Student Name: _____

SPECIALIST MATHEMATICS 2023

Unit 3

Key Topic Test 14 – Differentiation Applications
Technology Active

Recommended writing time*: 45 minutes
Total number of marks available: 30 marks

QUESTION BOOK

* The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions
- Students are permitted to bring into the room for this test: pens, pencils, highlighters, erasers, sharpeners and rulers, a CAS and/or scientific calculator
- Students are NOT permitted to bring into the room for this test: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 9 pages.

Instructions
- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.

SECTION A– Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question

Question 1

The coordinates of one of the local maximums on the graph of $y = \frac{1}{(\cos(x)+2)^2}$ is at:

A. $(-\pi, 1)$

B. $(2\pi, 1)$

C. $(0, 1)$

D. $\left(\frac{\pi}{2}, \frac{1}{2}\right)$

E. $\left(-\frac{\pi}{2}, \frac{1}{2}\right)$

Question 2

The coordinates of the point of inflection, correct to two decimal places, of
$y = \cos^{-1}(\log_e(x))$ is

A. $(2.72, 0)$

B. $(0.19, 0.91)$

C. $(1.86, 0.91)$

D. $(0.19, 0)$

E. $(1.86, -0.69)$

Question 3

The equation of the tangent to the curve $y = \frac{1}{\sec\left(\frac{x}{2}\right)+3}$ at $x = 0$ is

A. $y - 3 = 0$

B. $4y - 1 = 0$

C. $x = \frac{1}{4}$

D. $x = 0$

E. $y = 0$

Question 4

The x-intercept of the graph $f: \left[\frac{1}{2}, 2\right] \to R, f(x) = \tan\left(a \log_e(x)\right)$ is also a stationary point of $y = f(x)$.

The value of a is

A. -1
B. 0
C. 1
D. 2
E. $\frac{1}{3}$

Question 5

The maximum value of the function $f: [-1, 1] \to R, \ f(x) = \frac{2x}{(x^2+1)^2}$ is

A. $-\frac{3\sqrt{3}}{8}$
B. $\frac{3\sqrt{3}}{8}$
C. $-\frac{\sqrt{3}}{3}$
D. $\frac{\sqrt{3}}{3}$
E. $\frac{1}{2}$

Question 6

For the function $f(x) = e^{|x-2|} + 1$, which of the following statements is true?

A. The function is decreasing for all x.
B. The function has a horizontal asymptote at $x = 2$.
C. The function is not continuous at $x = 2$.
D. The function has a stationary point at $x = 2$.
E. The function is not differentiable at $x = 2$.

Question 7
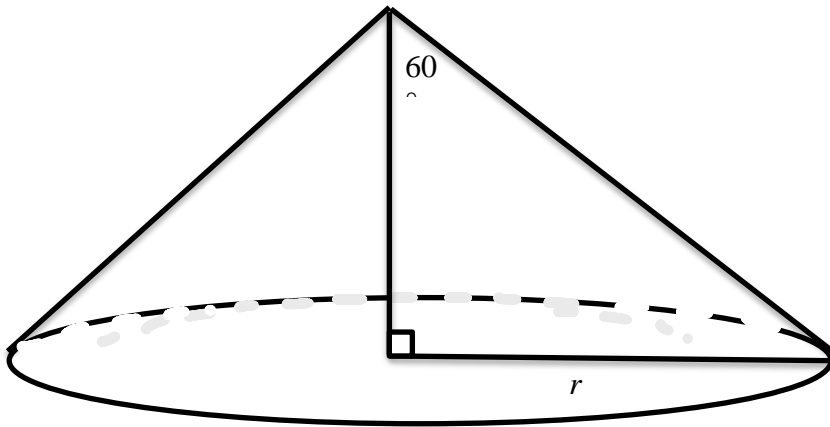
Cement falls from a concrete mixer to form a pile in the shape of a right circular cone with semi-vertex angle 60°.

Cement is added to the pile at a rate of 1.8 m^3 per minute. The rate at which the radius r metres of the pile is increasing, in metres per minute, when the radius of the pile is 0.9m, correct to two decimal places, is

A. 0.82

B. 0.91

C. 1.10

D. 1.23

E. 2.12

Section B Short-answer questions

Instructions for Section B

Answer each question in the space provided.

Please provide appropriate workings and use exact answers unless otherwise specified.

Question 1 (11 marks)
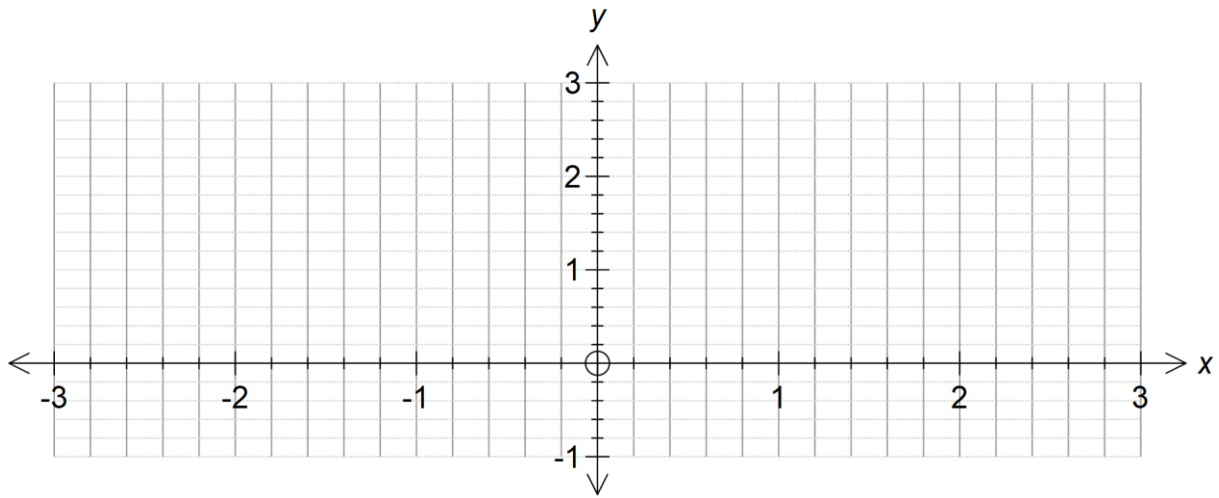
Let $f(x) = e^{\frac{2}{3}x^3}$.

a. Find $f'(x)$.

_____

_____

1 mark

b. Hence find the coordinates of the stationary point(s).

_____

_____

_____

_____

2 marks

c. Find $f''(x)$ and hence state the coordinates of any point(s) of inflection.

_____

_____

_____

_____

_____

3 marks

d. Sketch the graph of $y = f(x)$ on the axes below.
Label all key features.

3 marks

e. Sketch the graph of $y = f'(x)$ on the axes above.
You do not have to label any points.

2 marks

Question 2 (12 marks)

Let $f(x) = -(\ln(x))^2 - 2\ln(x) + 3$

a. Find the coordinates of any stationary point(s) of $f(x)$ and identify their nature.

_____

_____

_____

_____

_____

_____

2 marks

b. Find the coordinates of the inflection point. Hence find the equation of the tangent to the curve $y = f(x)$ at the inflection point.

_____

_____

_____

_____

_____

_____

3 marks

c. The tangent at a point (u, v) on $y = f(x)$ makes an angle of $120°$ with the positive direction of x-axis. Find all possible values of u and v.

_____

_____

_____

_____

_____

_____

_____

3 marks

d. Find the interval for which $f(x) > 2.5$.

_____

_____

_____

2 marks

e. Let $g: (a, \infty) \to R, \ g(x) = 3x^2 - 1$.

What is the minimum positive value of a for which the function $f\big(g(x)\big)$ is defined.

_____

_____

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_____

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_____

<div align="right">2 marks</div>

END OF KEY TOPIC TEST