Victorian Certificate of Education
2024

STUDENT NUMBER

Figures										Letter:
Words										

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 21 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (5 marks)

Given the differential equation $\frac{dy}{dx} = 2x\sqrt{16-9y^2}$, $y(1) = \frac{2}{3}$.

a. Solve the differential equation, expressing your answer in the form $y = f(x)$.

3 marks

b. Find the value of $\frac{d^2y}{dx^2}$ when $x = 1$ and $y = \frac{2}{3}$.

2 marks

Question 2 (4 marks)

Let $f : D \to R,\ f(x) = \dfrac{x^2 - 5x + 4}{x}$, where D is the maximal domain of f.

i. Determine the equations of all the asymptotes on the graph of $y = f(x)$.

1 mark

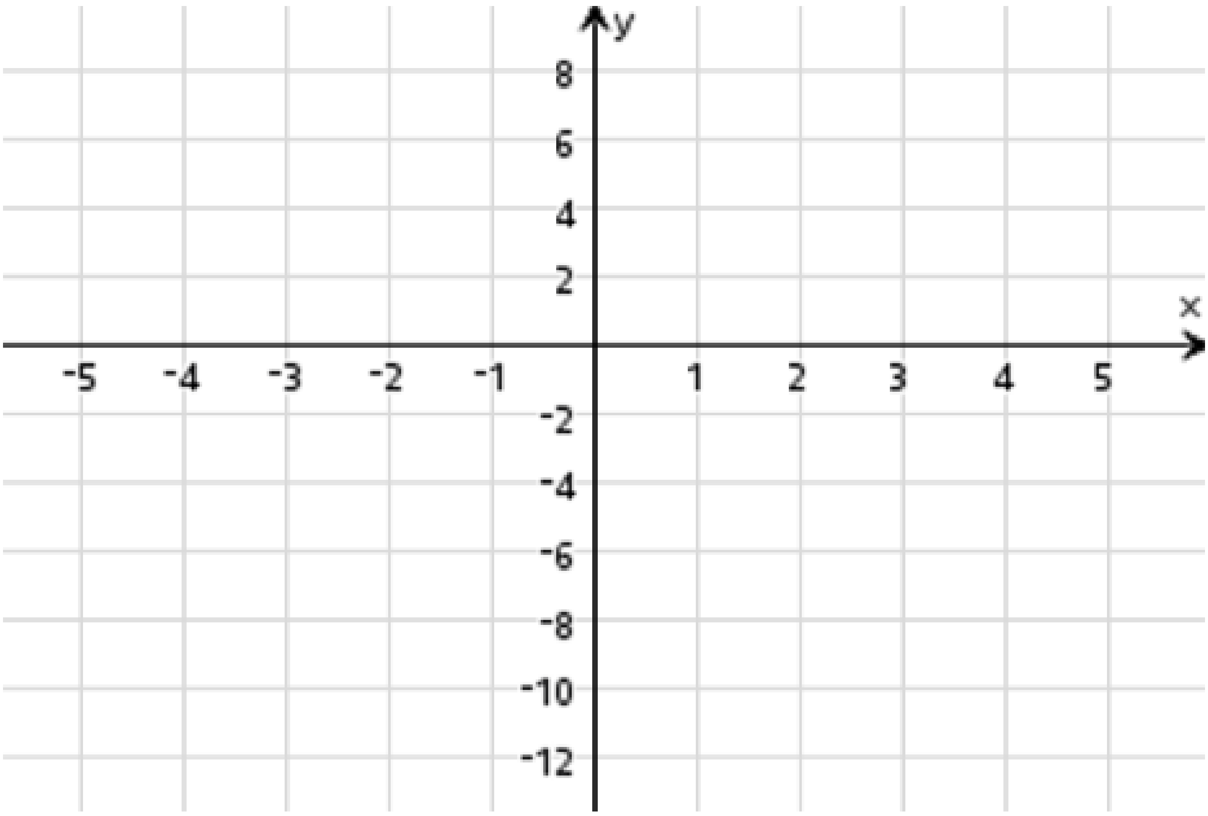
ii. Determine the coordinates of all the turning points on the graph of $y = f(x)$.

1 mark

iii. Show that the graph of $y = f(x)$ has no points of inflexion. 1 mark

iv. Sketch the graph of $y = f(x) = \dfrac{x^2 - 5x + 4}{x}$ on the axes below, labelling turning points, axial intercepts and the equations of all asymptotes.

1 mark



Question 3 (6 marks)

An ice block falls vertically from rest from a high cloud. Its acceleration is given by $a = 9.8 - 0.2v^2\ \text{m s}^{-2}$ where v is the velocity in m s^{-1} and x is the distance fallen in metres after a time t seconds.

a. Show that $v^2 = 49\left(1 - e^{-\frac{2x}{5}}\right)$.

3 marks

b. Show that $v=\dfrac{7\left(1-e^{-\frac{14t}{5}}\right)}{1+e^{-\frac{14t}{5}}}$.

3 marks

Question 4 (5 marks)

Given the three points $P(1,2,3)$, $Q(-1,2,-1)$, $R(-1,-2,5)$.

a. Show that the equation of the plane which passes through the points *P*, *Q* and *R* is given by $-4x+3y+2z=8$.

2 marks

b. The plane $-4x+3y+2z=8$ and the line $2-x=\frac{y-4}{2}=\frac{z-2}{c}$ intersect at an acute angle of $\cos^{-1}\left(\frac{5\sqrt{29}}{29}\right)$, determine the value of *c*.

3 marks

Question 5 (5 marks)

a. Solve the equation $z^5+1=0,\ z\in C,$ giving your answers in polar form.

2 marks

b. Verify that the equation $z^5+1=0$ can be expressed in the form $(z+1)Q(z)=0$ where $Q(z)=z^4-z^3+z^2-z+1$

1 mark

c. Let $u = z + \frac{1}{z}$ show that $Q(z) = 0$ can be expressed in the form $u^2 - u - 1 = 0$.

1 mark

d. Hence find the value $\cos\left(\frac{3\pi}{5}\right)$

1 mark

Question 6 (3 marks)

The weights of a carrots from a local farm are normally distributed with a mean of 70 grams with a standard deviation of 5 grams.

Z has the standard normal distribution and given that $\Pr(-1.2 < Z < 1.2) = 0.770$.

a. Determine the probability that the total weight of four independent carrots from the local farm will have a weight greater than 292 grams.
Give your answer correct to three decimal places.

2 marks

b. Find the probability that the mean weight of four carrots is between 67 and 73 grams.
Give your answer correct to three decimal places.

1 mark

Question 7 (4 marks)

Prove by induction that $47^n + 53 \times 47^{n-1}$ is divisible by 100 for $n \in N$.

Question 8 (5 marks)

a. Let $x = \log_e\left(\sec(2t) + \tan(2t)\right) - \sin(2t)$, show that $\dfrac{dx}{dt} = 2\tan(2t)\sin(2t)$

2 marks

b. A curve is given by the parametric equations $x = \log_e\left(\sec(2t) + \tan(2t)\right) - \sin(2t)$ and $y = \cos(2t)$. When part of the curve between $t = 0$ and $t = \frac{\pi}{4}$ is rotated about the x-axis it forms a volume of revolution, find the surface area of this volume of revolution.

3 marks

Question 9 (3 marks)

Find $\int t^3 \sin\left(t^2\right) dt$

End of question and answer book for the
2024 Kilbaha VCE Specialist Mathematics Trial Examination 1

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SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics formulas

Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\,\text{cis}(\theta)$	$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \text{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \text{cis}(n\theta)$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \text{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \frac{1}{2}(1 - \cos(2ax))$	$\cos^2(ax) = \frac{1}{2}(1 + \cos(2ax))$

Data analysis, probability and statistics

for independent random variables $X_1, X_2 \ldots X_n$	$E(aX_1+b)=aE(X_1)+b$ $E(a_1X_1+a_2X_2+\ldots+a_nX_n)$ $=a_1E(X_1)+a_2E(X_2)+\ldots+a_nE(X_n)$
	$\text{Var}(aX_1+b)=a^2\text{Var}(X_1)$ $\text{Var}(a_1X_1+a_2X_2+\ldots a_nX_n)$ $=a_1^2\text{Var}(X_1)+a_2^2\text{Var}(X_2)+\ldots+a_n^2\text{Var}(X_n)$
for independent identically distributed variables $X_1, X_2 \ldots X_n$	$E(X_1+X_2+\ldots+X_n)=n\mu$
	$\text{Var}(X_1+X_2+\ldots+X_n)=n\sigma^2$
approximate confidence interval for μ	$\left(\bar{x}-z\dfrac{s}{\sqrt{n}},\ \bar{x}+z\dfrac{s}{\sqrt{n}}\right)$

distribution of sample mean $\bar{X}$	mean	$E(\bar{X})=\mu$
	variance	$\text{Var}(\bar{X})=\dfrac{\sigma^2}{n}$

Vectors in two and three dimensions

$\underset{\sim}{r}(t)=x(t)\underset{\sim}{i}+y(t)\underset{\sim}{j}+z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert=\sqrt{x(t)^2+y(t)^2+z(t)^2}$
	$\dot{\underset{\sim}{r}}(t)=\dfrac{d\underset{\sim}{r}}{dt}=\dfrac{dx}{dt}\underset{\sim}{i}+\dfrac{dy}{dt}\underset{\sim}{j}+\dfrac{dz}{dt}\underset{\sim}{k}$
for $\underset{\sim}{r}_1=x_1\underset{\sim}{i}+y_1\underset{\sim}{j}+z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2=x_2\underset{\sim}{i}+y_2\underset{\sim}{j}+z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2=\lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$
	vector cross product $\underset{\sim}{r}_1\times\underset{\sim}{r}_2=\begin{vmatrix}\underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k}\\ x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\end{vmatrix}=(y_1z_2-y_2z_1)\underset{\sim}{i}+(x_2z_1-x_1z_2)\underset{\sim}{j}+(x_1y_2-x_2y_1)\underset{\sim}{k}$
vector equation of a line	$\underset{\sim}{r}(t)=\underset{\sim}{r}_1+t\underset{\sim}{r}_2=(x_1+x_2t)\underset{\sim}{i}+(y_1+y_2t)\underset{\sim}{j}+(z_1+z_2t)\underset{\sim}{k}$
parametric equation of line	$x(t)=x_1+x_2t \quad y(t)=y_1+y_2t \quad z(t)=z_1+z_2t$
vector equation of a plane	$\underset{\sim}{r}(s,t)=\underset{\sim}{r}_0+s\underset{\sim}{r}_1+t\underset{\sim}{r}_2$ $=(x_0+x_1s+x_2t)\underset{\sim}{i}+(y_0+y_1s+y_2t)\underset{\sim}{j}+(z_0+z_1s+z_2t)\underset{\sim}{k}$
parametric equation of a plane	$x(s,t)=x_0+x_1s+x_2t \quad y(s,t)=y_0+y_1s+y_2t \quad z(s,t)=z_0+z_1s+z_2t$
Cartesian equation of a plane	$ax+by+cz=d$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\left(\lvert x\rvert\right)+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\cot(ax)\right)=-a\,\mathrm{cosec}^2(ax)$	$\int \mathrm{cosec}^2(ax)\,dx=-\frac{1}{a}\cot(ax)+c$
$\frac{d}{dx}\left(\sec(ax)\right)=a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax)\,dx=\frac{1}{a}\sec(ax)+c$
$\frac{d}{dx}\left(\mathrm{cosec}(ax)\right)=-a\,\mathrm{cosec}(ax)\cot(ax)$	$\int \mathrm{cosec}(ax)\cot(ax)\,dx=-\frac{1}{a}\mathrm{cosec}(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(ax)\right)=\frac{1}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}(ax)\right)=\frac{-1}{\sqrt{1-(ax)^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}(ax)\right)=\frac{a}{1+(ax)^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1}\,dx=\frac{1}{a}\log_e\left(\lvert ax+b\rvert\right)+c$

Calculus- continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf\left(x_n, y_n\right)$
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
surface area Cartesian about the x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$
surface area Cartesian about the y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$
surface area parametric about the x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
surface area parametric about the y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Kinematics

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}t^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u+v)t$

END OF FORMULA SHEET