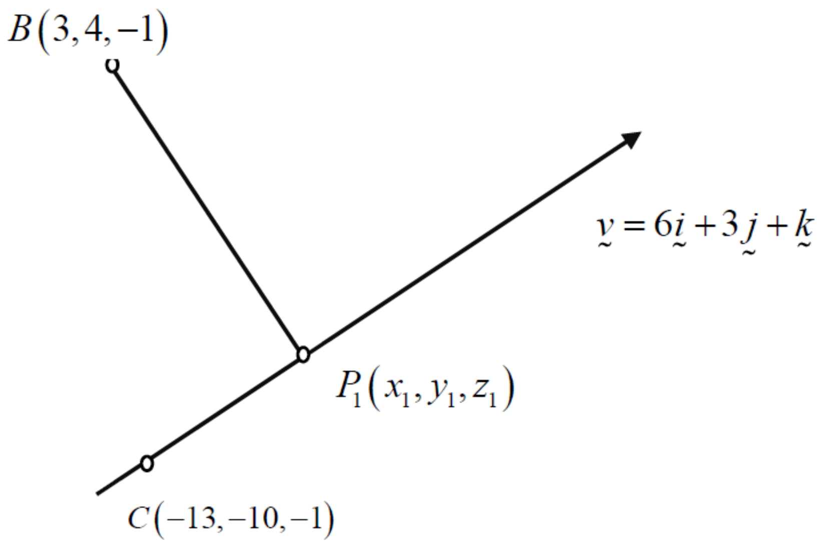
solving $\lambda = 1$

substituting $\lambda = 1$ gives $P_0(3,4,-1)$ and

$\overrightarrow{AP_0} = -2\underset{\sim}{i} + 5\underset{\sim}{j} - 3\underset{\sim}{k},$

$\left|\overrightarrow{AP_0}\right| = \sqrt{4+25+9} = \sqrt{38}$ A1

b.



The line $\underset{\sim}{r}(t) = (-13+6t)\underset{\sim}{i} + (-10+3t)\underset{\sim}{j} + (-1+t)\underset{\sim}{k}$ has direction $\underset{\sim}{v} = 6\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}$

and in parametric form is $x = -13+6t, \quad y = -10+3t, \quad z = -1+t$

let the point $P_1(x_1, y_1, z_1)$ be the point on the line which is closest to the point $B(3,4,-1)$

The vector $\overrightarrow{BP_1} = (x_1 - 3)\underset{\sim}{i} + (y_1 - 4)\underset{\sim}{j} + (z_1 + 1)\underset{\sim}{k}$ is perpendicular to the direction of the line,

so that $\overrightarrow{BP_1}.\underset{\sim}{v} = 0$, so that $(4) \quad 6(x_1 - 3) + 3(y_1 - 4) + (z_1 + 1) = 0$ M1

The point $C(-13,-10,-1)$ also lies on the line and the vector

$\overrightarrow{CP_1} = (x_1 + 13)\underset{\sim}{i} + (y_1 + 10)\underset{\sim}{j} + (z_1 + 1)\underset{\sim}{k}$ is parallel to the direction of the line $\underset{\sim}{v}$ so that

$\overrightarrow{CP_1} = \mu\underset{\sim}{v}$ and $\underset{\sim}{i}\ (5)\ \ x_1 + 13 = 6\mu, \quad \underset{\sim}{j}\ (6)\ \ y_1 + 10 = 3\mu, \quad \underset{\sim}{k}\ (7)\ \ z_1 + 1 = \mu$ A1

rearranging $x_1 = -13+6\mu, \quad y_1 = -10+3\mu, \quad z_1 = -1+\mu$ substitute into (4)

$(4) \quad 6(6\mu - 16) + 3(3\mu - 14) + \mu = 0$

$36\mu - 96 + 9\mu - 42 + \mu = 0$

$46\mu = 138$ M1

solving $\mu = 3$

substituting $\mu = 3$ gives $P_1(5,-1,2)$ and

$\overrightarrow{BP_1} = 2\underset{\sim}{i} - 5\underset{\sim}{j} + 3\underset{\sim}{k}$,

$\left|\overrightarrow{P_1B}\right| = \sqrt{4+25+9} = \sqrt{38}$ A1

c. $\underset{\sim}{r}(t) = (-13+6t)\underset{\sim}{i} + (-10+3t)\underset{\sim}{j} + (-1+t)\underset{\sim}{k}$, and the plane $-2x+5y-3z=17$.

Now the direction of the line $\underset{\sim}{v} = 6\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}$ and the normal to the plane $\underset{\sim}{n} = -2\underset{\sim}{i} + 5\underset{\sim}{j} - 3\underset{\sim}{k}$ are perpendicular since $\underset{\sim}{v}.\underset{\sim}{n} = -12+15-3=0$,

therefore the line and the plane do not intersect. A1

The point $A(5,-1,2)$ is the same point as $P_1(5,-1,2)$ and lies on the line and

The point $B(3,4,-1)$ is the same point as $P_0(3,4,-1)$ and lies on the plane.

Since from **a.** the closest distance from the plane to the point A was $\sqrt{38}$ and from **b.** the closest distance from the line to the point B was also $\sqrt{38}$,

it follows that the distance between the line and the plane is also $\sqrt{38}$ A1

d. the line $\frac{x+13}{6} = \frac{y+10}{b} = z+1 = t$

$x=-13+6t, \quad y=-10+bt, \quad z=-1+t$ substitute into the plane

$-2x+5y-3z=d$

$-2(-13+6t)+5(-10+bt)-3(-1+t)=d$

$26-12t-50+5bt+3-3t=d$ M1

$(5b-15)t = d+21$

i. for a unique point of intersection

$b \neq 3, \quad d \in R$

ii. an infinite number of points of intersection

$b=3, \quad d=-21$ A1

Question 6

a.i. $x = 20\cos(t), \quad y = 10\sin(t)$

rotate around the x-axis

$$S_x = 2\pi \int_{t_1}^{t_2} y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\frac{dx}{dt} = -20\sin(t), \quad \frac{dy}{dt} = 10\cos(t)$$

by symmetry

$$S_x = 4\pi\int_0^{\frac{\pi}{2}} 10\sin(t)\sqrt{400\sin^2(t) + 100\cos^2(t)}\; dt = 2147.84$$ A1

Define $x1(t) = 20\cdot\cos(t)$ — Done

Define $y1(t) = 10\cdot\sin(t)$ — Done

$4\cdot\pi\cdot\int_0^{\frac{\pi}{2}}\left(y1(t)\cdot\sqrt{\left(\frac{d}{dt}(x1(t))\right)^2+\left(\frac{d}{dt}(y1(t))\right)^2}\right)dt$ — 2147.84353

ii. $V_x = \pi\int_{y_1}^{y_2} x^2\, dy$

$$\frac{x^2}{400} + \frac{y^2}{100} = 1$$

$$x^2 = 400\left(1 - \frac{y^2}{100}\right)$$

by symmetry

$$V_x = 2\pi\int_0^{10} 400\left(1 - \frac{y^2}{100}\right)dy = 16755.16$$ A1

$\pi\cdot\int_{-10}^{10}\left(400\cdot\left(1-\frac{y^2}{100}\right)\right)dy$ — 16755.16082

$2\cdot\pi\cdot\int_{0}^{10}\left(400\cdot\left(1-\frac{y^2}{100}\right)\right)dy$ — 16755.16082

b. watermelons $W \overset{d}{=} N\left(3, 0.4^2\right)$

cantaloupes $C \overset{d}{=} N\left(1.5, 0.2^2\right)$

$3\cdot(0.4)^2 + 2\cdot(0.2)^2$ — 0.5600

normCdf$\left(11, \infty, 12, \sqrt{0.56}\right)$ — 0.9093

total of three independent watermelons and two independent cantaloupes

$T = W_1 + W_2 + W_3 + C_1 + C_2 \qquad E(T) = 3E(C) + 2E(C) = 3\times 3 + 2\times 1.5 = 12$

$\text{Var}(T) = 3\text{Var}(W) + 2\,\text{Var}(C) = 3\times 0.4^2 + 2\times 0.2^2 = 0.56, \qquad T \overset{d}{=} N(12, 0.56)$

$\Pr(T > 11) = 0.9093$ A1

c. $CI:\ \bar{x} \pm z\frac{\sigma}{\sqrt{n}} = (1.45, 1.55)$

$\bar{x} = 1.5, \quad \sigma = 0.2, \quad 95\%, \quad z = 1.96$

solving $\frac{1.96\times 0.2}{\sqrt{n}} = 0.05$

solve$\left(\frac{1.96\cdot 0.2}{\sqrt{n}} = 0.05, n\right)$ — $n = 61.4656$

zInterval 0.2,1.5,62,0.95: *stat.results*

"Title"	"z Interval"
"CLower"	1.4502
"CUpper"	1.5498
"$\bar{x}$"	1.5000
"ME"	0.0498
"n"	62.0000

gives $n = 61.46$ so accept $n = 61$ or 62 A1

d. $H_0: \ \mu = 3$

$H_1: \ \mu < 3$ one-sided test A1

e. $n = 36, \ \ \bar{W} \overset{d}{=} N\left(3, \dfrac{0.4^2}{36}\right), \ \ \sigma_{\bar{W}} = \dfrac{0.4}{6}$

$\text{normCdf}\left(-\infty, 2.9, 3, \dfrac{0.4}{6}\right)$ 0.0668

zTest 3,0.4,2.9,36,−1: *stat.results*

"Title"	"z Test"
"Alternate Hyp"	"μ < μ0"
"z"	−1.5000
"PVal"	0.0668
"x̄"	2.9000
"n"	36.0000
"σ"	0.4000

p value $= \Pr\left(\bar{W} < 2.9 \mid \mu = 3\right)$

$= 0.0668$ A1

Since the p value $0.0668 > 0.05$

there is no evidence to support the

alternative hypothesis A1

f. $\Pr\left(W < w^*\right) = 0.05$

$Z \overset{d}{=} N(0,1)$

$\Pr\left(Z < -0.05\right) = -1.6449$

$-1.6449 = \dfrac{w^* - 3}{\frac{0.4}{6}}$

$w^* = 2.89034$

so round up $w^* \geq 2.8904$ A1

$\text{invNorm}(0.05, 0, 1)$ −1.64485

$\text{solve}\left(\dfrac{w-3}{\frac{0.4}{6}} = -1.64485, w\right)$ $w = 2.89034$

$\text{invNorm}\left(0.05, 3, \dfrac{0.4}{6}\right)$ 2.89034

$\text{normCdf}\left(-\infty, 2.8903, 3, \dfrac{0.4}{6}\right)$ 0.04993

$\text{normCdf}\left(-\infty, 2.8904, 3, \dfrac{0.4}{6}\right)$ 0.05009

g. $n = 36, \ \ \bar{S} \overset{d}{=} N\left(2.8, \dfrac{0.4^2}{36}\right) \ \ \sigma_{\bar{S}} = \dfrac{0.4}{6}$

$\text{normCdf}\left(2.89034, \infty, 2.8, \dfrac{0.4}{6}\right)$ 0.08769

$\beta = \Pr\left(\bar{S} > 2.89034 \mid \mu = 2.8\right) = 0.08769$ A1

8.8%

End of detailed answers for the
2024 Kilbaha VCE Specialist Mathematics Trial Examination 2

Kilbaha Education (Est. 1978) (ABN 47 065 111 373)	**PayID: 47065111373**
PO Box 3229	**Email: kilbaha@gmail.com**
Cotham Vic 3101	**Tel: (03) 9018 5376**
Australia	**Web: https://kilbaha.com.au**