TSSM
Creating VCE Success

2023 Trial Examination

Letter

STUDENT NUMBER

SPECIALIST MATHEMATICS

Unit 1 Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION & ANSWER BOOK

Structure of book

Number of questions	Number of questions to be answered	Number of marks
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied
- Question and answer book of 9 pages.

Instructions
- Print your student number in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagram in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude g ms^{-2} , where $g = 9.8$.

Question 1 (3 marks)

Consider the following sets of numbers:

$A = \{1, 3, 4, 5, 8\}$, $B = \{2, 3, 6, 7, 8, 9\}$, $C = \{\sqrt{2}, \sqrt{5}\}$, $D = \{\frac{1}{2}, \frac{5}{3}, 2\frac{3}{4}\}$

a. Identify the set $A \cap B$.

_____

_____

1 mark

b. Which set or sets contain rational numbers?

_____

_____

1 mark

c. Which set or sets contain natural numbers?

_____

_____

1 mark

TURN OVER

Question 2 (5 marks)

a. Factorise fully: $x^3 + 3x^2 + 2x$

_____

_____

2 marks

b. Hence prove that $x^3 + 3x^2 + 2x$ is a multiple of 6 given $x \in N$.

_____

_____

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3 marks

Question 3 (6 marks)

a. Complete the following truth table relating to the three Boolean variables A, B, C

A	B	C	A'	$A' \wedge B$	$A' \wedge C$	$(A' \wedge B) \vee (A' \wedge C)$	$B \vee C$	$A' \wedge (B \vee C)$
1	1	1	0	0	0	0	1	0
1	1	0						
1	0	1						
1	0	0						
0	1	1						
0	1	0						
0	0	1						
0	0	0						

2 marks

b. The truth table in part a. confirms that $(A' \wedge B) \vee (A' \wedge C)$ simplifies to _____

1 mark

c. Suppose the parents of a family of three children will only give authority to view the family jewels if their son, **A** is not present and both daughters **B,C** are present.

Complete the following truth table, consistent with these restrictions. For the Boolean variables, let $1 = $ present and $0 = $ absent.

A	B	C	AUTHORITY
1	1	1	
1	1	0	
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	

1 mark

d. Draw a logic gate diagram to represent the outcomes from **part c**.

2 marks

$2 + 1 + 1 + 2 = 6$ marks

Question 4 (5 marks)

Consider the matrices:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, \ B = \begin{bmatrix} 0 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix}, \ C = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}, D = \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}$$

a. What is the element in row 2, column 1 for matrix B?

_____

1 mark

b. Calculate $2A^2$.

2 marks

c. Explain why $B \times (C + D)$ exists but $(C + D) \times B$ does not. Hence find $B \times (C + D)$

2 marks

$1 + 2 + 2 = 5$ marks

Question 5 (3 marks)

A ball is dropped from a height of $2m$ then returns to 80% of its previous height after each bounce. Express the distance the ball travels as the sum of two geometric series hence find the **total** distance that the ball travels.

_____

_____

_____

_____

3 marks

Question 6 (8 marks)

A graph, *A*, has the following adjacency matrix:

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \end{array}$$

a. How many edges end at v_3?

_____

1 mark

b. Which, if any, vertex or vertices in graph *A* have a loop?

_____

1 mark

c. Draw graph *A*.

2 marks

d. Explain why graph *A* has an Euler path and identify one such path.

_____

_____

_____

2 marks

TURN OVER

e. Explain why graph A does not have an Euler circuit. Redraw graph A with the inclusion of an extra edge so that an Euler circuit now exists and identify one such circuit.

_____

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_____

2 marks

Question 7 (6 marks)

a. Find the 20th term of the relation $t_{n+1} = t_n + 6, t_1 = 8, n \geq 1$.

_____

_____

_____

2 marks

b. Identify the first 3 terms of the relation $P_{n+1} = 10P_n - 2, P_1 = 12, n \geq 1$.

_____

_____

_____

1 mark

c. Specify the recurrence relation for a bacterial infection U_n, which initially has 10 spores on the first day then increases by a factor of 5 on each successive day. Hence write the nth term in the form $x \times y^n$ where $x, y \in N$

_____

_____

_____

_____

3 marks

Question 8 (4 marks)

The letters $ABCDE$ are arranged randomly in a straight line.

a. How many different arrangements are possible?

_____

1 mark

b. How many arrangements are possible with the A next to the B?

_____

_____

1 mark

c. The five letters are now randomly split into two groups (X and Y). Each group has at least one letter. Letters within a given group are selected but not arranged. How many combinations are possible?

_____

_____

_____

_____

2 marks

END OF QUESTION AND ANSWER BOOK