The Mathematical Association of Victoria

Trial Exam 2023

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 18 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the space provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2} where $g = 9.8$.

Question 1 (3 marks)

a. Find the vector (cross) product of the vectors $2\underset{\sim}{i} - \underset{\sim}{k}$ and $\underset{\sim}{j} + 2\underset{\sim}{k}$. 2 marks

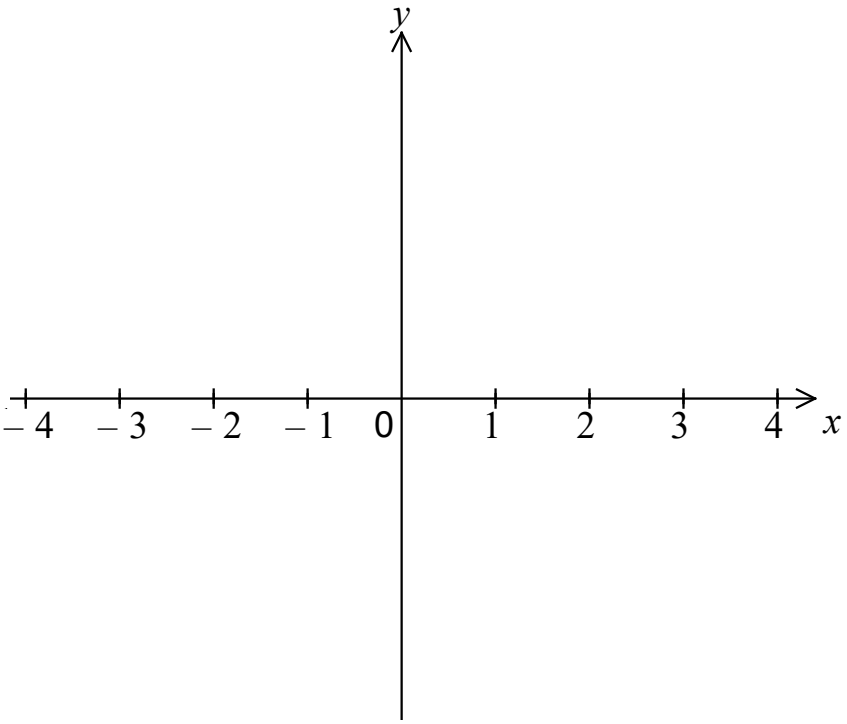
The vectors $2\underset{\sim}{i} - \underset{\sim}{k}$ and $\underset{\sim}{j} + 2\underset{\sim}{k}$ are parallel to a plane that contains the point $(3,\ 2,\ -1)$.

b. Find a cartesian equation of this plane. 1 mark

TURN OVER

Question 2 (4 marks)

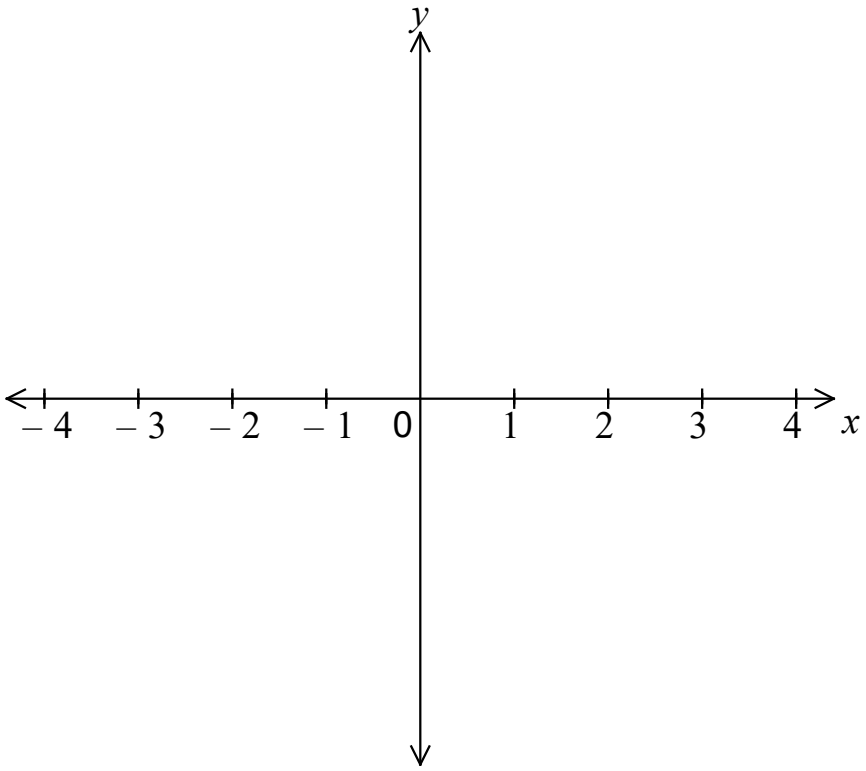
a. Sketch the graph of $y = \dfrac{x^3 + 1}{x^2 - 4}$ on the axes provided below, labelling any asymptotes with their equations and any axial intercepts with their coordinates. You can assume that there are turning points at the points where $x = 0$ and $x \approx -0.314$. 3 marks



Working space

Question 2 cont'd

b. Hence sketch the graph of $y = \dfrac{\left|x^3 + 1\right|}{x^2 - 4}$ on the axes provided below, labelling any asymptotes with their equations and any axial intercepts with their coordinates. 1 mark



Working space

TURN OVER

Question 3 (4 marks)

a. Find the square roots of $1-i\sqrt{3}$. Give your answers in the form $a+ib$ where $a,b \in R$. 2 marks

Question 3 cont'd

b. Hence find all solutions to $4z^3 + 4z^2 = -i\sqrt{3}z$, where $z \in C$. Give your answers in the form $a + ib$ where $a, b \in R$. 2 marks

Question 4 (3 marks)

The *Gratuitous Citrus Farm* grows oranges. The mass, in grams, of the oranges is known to be normally distributed with a standard deviation of 5 grams. A bag of oranges is collected and used to calculate an approximate 95% confidence interval for the mean mass, measured in grams, of all oranges grown on the farm. The confidence interval was found to be (198.6, 201.4).

Find the total mass of oranges in the bag. Use $\Pr(-1.96 < Z < 1.96) = 0.95$ and give your answer correct to the nearest gram.

Question 5 (3 marks)

Use proof by induction to prove that $3^{2n}+7$ is divisible by 8 $\forall n \in N$.

TURN OVER

Question 6 (3 marks)

Find the area of the surface generated by rotating the curve $y = \frac{e^x + e^{-x}}{2}$, $x \in \left[0,\ \log_e(2)\right]$, about the x-axis.

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TURN OVER

Question 7 (6 marks)

Consider the function f with rule $f(x) = \sin^{-1}(2x-1) + \frac{\pi}{2}$.

a. State the maximal domain and range of f. 1 mark

b. State the coordinates of the point of inflection of f. 1 mark

c. Find a rule for f^{-1}, the inverse of f. 1 mark

Question 7 cont'd

d. The region bounded by the graph of $y = f(x)$ and the lines $x = 0$ and $y = \frac{\pi}{2}$ is rotated about the *y*-axis to form a solid of revolution. Find the volume of this solid. 3 marks

Question 8 (6 marks)

The velocity of an object at time t is given by $\underset{\sim}{v}(t) = \cos(t)\underset{\sim}{i} + \left(\sin(2t) - 1\right)\underset{\sim}{j}$, where $t \geq 0$.

The initial position of the object is $\underset{\sim}{i} + 2\underset{\sim}{j}$.

a. Find the position vector of the object. 2 marks

Question 8 cont'd

b. Find the first time at which the scalar (dot) product of the velocity and acceleration of the object is equal to $-\frac{1}{2}$. 4 marks

Question 9 (5 marks)

a. Find an anti-derivative of $x\arctan\left(\frac{1}{x}\right)$. 3 marks

Question 9 cont'd

b. The acceleration of an object moving in a straight line is given by $x\arctan\left(\frac{1}{x}\right)$ ms^{-2} where x is the displacement in meters of the object from an origin after t seconds.

Given that the speed of the object is $1\ \text{ms}^{-1}$ when $x=1$, find the speed of the object when $x=\sqrt{3}$.

Give your answer in the form $\sqrt{\sqrt{a}+\frac{\pi}{b}}$ where $a,b\in Z^{+}$. 2 marks

TURN OVER

Question 10 (3 marks)

Let $\sin(3\theta) = \sin(2\theta)$.

Find all possible values of $\cos(\theta)$, giving two of your answers in the form $\frac{1 \pm \sqrt{a}}{b}$ where $a, b \in Z^+$.

END OF QUESTION AND ANSWER BOOK