NAME: ______________________________

VCE® SPECIALIST MATHEMATICS

Units 3 & 4 Practice Written Examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	20	20	20
2	6	60	60
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and Answer Book of 26 pages.
- Formula Sheet.
- Answer Sheet for Multiple-Choice Questions.

Instructions

- Write your **name** in the space provided above on this page.
- Write your **name** on the Multiple-Choice Answer Sheet.
- All written responses must be in English.

At the end of the examination

- place the answer sheet for Multiple-Choice Questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A - MULTIPLE-CHOICE

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for Multiple-Choice Questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, and an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

Pseudocode for a particular procedure is shown below.

```
algorithm(n, f, t1, t2)
f ← 0
t1 ← 1
t2 ← 2
n ← 4
repeat n times
    f = 3 x t1 - t2
    t2 = f
end loop
return f
```

What is the output for this procedure?

A. 0

B. -1

C. 1

D. 2

E. 4

Question 2

Which of the following statements can be disproved using a counterexample?

A. All numbers of the form $n^3 - n$, where $n \in N$, are divisible by 3.

B. If $a + b > 12$, where $a, b \in R$, then either $a > 6$ or $b > 6$.

C. $x^2 + 5y^2 \geq 2xy$, $x, y \in R$.

D. $|a + b| \leq |a| + |b|$, $a, b \in R$.

E. All prime numbers can be expressed in the form $p = 6n \pm 1$, $n \in N$.

Question 3

A solution to the equation $z\bar{z} = z + \bar{z}$, where $z \in C$, is

A. $z = -1 - i$

B. $z = -1 + i$

C. $z = 1 + i$

D. $z = i$

E. $z = -i$

Question 4

$\dfrac{(1-i)^5}{(1+i\sqrt{3})^3}$ is equal to

A. $\dfrac{1}{\sqrt{2}}(1+i)$

B. $\dfrac{1}{2}(1-i)$

C. $\dfrac{1}{\sqrt{2}}(1-i)$

D. $\sqrt{2}\operatorname{cis}\left(-\dfrac{\pi}{4}\right)$

E. $\dfrac{1}{\sqrt{2}}\operatorname{cis}\left(\dfrac{\pi}{4}\right)$

Question 5

The solutions of $(z+1)^3 - 8i = 0$, where $z \in C$, are

A. $z = -1 + 2i$

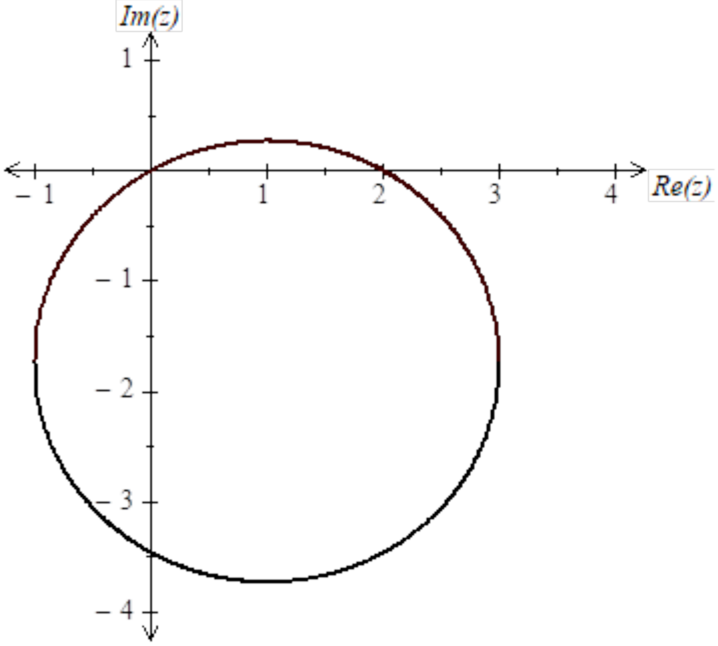
B. $z = -1 - 2i$

C. $z = 1 - 2i$

D. $z = -1 - 2i,\ -1 \pm \sqrt{3} + i$

E. $z = -1 - 2i,\ \pm\sqrt{3}i$

Question 6



The graph of the circle shown on the Argand plane above passes through (0, 0) and (2, 0).

A possible equation for this circle is

A. $|z - 1 + \sqrt{3}i| = 2,\ z \in C$

B. $|z + 1 - \sqrt{3}i| = 4,\ z \in C$

C. $|z + 1 + \sqrt{3}i| = 4,\ z \in C$

D. $|z - 1 - \sqrt{3}i| = 2,\ z \in C$

E. $|z + 1 + \sqrt{3}i| = 2,\ z \in C$

Question 7

Euler's method with a step size of 0.1 is used to find an approximate solution to the differential equation $\frac{dy}{dx} = \tan^{-1}\sqrt{1+x}$, $y(0)=0$. The value of $y(0.2)$ given by Euler's method, correct to three decimal places, is

A. 0.159

B. 0.237

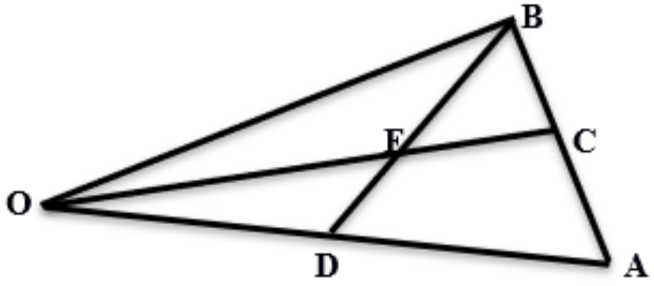
C. 0.346

D. 0.809

E. 0.831

Question 8

In the triangle OAB shown below, $\overrightarrow{\text{OA}} = \underset{\sim}{a}$, $\overrightarrow{\text{OB}} = \underset{\sim}{b}$, $\overrightarrow{\text{OE}} = \frac{2}{3}\overrightarrow{\text{OC}}$ and $\overrightarrow{\text{EB}} = \frac{3}{5}\overrightarrow{\text{DB}}$.



If $\overrightarrow{\text{OD}} = m\underset{\sim}{a}$ and $\overrightarrow{\text{AC}} = n\left(\underset{\sim}{b} - \underset{\sim}{a}\right)$, the values of m and n are

A. $m = \frac{3}{5}$, $n = \frac{2}{3}$

B. $m = \frac{2}{3}$, $n = \frac{3}{5}$

C. $m = \frac{4}{9}$, $n = \frac{3}{5}$

D. $m = \frac{2}{5}$, $n = \frac{4}{9}$

E. $m = \frac{4}{9}$, $n = \frac{2}{3}$

Question 9

The equation of a line L is $\utilde{r}_1 = \utilde{i} + 2\utilde{j} - 2\utilde{k} + t\left(\utilde{i} - \utilde{j} + \utilde{k}\right),\ t \in R$. An equation of a line that passes through the point (3, 2, 1) and is perpendicular to L is

A. $\utilde{r}_2 = s\left(\utilde{j} + \utilde{k}\right),\ s \in R$

B. $\utilde{r}_2 = 3\utilde{i} + 2\utilde{j} + \utilde{k} + s\left(\utilde{j} + \utilde{k}\right),\ s \in R$

C. $\utilde{r}_2 = \utilde{i} + 2\utilde{j} - 2\utilde{k} + s\left(\utilde{j} + \utilde{k}\right),\ s \in R$

D. $\utilde{r}_2 = 3\utilde{i} + 2\utilde{j} + \utilde{k} + s\left(\utilde{i} - \utilde{j} + \utilde{k}\right),\ s \in R$

E. $\utilde{r}_2 = 3\utilde{i} + 2\utilde{j} - 2\utilde{k} + s\left(\utilde{i} - \utilde{j} + \utilde{k}\right),\ s \in R$

Question 10

A vector perpendicular to the lines $\underset{\sim}{r}_1 = \underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k} + t\left(\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}\right)$, $t \in R$, and $\underset{\sim}{r}_2 = 2\underset{\sim}{\mathbf{i}} + \underset{\sim}{\mathbf{k}} + s\left(\underset{\sim}{\mathbf{i}} + \underset{\sim}{\mathbf{j}}\right)$, $s \in R$, is given by

A. $\begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ -1 & -1 & 0 \\ 1 & -1 & -1 \end{vmatrix}$

B. $\begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 2 & 0 & 1 \\ 1 & -2 & -1 \end{vmatrix}$

C. $\begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{vmatrix}$

D. $\begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$

E. $\begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & -1 & -1 \\ 2 & 0 & 1 \end{vmatrix}$

Question 11

A plane is perpendicular to the vector $\underset{\sim}{n} = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ and contains the point (1, 1, 2).

A Cartesian equation of this plane is

A. $2x + y - 2z = 1$

B. $x + y + 2z = -2$

C. $-2x - y + 2z = 1$

D. $-2x - y + 2z = 2$

E. $-2x + y - 2z = 1$

Question 12

The lines $\underset{\sim}{r_1} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k} + t\left(\underset{\sim}{i} - \underset{\sim}{j}\right)$, $t \in R$, and $\underset{\sim}{r_2} = 2\underset{\sim}{i} - 4\underset{\sim}{j} - \beta\underset{\sim}{k} + s\left(\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}\right)$, $s \in R$ and β is a real constant, are skew provided that

A. $\beta = 3$

B. $\beta \neq 3$

C. $\beta = 1$

D. $\beta \neq 1$

E. The lines can never be skew.

Question 13

A vector equation of the line connecting the points $A(1, 1, 1)$ and $B(-2, 0, 2)$ is

A. $\underset{\sim}{r} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k} + t\left(-3\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right),\ t \in R$

B. $\underset{\sim}{r} = -2\underset{\sim}{i} + 2\underset{\sim}{k} + t\left(3\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right),\ t \in R$

C. $\underset{\sim}{r} = -2\underset{\sim}{i} + 2\underset{\sim}{k} + t\left(-3\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}\right),\ t \in R$

D. $\underset{\sim}{r} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k} + t\left(3\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right),\ t \in R$

E. $\underset{\sim}{r} = t\left(-3\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}\right),\ t \in R$

Question 14

The position vectors of two objects are given by $\underset{\sim}{r_1}(t) = \underset{\sim}{i} + \underset{\sim}{k} + t\left(\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}\right)$ and $\underset{\sim}{r_2}(t) = \underset{\sim}{i} + \underset{\sim}{k} + t\left(2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}\right)$ where $t \in R$ is measured in minutes. After 4 minutes the area of the triangle formed by the respective positions of each object and their starting point is

A. $\frac{1}{2}\sqrt{5169}$

B. $22\sqrt{2}$

C. $6\sqrt{26}$

D. $5\sqrt{29}$

E. $\frac{1}{2}\sqrt{3305}$

Question 15

The area between the curves $y = \dfrac{1}{\sqrt{10-4x^2}}$ and $y = \dfrac{1}{1+x^2}$ is closest to

A. 1.6792

B. 0.7825

C. 0.8912

D. 0.8944

E. 0.8967

Question 16

A car travelling at $10\,\text{ms}^{-1}$ hits a soft barrier and slows down with an acceleration equal to $-5v^2\ \text{ms}^{-2}$ where $v\ \text{ms}^{-1}$ is its speed after hitting the barrier.

What distance, in meters, does the car travel while slowing down to $5\,\text{ms}^{-1}$?

A. 0.5

B. $\log_e(2)$

C. $0.2\log_e(2)$

D. $5\log_e(2)$

E. 0.2

Question 17

The amount of coffee, X ml, dispensed in a cup by the coffee machines at Starving John's is a random variable that has the probability density function

$$f(x)=\begin{cases}\dfrac{1}{10} & \text{if } 195<x<205\\ 0 & \text{elsewhere}\end{cases}$$

The probability that the average amount of coffee dispensed by the machine in 80 random cups is less than 197 ml is closest to

A. 0.1145

B. 0.2

C. 0.4976

D. 0.4465

E. 0.3148

Question 18

On a particular test the results are scaled using the formula $S=aX+b$ where S is the scaled result, X is the unscaled result, $\mu_X=24$, $\sigma_X^2=16$, $\mu_S=32$ and $\sigma_S^2=25$.
What is the unscaled result coresponding to the scaled result $S=27$?

A. 20

B. 25

C. 30

D. 35

E. 36

Question 19

A sample of size 80 is randomly selected from a population and used to calculate an approximate C% confidence interval for the population mean.

What is the value of C, correct to the nearest whole number, if the standard deviation of the sample was 4.34 and the confidence interval was calculated to be (28.712, 29.928)?

A. 99

B. 95

C. 86

D. 79

E. 76

Question 20

The Minister for Aged Care wishes to estimate the diastolic blood pressure, measured in units of mmHg (millimeters of mercury), of patients in federally-funded aged care facilities by collecting a random sample of patient blood pressures and calculating a 95% confidence interval.

The Chief Medical Officer advises that the standard deviation of blood pressure can be taken to be 12 mmHg.

What is the minimum sample size needed to ensure that the sample mean differs from the population mean by no more than 1.2 mmHg?

A. 385

B. 400

C. 120

D. 19

E. 20

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (10 marks)

a. Express the relation $|z+1||\bar{z}+1| = 3$, where $z \in C$, in Cartesian form. 1 mark

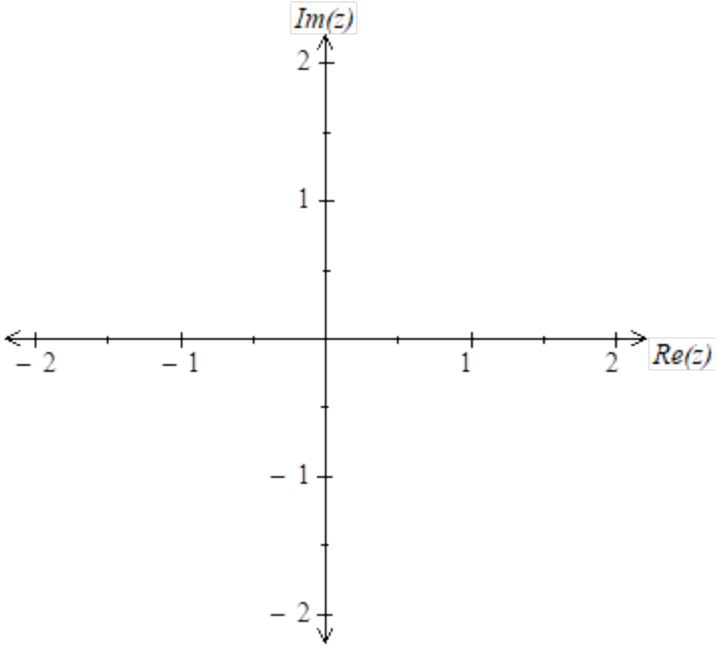
____

____

____

b. Sketch the graphs of $|z+1||\bar{z}+1| = 3$ and $\text{Arg}(z-1) = \dfrac{4\pi}{3}$ on the Argand diagram below.

2 marks



Working space

c. Express the relation $\operatorname{Arg}(z-1)=\frac{4\pi}{3}$ in Cartesian form. 2 marks

d. Find the value(s) of z such that $|z+1||\bar{z}+1|=3$ and $\operatorname{Arg}(z-1)=\frac{4\pi}{3}$. 2 marks

e. Find, correct to four decimal places, the smaller of the two areas bounded by the graphs of $|z+1||\bar{z}+1|=3$ and $\text{Re}(z)=0$. 3 marks

Question 2 (9 marks)

The planes $\prod_1$ and $\prod_2$ are defined by the Cartesian equations $2x+2y-z=3$ and $x-y+z=3$ respectively.

a. Find the minimum distance of $\prod_1$ from the point $P(1,\ -1,\ 2)$. 3 marks

b. Find the angle between $\prod_1$ and $\prod_2$. Give your answer in degrees, correct to one decimal place. 2 marks

c. Find an equation for $\prod_1$ in vector form. 2 marks

d. Find in vector form an equation for the line of intersection of $\prod_1$ and $\prod_2$. 2 marks

Question 3 (15 marks)

The position vector of a weather balloon is given by

$$\underset{\sim}{r}(t) = 12t\,\underset{\sim}{i} - 5t\,\underset{\sim}{j} + \left(2 - \cos^2(2\pi t)\right)\underset{\sim}{k},\ t \geq 0$$

where t is measured in hours and components are measured in km. The unit vectors $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$ point in the directions east, north, and upwards respectively.

a. **i.** At what time, in hours, does the balloon first reach a height of 1.5 km? 1 mark

ii. Find how far from its starting point the balloon is at that time. Give your answer in km and in the form $\dfrac{\sqrt{a}}{b}$ where $a, b \in N$. 2 marks

b. Find the maximum speed of the balloon, in km/hour and correct to two decimal places, and the time in hours at which this speed first occurs. 3 marks

At $t = 1$ the weather balloon stops sending information due to a malfunction and a repair person is sent up via jetpack to repair it.

c. The position vector of the repair person is $\underset{\sim}{r_J} = (at + 2)\underset{\sim}{\text{i}} + bt\,\underset{\sim}{\text{j}} + \left(20t - \frac{1}{2}gt^2\right)\underset{\sim}{\text{k}}$,

where $a, b \in R$.

i. Find the time taken for the repair person to reach the balloon, correct to the nearest second, after they are sent up. 2 marks

ii. Hence find the values of *a* and *b*, correct to three decimal places. 2 marks

When the repairs are finished, the repair person switches off the jetpack and falls directly downwards. Once they reach a speed of $10\,\text{ms}^{-1}$, a parachute opens and the repair person continues to fall downwards with an acceleration equal to $g-0.8v^2\ \text{ms}^{-2}$ where $v\ \text{ms}^{-1}$is their speed after the parachute opens.

d. Find the distance, correct to the nearest centimeter, that the repair person falls while slowing down from $10\,\text{ms}^{-1}$ to $5\,\text{ms}^{-1}$. 3 marks

e. Find the limiting speed (terminal velocity) of the falling repair person. 2 marks

Question 4 (9 marks)

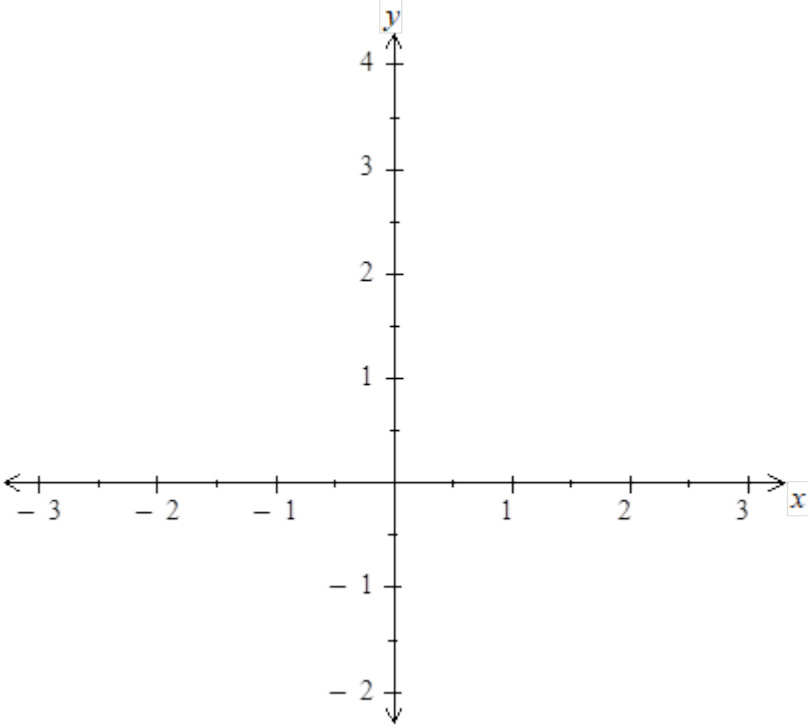
A function $y(x)$ is defined by $\frac{dy}{dx} = -3x\sin\left(x^2\right)$, where $x \in [-1, 2]$ and $y(0) = 2.5$.

a. Show by solving the above differential equation that $y(x) = \frac{3}{2}\cos(x^2) + 1$. 2 marks

b. Find the coordinates of all turning points of $y(x)$. 2 marks

c. Find the coordinates of all points of inflection of $y(x)$. Give your answer correct to three decimal places. 2 marks

d. Sketch the graph of $y = y(x)$ on the set of axes below. Label all endpoints, turning points and axes intercepts with their coordinates. Give all approximate values correct to three decimal places. 3 marks



Working space

Question 5 (8 marks)

A function $y = f(x)$ is defined by the parametric equations $x = a - \sqrt[3]{t}$ and $y = \sqrt{t}$ where $t \geq 0$.

a. Show that $f(x) = (a - x)^{\frac{3}{2}}$ and include any domain restrictions. 2 marks

b. Prove that the graph of $y = f(x)$ has no points of inflection. 1 mark

c. The region bounded by the graph of $y = f(x)$ and the lines $y = 0$ and $x = a$ is rotated around the *x*-axis to form a solid of revolution. Find the volume of this solid. 2 marks

d. Let $a = 3$ and $0 \le t \le 8$.

Find, correct to two decimal places, the area of the surface generated by rotating the curve about the *x*-axis. 3 marks

Question 6 (9 marks)

The Sunshine Solar Company sells solar panels. It claims that the energy efficiency of their panels is normally distributed with a mean of 40%. The Minister for Consumer Affairs decides to check this claim by conducting a one-tailed statistical test at the 5% level of significance.

A random sample of 40 panels is tested and the sample mean is found to be 38%. Assume that the standard deviation of the energy efficiency is 6%.

a. Write down suitable hypotheses H_0 and H_1 for this test. 1 mark

b. Find the *p* value for this test, correct to four decimal places. 1 mark

c. State the probability of a type 1 error for this test. 1 mark

d. State with a reason if the Minister for Consumer Affairs should reject or confirm the claim made by the company. 1 mark

The lives of solar panels are normally distributed with a mean of 9.5 years and a standard deviation of 1.5 years. The Sunshine Solar Company claims that its solar panels have a longer life than other panels on the market.

A random sample of 25 Sunshine Solar Company panels is collected and the lives of the panels tested. A two-tailed test at the 5% level of significance is conducted.

e. Find, correct to two decimal places, the minimum and maximum values of the sample mean for H_0 to be rejected. 2 marks

f. If the true mean life of Sunshine Solar Company solar panels is 9 years, find the probability of a type 2 error for this test. Give your answer correct to four decimal places. 3 marks

END OF EXAMINATION

Multiple-Choice Answer Sheet

Student Name: ____

Shade the letter that corresponds to each correct answer.

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E

VCE® Specialist Mathematics

Practice Written Examination 2

ADVICE FOR TEACHERS

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The VCAA Formula Sheet can be downloaded and printed from:

https://www.vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Specialist-Mathematics.aspx

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- Insert your school logo/identification on page 1 and other pages at your discretion.

Solution Pathway

Below are sample answers and solutions. Please consider the merit of alternative responses.

Specialist Mathematics Examination 2: Marking Scheme

Section A: Multiple-Choice Questions - Answers

1.	D	**6.**	A	**11.**	C	**16.**	C
2.	E	**7.**	A	**12.**	D	**17.**	D
3.	C	**8.**	C	**13.**	D	**18.**	A
4.	B	**9.**	B	**14.**	B	**19.**	D
5.	D	**10.**	A	**15.**	E	**20.**	A

Section A : Multiple-Choice Questions - Solutions

MCQ 1	$f_0 = 0.$ $f_1 = 3\times1-2=1.$ $f_2 = 3\times1-\underbrace{f_1}_{t_2=f}=3\times1-1=2.$ $f_3 = 3\times1-\underbrace{f_2}_{t_2=f}=3\times1-2=1.$ $f_4 = 3\times1-\underbrace{f_3}_{t_2=f}=3\times1-1=2.$	**D**
MCQ 2	Options A, B, C and D are all true. Option E is false and has counter-examples of 2 and 3.	**E**
MCQ 3	Let $z=a+ib$, $a,b\in R$, be a solution: $z\bar{z}=a^2+b^2$ and $z+\bar{z}=2a$ therefore $z\bar{z}=z+\bar{z} \quad \Rightarrow a^2+b^2=2a$. Eliminate incorrect options by testing $a^2+b^2=2a$. Option C is the only correct option: $z=1+i \Rightarrow a=b=1$ ✓.	**C**

MCQ 4	Use a CAS: $1-i=\sqrt{2}\text{cis}\left(-\frac{\pi}{4}\right) \quad \Rightarrow (1-i)^5=\left(\sqrt{2}\right)^5 \text{cis}\left(-5\times\frac{\pi}{4}\right)=4\sqrt{2}\text{cis}\left(-\frac{5\pi}{4}\right).$ $1+i\sqrt{3}=2\text{cis}\left(\frac{\pi}{3}\right) \quad \left(1-i\sqrt{3}\right)^3=2^3\text{cis}\left(3\times\frac{\pi}{3}\right)=8\text{cis}(\pi).$ Therefore: $\frac{(1-i)^5}{(1+i\sqrt{3})^3}=\frac{4\sqrt{2}\text{cis}\left(-\frac{5\pi}{4}\right)}{8\text{cis}(\pi)} \quad =\frac{1}{\sqrt{2}}\text{cis}\left(-\frac{5\pi}{4}-\pi\right) \quad =\frac{1}{2}(1-i).$	**B**
MCQ 5	Use a CAS to solve $(z+1)^3-8i=0$: $z=-2i$, $-1\pm\sqrt{3}+i$.	**D**
MCQ 6	The given circle has radius 2 and centre at $z=1-i\sqrt{3}$. Compare with the standard form $\lvert z-z_1 \rvert = r$ of a circle with radius r and centre at $z=z_1$.	**A**
MCQ 7	$y_0=0$, $x_0=0$ and step size $h=0.1$. $x_0=0$, $x_n=0.2$ and $h=0.1$ therefore the number of iterations is $n=2$ and $y(0.2)\approx y_2$. Execute Euler's method on a CAS: $y_2\approx 0.159$.	**A**

MCQ 8	• $\overrightarrow{EB} = \frac{3}{5}\overrightarrow{DB}$. (1) Substitute $\overrightarrow{DB} = \overrightarrow{DO} + \overrightarrow{OB} = -m\underset{\sim}{a} + \underset{\sim}{b}$ into equation (1): $\overrightarrow{EB} = \frac{3}{5}\left(\underset{\sim}{b} - m\underset{\sim}{a}\right) = \frac{3}{5}\underset{\sim}{b} - \frac{3m}{5}\underset{\sim}{a}$. (2) • $\overrightarrow{EB} = \overrightarrow{EO} + \overrightarrow{OB} = -\overrightarrow{OE} + \underset{\sim}{b}$. (3) Substitute $\overrightarrow{OE} = \frac{2}{3}\overrightarrow{OC}$ into equation (3): $\overrightarrow{EB} = -\frac{2}{3}\overrightarrow{OC} + \underset{\sim}{b}$. (4) Substitute $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \underset{\sim}{a} + n\left(\underset{\sim}{b} - \underset{\sim}{a}\right) = n\underset{\sim}{b} + (1-n)\underset{\sim}{a}$ into equation (4): $\overrightarrow{EB} = \left(1 - \frac{2n}{3}\right)\underset{\sim}{b} - \frac{2}{3}(1-n)\underset{\sim}{a}$. (5) Equate the two expressions for $\overrightarrow{EB}$ in equation (2) and equation (5): Coefficients of $\underset{\sim}{b}$: $\frac{3}{5} = 1 - \frac{2n}{3} \quad \Rightarrow n = \frac{3}{5}$. Coefficients of $\underset{\sim}{a}$: $-\frac{3m}{5} = -\frac{2}{3}(1-n) \quad = -\frac{2}{3}\left(1 - \frac{3}{5}\right) \quad \Rightarrow m = \frac{4}{9}$.	**C**
MCQ 9	• Eliminate incorrect options by testing whether the vector $\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ in the direction of *L* is perpendicular to the direction of the line in each option: Options D and E are eliminated because the dot product of $\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ with their lines is not zero. • Eliminate incorrect options from options A, B and C by testing whether the point (3, 2, 1) lies on their line: Options A and C are eliminated because their lines never pass through points with an *x*-coordinate $x = 3$.	**B**

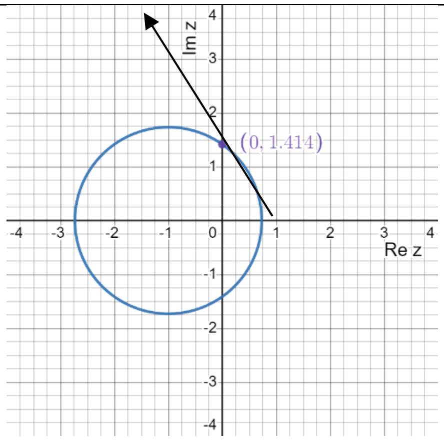
MCQ 10	Vectors in the direction of each line are $\underset{\sim}{v}_1 = \underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{v}_2 = \underset{\sim}{i} + \underset{\sim}{j}$ therefore, vectors perpendicular to each line are given by $\pm\underset{\sim}{v}_1 \times \underset{\sim}{v}_2$ and $\pm\underset{\sim}{v}_2 \times \underset{\sim}{v}_1$. Identify which option has one of these forms. Option A $\begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ -1 & -1 & 0 \\ 1 & -1 & -1 \end{vmatrix}$ corresponds to $-\underset{\sim}{v}_2 \times \underset{\sim}{v}_1$ ✓	**A**
MCQ 11	The vector $\underset{\sim}{n} = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ is normal to the plane therefore $2x + y - 2z = d$. Substitute the point (1, 1, 2): $d = -1$. Therefore $2x + y - 2z = -1 \quad \Rightarrow -2x - y + 2z = 1$.	**C**
MCQ 12	The lines are skew if they are not parallel and do not intersect. The lines will intersect if there are values of *t* and *s* such $\underset{\sim}{r}_1 = \underset{\sim}{r}_2$. Equate $\underset{\sim}{\mathbf{i}}$-components: $t + 1 = s + 2$. (1) Equate $\underset{\sim}{\mathbf{j}}$-components: $1 - t = s - 4$. (2) Solve equations (1) and (2) simultaneously: $t = 3$, $s = 2$. Equate $\underset{\sim}{\mathbf{k}}$-components: $1 = s - \beta \quad \Rightarrow 1 = 2 - \beta \quad \Rightarrow \beta = 1$. For the lines to be skew it is therefore required that $\beta \neq 1$.	**D**
MCQ 13	A vector in the direction of the line is $\overrightarrow{AB} = -3\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$. Using point *A* gives $\underset{\sim}{r} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k} + s\,\overrightarrow{AB}$, $s \in R$. This is equivalent to $\underset{\sim}{r} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k} + t\,\overrightarrow{BA} \quad (t = -s) \qquad = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k} + t\left(3\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right)$, $t \in R$.	**D**

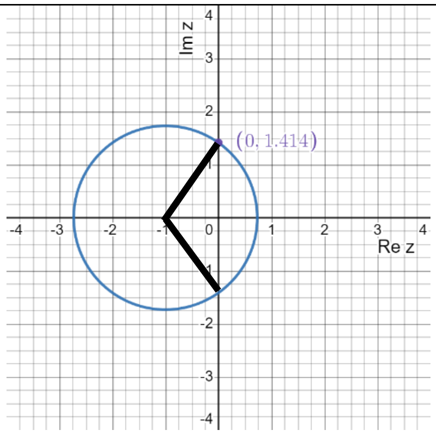
MCQ 14	At $t=0$ each object is at the point $A(1,\ 0,\ 1)$. At $t=4$: $\underset{\sim}{r}_1(4) = 5\underset{\sim}{i} - 4\underset{\sim}{j} - 3\underset{\sim}{k}$ which corresponds to the point $B(5,\ -4,\ -3)$. $\underset{\sim}{r}_2(4) = 8\underset{\sim}{i} + 4\underset{\sim}{j} + 5\underset{\sim}{k}$ which corresponds to the point $C(8,\ 4,\ 5)$. Relative to an origin O: $\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB} = (5-1)\underset{\sim}{i} - 4\underset{\sim}{j} + (-3-1)\underset{\sim}{k} = 4\underset{\sim}{i} - 4\underset{\sim}{j} - 4\underset{\sim}{k}$. $\overrightarrow{AC} = -\overrightarrow{OA} + \overrightarrow{OC} = (8-1)\underset{\sim}{i} + (4-0)\underset{\sim}{j} + (5-1)\underset{\sim}{k} = 7\underset{\sim}{i} + 4\underset{\sim}{j} + 4\underset{\sim}{k}$. Use a CAS to evaluate cross product: $\text{Area} = \frac{1}{2}\left\lvert \overrightarrow{AB} \times \overrightarrow{AC} \right\rvert = 22\sqrt{2}$.	**B**
MCQ 15	The x-coordinates of the intersection points are found by solving the equation $\frac{1}{\sqrt{10-4x^2}} = \frac{1}{1+x^2}$: $x = \pm\sqrt{3\sqrt{2}-3}$ (using a CAS). $\text{Area} = \int_{-\sqrt{3\sqrt{2}-3}}^{\sqrt{3\sqrt{2}-3}} \left\lvert \frac{1}{\sqrt{10-4x^2}} - \frac{1}{1+x^2} \right\rvert dx \quad \approx 0.8967$ (using a CAS).	**E**
MCQ 16	$a = v\frac{dv}{dx} = -5v^2 \quad \Rightarrow \frac{dv}{dx} = -5v \quad \Rightarrow \frac{dx}{dv} = -\frac{1}{5v}$ where $v = 10$ when $x = 0$. The integral solution is $x = -\int_{10}^{5} \frac{1}{5v}\, dv + 0$. Evaluate using a CAS: $x = \frac{1}{5}\log_e(2)$.	**C**

MCQ 17	The sample size is sufficently large to justify $\overline{X} \sim \text{Normal}\left(\mu_{\overline{X}} = \mu,\ \sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}}\right)$ even though the distribution of the population is uniform and therefore extremely non-normal. • $\mu = E(X) = \displaystyle\int_{195}^{205} \frac{x}{10}\, dx = 200$ (using a CAS or 'by hand') • $\sigma^2 = E\left(X^2\right) - \left(E(X)\right)^2 = \displaystyle\int_{195}^{205} \frac{x^2}{10}\, dx - 200^2$ $\Rightarrow \sigma_{\overline{X}} = \dfrac{\sqrt{\displaystyle\int_{195}^{205} \frac{x^2}{10}\, dx - 200^2}}{\sqrt{80}} \approx 22.307$ (using a CAS). Therefore $\overline{X} \sim \text{Normal}\left(\mu_{\overline{X}} = 200,\ \sigma_{\overline{X}} = 22.307\right)$. Use a CAS to evaluate $\Pr\left(\overline{X} < 197\right)$: 0.4465.	**D**
MCQ 18	$\mu_S = a\mu_x + b \quad \Rightarrow 24a + b = 32.$ (1) ${\sigma_S}^2 = a^2{\sigma_X}^2 \quad \Rightarrow 25 = 16a^2 \quad \Rightarrow a = \dfrac{5}{4}.$ Substitute into equation (1) and solve for *b*: $b = 2$. Therefore $S = \dfrac{5}{4}X + 2.$ Substitute $S = 27$ and solve for *X*: $X = 20$.	**A**

MCQ 19	Substitute $n = 80$, $\sigma = s = 4.34$ and the endpoints of the given confidence interval (28.712, 29.928) into the confidence interval formulae: $\bar{x} - z_{\alpha/2}\dfrac{4.34}{\sqrt{80}} = 28.712$. …. (1) $\bar{x} + z_{\alpha/2}\dfrac{4.34}{\sqrt{80}} = 29.928$. …. (2) Use a CAS to solve equations (1) and (2) for the critical value $z_{\alpha/2}$: $z_{\alpha/2} = 1.25302$. Substitute $z_{\alpha/2} = 1.25302$ into $\Pr\left(-z_{\alpha/2} < Z < z_{\alpha/2}\right) = 1 - \alpha$ (where Z is the standard norrmal variable): $\Pr\left(-1.25302 < Z < 1.25302\right) = 1 - \alpha$. Use a CAS to do the inverse normal calculation: $1 - \alpha = 0.789794$. $C = 100(1 - \alpha) = 78.9$.	**D**
MCQ 20	It is required that the margin of error $z_c \dfrac{\sigma}{\sqrt{n}} \le 1.2$. For a 95% confidence interval the critical value is $z_c = 1.96$. Use a CAS to solve $1.96 \times \dfrac{12}{\sqrt{n}} \le 1.2$: $n \ge 384.16$. Therefore, the minimum sample size is 385. **Note:** Students who use $z_c = 2$ will get 400 (option B).	**A**

Section B: Solutions

1a.	$\lvert z+1\rvert\lvert \bar{z}+1\rvert=3 \quad \Rightarrow \lvert z+1\rvert\lvert\overline{z+1}\rvert=3 \quad \Rightarrow \lvert z+1\rvert^2=3 \quad \Rightarrow \lvert z+1\rvert=\sqrt{3}$. **Answer:** $(x+1)^2+y^2=3$.	**1 mark**
1b.	 $\lvert z+1\rvert\lvert\bar{z}+1\rvert=3$ is a circle of radius $\sqrt{3}$ and centre at $z=-1$. $\text{Arg}(z-1)=\frac{4\pi}{3}$ is a ray extending from $z=1$ at an angle $\frac{4\pi}{3}$ relative to the positive real axis.	**1 mark** **1 mark**
1c.	The ray lies on the line with cartesian equation $y=\tan\left(\frac{4\pi}{3}\right)(x-1)$. **Rule:** $y=\sqrt{3}(1-x)$. The ray is the part of the line $y=\sqrt{3}(1-x)$ for which $x<1$ (see **part b.**) **Domain:** $x<1$.	**1 mark** **1 mark**
1d.	The required value(s) of z correspond to the intersection point(s) of the graphs of $(x+1)^2+y^2=3$ and $y=\sqrt{3}(1-x)$ subject to the restriction $x<1$. Use a CAS to solve $(x+1)^2+y^2=3$ and $y=\sqrt{3}(1-x)$ where $x<1$: $x=\frac{1}{2}$ and $y=\frac{\sqrt{3}}{2}$. **Answer:** $z=\frac{1}{2}+\frac{\sqrt{3}}{2}i$. 'By hand' solution: Substitute $y=\sqrt{3}(1-x)$ into $(x+1)^2+y^2=3$. $(x+1)^2+3(1-x)^2=3 \quad \Rightarrow 4x^2-4x+1=0 \quad \Rightarrow (2x-1)^2=0 \quad \Rightarrow x=\frac{1}{2}$.	**1 mark** **1 mark**

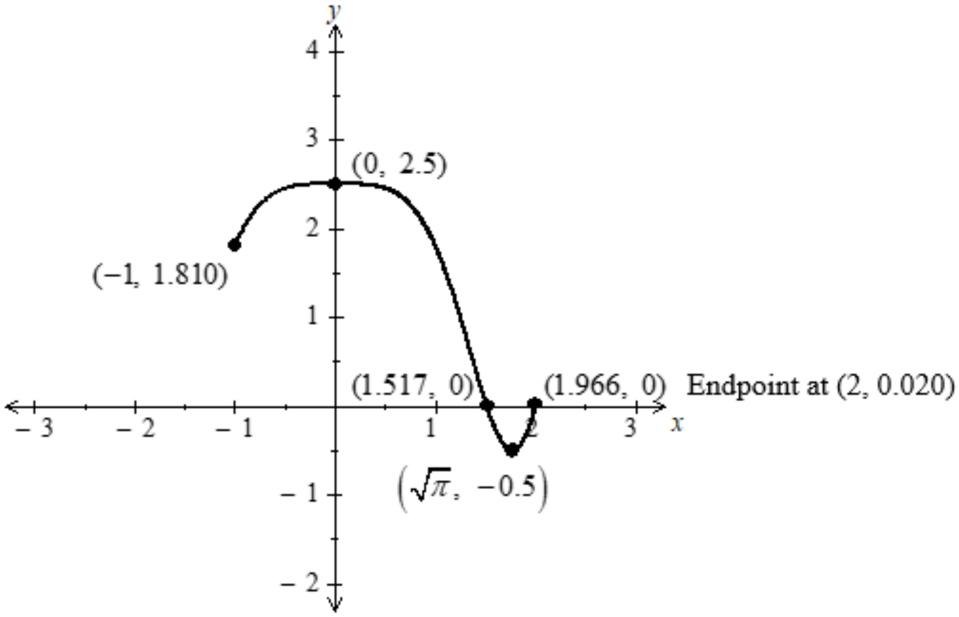
1e.	 The area of the minor sector bounded by the points $(0,\ \pm\sqrt{2})$ and $(-1,0)$ is $\frac{1}{2}r^2(2\theta)$ where $\theta = \arctan\left(\sqrt{2}\right)$ and $r^2=3$: $A_{sector} = 3\arctan\left(\sqrt{2}\right)$. $A_{triangle} = \sqrt{2}$. Therefore $A_{segment} = 3\arctan\left(\sqrt{2}\right)-\sqrt{2}$. $\cong 1.4517$.	**1 mark** **1 mark** **1 mark**
2a.	A unit vector normal to the plane Π_1 is given by $\hat{\underset{\sim}{n}}_1 = \frac{1}{3}\left(2\underset{\sim}{i}+2\underset{\sim}{j}-\underset{\sim}{k}\right)$. Q is a point on the plane: $\overrightarrow{PQ} = \overrightarrow{OQ}-\overrightarrow{OP} = (x-1)\underset{\sim}{i}+(y+1)\underset{\sim}{j}+(z-2)\underset{\sim}{k}$. Distance from $P = \left\|\overrightarrow{PQ}.\hat{\underset{\sim}{n}}_1\right\| = \frac{1}{3}\|2x-2+2y+2-z+2\|$. Substitute $2x+2y-z=3$: $\left\|\overrightarrow{PQ}.\hat{\underset{\sim}{n}}_1\right\| = \frac{1}{3}\|3+2\|=\frac{5}{3}$. **Answer:** $\frac{5}{3}$. Alternative: Substitute the point $\underbrace{(1,\ -1,\ 2)}_{(x_0,\,y_0,\,z_0)}$ and the Cartesian equation $\underbrace{2x+2y-z=3}_{ax+by+cz=d}$ into the formula $D=\frac{\|ax_0+by_0+cz_0-d\|}{\sqrt{a^2+b^2+c^2}}$.	**1 mark** **1 mark** **1 mark**
2b.	Angle between planes = angle between normals. A unit vector normal to the plane Π_2 is given by $\hat{\underset{\sim}{n}}_2 = \frac{1}{\sqrt{3}}\left(\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{k}\right)$. $\hat{\underset{\sim}{n}}_1.\hat{\underset{\sim}{n}}_2 = \frac{2-2-1}{3\sqrt{3}} = \frac{-1}{3\sqrt{3}} \quad \Rightarrow \theta = \arccos\left(\frac{-1}{3\sqrt{3}}\right)$. **Answer:** 101.1°	**1 mark** **1 mark**

2c.	Choose any three points which satisfy $2x+2y-z=3$. For example: $A(0,\ 0,\ -3)$, $B(0,\ -1,\ -1)$ and $C(2,\ 0,\ 1)$. $\overrightarrow{AB}=-\underset{\sim}{j}+2\underset{\sim}{k}$, $\overrightarrow{AC}=2\underset{\sim}{i}+2\underset{\sim}{k}$. So a vector equation of the plane $\prod_1$ is: $\underset{\sim}{r}=\overrightarrow{OA}+s\,\overrightarrow{AB}+t\,\overrightarrow{AC}$, $s,t\in R$. **Answer:** $\underset{\sim}{r}=s\left(-\underset{\sim}{j}+2\underset{\sim}{k}\right)+t\left(2\underset{\sim}{i}+2\underset{\sim}{k}\right)-3\underset{\sim}{k}$, $s,t\in R$. This answer is not unique since any three points in $\prod_1$ can be chosen. Full marks should be given if the above process is correctly followed.	**1 mark** **1 mark**
2d.	Use a CAS to solve $\prod_1$: $2x+2y-z=3$. (1) $\prod_2$: $x-y+z=3$. (2) simultaneously: $x=\lambda$, $y=6-3\lambda$, $z=9-4\lambda$, $\lambda\in R$. This solution gives the parametric equations of the required line. The equation of the line must be expressed in vector form. **Answer:** $\underset{\sim}{r}=6\underset{\sim}{j}+9\underset{\sim}{k}+\lambda\left(\underset{\sim}{i}-3\underset{\sim}{j}-4\underset{\sim}{k}\right)$. This answer is not unique since the parametric solution of equations (1) and (2) is not unique. For example, an equivalent solution is $x=\dfrac{6-\beta}{3}$, $y=\beta$, $z=\dfrac{3+4\beta}{3}$, $\beta\in R$. Full marks should be given if the above process is correctly followed. Solving equations (1) and (2) 'by-hand: Equation (1) + equation (2): $3x+y=6$. Let $x=\lambda$, $\lambda\in R$: $y=6-3\lambda$. Substitute into $y=6-3\lambda$ into $2x+2y-z=3$: $2\lambda+12-6\lambda-z=3$ $\Rightarrow z=9-4\lambda$.	**1 mark** **1 mark**

3ai.	It is required that the $\underset{\sim}{\mathrm{k}}$-component of $\underset{\sim}{r}$ is equal to 1.5. Use a CAS to solve $2-\cos^2(2\pi t)=1.5$ (use a domain for which only the first positive solution is given). **Answer:** $t=\frac{1}{8}$ hours.	**1 mark**
3aii.	$\underset{\sim}{r}\left(\frac{1}{8}\right)-\underset{\sim}{r}(0)=\frac{3}{2}\underset{\sim}{\mathrm{i}}-\frac{5}{8}\underset{\sim}{\mathrm{j}}+\frac{1}{2}\underset{\sim}{\mathrm{k}}$. Use a CAS to calculate $D=\left\lvert\frac{3}{2}\underset{\sim}{\mathrm{i}}-\frac{5}{8}\underset{\sim}{\mathrm{j}}+\frac{1}{2}\underset{\sim}{\mathrm{k}}\right\rvert$. **Answer:** $\frac{\sqrt{185}}{8}$ km.	**1 mark** **1 mark**
3b.	$\underset{\sim}{v}=\frac{d\underset{\sim}{r}}{dt}=12\underset{\sim}{\mathrm{i}}-5\underset{\sim}{\mathrm{j}}+2\pi\sin(4\pi t)\underset{\sim}{\mathrm{k}}$. $v(t)=\sqrt{169+4\pi^2\sin^2 4\pi t}$. By inspection, the maximum speed occurs when $\sin(4\pi t)=1 \Rightarrow t=\frac{1}{8}$ hour. **Answer:** $t=\frac{1}{8}$. $v\left(\frac{1}{8}\right)=\sqrt{169+4\pi^2} \quad \approx 14.44$ km/hour. **Answer:** 14.44.	**1 mark** **1 mark** **1 mark**
3ci.	When the repair person takes off the position vector of the balloon is $\underset{\sim}{r}(t)=12(t+1)\underset{\sim}{\mathrm{i}}-5(t+1)\underset{\sim}{\mathrm{j}}+\left(2-\cos^2 2\pi(t+1)\right)\underset{\sim}{\mathrm{k}},\ t\geq 0$. Equate the $\underset{\sim}{\mathrm{k}}$-components of the position vectors of balloon and repair person: $2-\cos^2\left(2\pi(t+1)\right)=20t-\frac{1}{2}gt^2 \quad =20t-4.9t^2$. Use a CAS to solve for t (use a domain for which only the first positive solution is given). **Answer:** $t\approx 0.0569$ hours $=205$ seconds.	**1 mark** **1 mark**

3cii.	Equate the $\underset{\sim}{j}$-components of the position vectors of balloon and repair person when $t \approx 0.0569$ and use a CAS to solve for *b*: **Answer:** $b \approx -5.285$. Equate the $\underset{\sim}{i}$-components of the position vectors of balloon and repair person when $t \approx 0.0569$ and use a CAS to solve for a: **Answer:** $a \approx 10.683$.	**1 mark** **1 mark**
3d.	$a = g - 0.8v^2 = v\dfrac{dv}{dx}$ $\Rightarrow \dfrac{dx}{dv} = \dfrac{v}{9.8 - 0.8v^2}$. The integral solution is $x = \int_{10}^{5} \dfrac{v}{9.8 - 0.8v^2}\,dv + 0$. Evaluate using a CAS: $x \approx 1.2056$ meters. **Answer:** 1.21 meters $= 121$ cm.	**1 mark** **1 mark** **1 mark**
3e.	As acceleration approaches zero, the speed approaches its limiting value: $0 = 9.8 - 0.8v^2$. Use a CAS to solve for *v* (positive solution required). **Answer:** 3.5 ms^{-1}.	**1 mark** **1 mark**

<table>
<tr>
<td>4a.</td>
<td>$y=-\int 3x\sin\left(x^2\right)\,dx$. Substitute $u=x^2 \Rightarrow dx=\frac{du}{2x}$:
$y=-\int 3x\sin(u)\frac{du}{2x} \quad =-\frac{3}{2}\int \sin(u)\,du \quad =\frac{3}{2}\cos(u)+c$
$=\frac{3}{2}\cos\left(x^2\right)+c$.
Substitute $y=2.5$ when $x=0$: $2.5=1.5+c \quad \Rightarrow c=1$.</td>
<td>1 mark
All working is required (because this is a “Show …” question)
1 mark</td>
</tr>
<tr>
<td>4b.</td>
<td>Stationary points: Use a CAS to solve $\frac{dy}{dx}=0,\ x\in[-1,2]$:
$\frac{dy}{dx}=-3x\sin\left(x^2\right)=0 \quad \Rightarrow x=0$ and $x=\sqrt{\pi}$.
Evidence of testing the nature of the stationary points (sign test or double derivative test).
Answer: (0, 2.5) and $\left(\sqrt{\pi},\ -0.5\right)$.</td>
<td>1 mark
1 mark</td>
</tr>
<tr>
<td>4c.</td>
<td>Use a CAS to find potential points of inflection by solving
$\frac{d^2y}{dx^2}=3\sin(x^2)+6x^2\cos(x^2)=0,\ x\in[-1,2]$:
$x=0$ and $x\approx 1.355$.
$\frac{d^2y}{dx^2}=0$ is a necessary but not sufficient condition. Change in concavity must be investigated by investigating the change in sign of $\frac{d^2y}{dx^2}$ (use a CAS).
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td><td>1.355</td><td>2</td></tr>
<tr><td>$\frac{d^2y}{dx^2}$</td><td>5.766</td><td>0</td><td>5.766</td><td>0</td><td>−17.958</td></tr>
</table>
There is no point of inflection at $x=0$ because the sign of $\frac{d^2y}{dx^2}$ does not change therefore there is no change in concavity.
There is a point of inflection at $x\approx 1.355$ because the sign of $\frac{d^2y}{dx^2}$ changes therefore there is a change in concavity.
Answer: (1.355, 0.605).</td>
<td>1 mark
Evidence that change in concavity is tested.
1 mark</td>
</tr>
</table>

4d.		**1 mark** Correct shape **2 marks** Correct endpoints Deduct 1 mark for any missing coordinates of intercepts

5a.	$y^2 = t$ and $(a-x)^3 = t$ therefore $y = (a-x)^{\frac{3}{2}}$. Domain: $a - x \ge 0 \quad \Rightarrow x \le a$.	**1 mark** **1 mark**
5b.	Use a CAS: $\frac{d^2y}{dx^2} = \frac{3}{4\sqrt{a-x}}$ therefore the second derivative can never be zero.	**1 mark**
5c.	$V = \pi\int_0^a y^2 dx = \pi\int_0^a (a-x)^3 dx$. Evaluate the integral using a CAS. **Answer:** $\frac{\pi a^4}{4}$.	**1 mark** **1 mark**
5d.	It is possible but inconvenient to use the parametric equations. It is easier to use the Cartesian equation. $t = 0 \Rightarrow x = 3$. $\quad t = 8 \Rightarrow x = 1$. $A = 2\pi\int_1^3 \sqrt{1+\left(\frac{dy}{dx}\right)^2} dx$. Use a CAS: $\frac{dy}{dx} = -\frac{3}{2}\sqrt{3-x} \quad \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{9}{4}(3-x)$. $A = 2\pi\int_1^3 \sqrt{1+\frac{9}{4}(3-x)} dx$. Evaluate the integral using a CAS. **Answer:** 22.15.	**1 mark** **1 mark** **1 mark**

6a.	H_0: Average efficiency of solar panels is 40% H_1: Average efficiency of solar panels is less that 40%	**1 mark**
6b.	$\overline{X} \sim \text{Normal}\left(\mu_{\overline{X}} = 40,\ \sigma_{\overline{X}} = \frac{6}{\sqrt{40}}\right)$ under H_0. Use a CAS to evaluate $p = \Pr\left(\overline{X} \le 38\right)$. **Answer:** 0.0175.	**1 mark**
6c.	The probability of a type 1 error is the level of significance (the probability of rejecting H_0 when it is true). **Answer:** 0.05.	**1 mark**
6d.	0.0175 (p value) < 0.05 (level of significance) therefore reject H_0.	**1 mark**
6e.	$\overline{Y} \sim \text{Normal}\left(\mu_{\overline{Y}} = 9.5,\ \sigma_{\overline{Y}} = \frac{1.5}{\sqrt{25}}\right)$ under H_0. Let c_1^* be the minimum value of the sample mean for H_0 to be rejected. $\Pr\left(\overline{Y} \le c_1^*\right) = 0.025$. Use a CAS to do the inverse normal calculation: $c_1^* = 8.65789$. **Answer:** 8.66.	**1 mark**
	Let c_2^* be the maximum value of the sample mean for H_0 to be rejected. $\Pr\left(\overline{Y} \ge c_2^*\right) = 0.025$. Use a CAS to do the inverse normal calculation: $c_2^* = 10.34211$. **Answer:** 10.34. **Note:** $c_2^* = 9.5 + \left(9.5 - c_1^*\right) = 10.34$ (by symmetry of the normal distribution).	**1 mark**

6f.	The probability of a type 2 error is the probability of **not** rejecting H_0 when it is false,	**1 mark**
	and is therefore equal to $\Pr\left(c_1^* \le \overline{Y} \le c_2^* \mid \mu = 9\right)$.	**1 mark**
	Substitute the values of c_1^* and c_2^* from **part e.**:	
	$\Pr\left(8.65789 \le \overline{Y} \le 10.34211 \mid \mu = 9\right)$. (Final answer is consequential)	
	Use a CAS to evaluate.	
	Answer: 0.8729.	**1 mark**