

Student Name: _____



SPECIALIST MATHEMATICS 2023

Unit 3

Key Topic Test 13 – Differentiation Applications

Technology Free

Recommended writing time*: 45 minutes

Total number of marks available: 30 marks

QUESTION BOOK

* The recommended writing time is a guide to the time students should take to complete this test. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this test: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the room for this test: any calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 7 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this test.

Question 1 (6 marks)

Let $f(x) = \ln(x^2 + 4)$

a. Find $f'(x)$

1 mark

b. Hence, find the coordinates of the stationary point of $f(x)$ and show that this stationary point is a point of local minimum.

3 marks

c. Find the coordinates of any point(s) of inflection of $f(x)$.

2 marks

Question 2 (4 marks)

Find the gradient of the normal to the curve $\sin^2(x) + \cos^2(y) = \frac{x}{y}$ at the point $\left(\frac{\pi}{4}, \frac{\pi}{3}\right)$.

Question 3 (5 marks)

Find the point of intersection of the tangents to the curve given by $2x^2y + 3x = 2y$ at $y = 1$.

Question 4 (4 marks)

Petroleum is leaking at a constant rate of 9.5 mm^3 per minute to form a circular puddle on the floor.

The puddle spreads out evenly, with constant depth of 2 mm.

The volume, $V \text{ mm}^3$, of petroleum in the puddle is given by $V = \frac{a}{2} \pi r^2$, where r is the radius in mm and a is a real constant.

- a.** Show that $a = 4$.

1 mark

- b.** Find the exact rate of change of the radius of the puddle when the radius is 25 mm.

2 marks

- c.** Find the rate at which the volume is changing with change in radius.

1 mark

Question 5 (8 marks)

Let $f(x) = x^2e^x$

- a.** Find the coordinates of the stationary point(s) of the curve $y = x^2e^x$.

2 marks

- b.** For what value(s) of x is $f(x)$ strictly increasing?

3 marks

