2023 VCE Specialist Mathematics Year 12 Trial Examination 1

Kilbaha Education

Quality educational content

Kilbaha Education **PO Box 2227** **Kew Vic 3101** **Australia**	**Tel: (03) 9018 5376** **kilbaha@gmail.com** **https://kilbaha.com.au**

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Victorian Certificate of Education
2023

STUDENT NUMBER

Figures										Letter
Words										

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 22 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (4 marks)

Solve the equation $z^4 + 16 = 0$, $z \in C$, giving your answers in rectangular $a + bi$ form.

Question 2 (3 marks)

Solve the differential equation $\frac{dy}{dx} = \frac{x\left(9+4y^2\right)}{6}$ given $y(0) = \frac{3}{2}$.

Express your answer in the form $y = f(x)$.

Question 3 (3 marks)

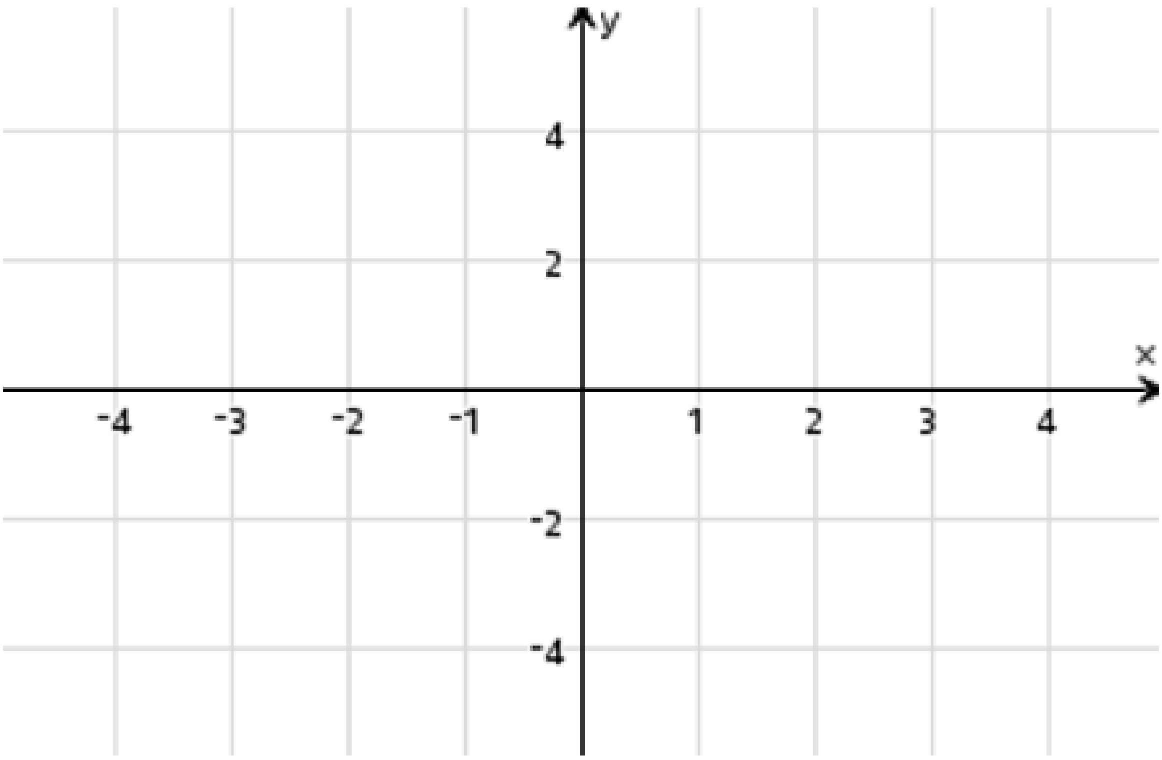
Let $f: D \to R,\ f(x) = \dfrac{x}{x^2 - 4}$, where D is the maximal domain of *f*.

i. Find $f'(x)$ and hence show that the graph of f has no turning points.

1 mark

ii. Given that $f''(x) = \dfrac{2x\left(x^2 + 12\right)}{\left(x^2 - 4\right)^3}$ find the coordinates of any points of inflexion and sketch the graph of $f(x) = \dfrac{x}{x^2 - 4}$ on the axes below, labelling the equations of all asymptotes.

2 marks



Question 4 (3 marks)

A particle moves in a straight line. At a time t seconds, the particle has a displacement of x metres and a velocity of $v \text{ ms}^{-1}$ and an acceleration $a \text{ ms}^{-2}$. Initially the particle is at the origin and has a velocity of 2 ms^{-1}. If the acceleration $a = -\frac{4}{3}e^{-\frac{2x}{3}}$ and the velocity of the particle is always positive.

a. Show that $v = 2e^{-\frac{x}{3}}$

1 mark

b. After three seconds, the particle has a displacement of $\log_e(s)$, find the value of s.

2 marks

Question 5 (3 marks)

The weights of a piece of corn are normally distributed with a mean of 100 gm with a variance of 12 gm^2. Z has the standard normal distribution and given that $\Pr(Z < 1.5) = 0.933$.

a. Determine the probability that the total weight of three independent pieces of corn will be between 291 and 300 gms. Give your answer correct to three decimal places.

2 marks

b. An **approximate** 95% confidence interval for *n* pieces of corn was found to be 99 to 101 gm. Given that $\Pr(-2 < Z < 2) = 0.95$, where Z is a standard normal random variable.

Determine the value of *n*.

1 mark

Question 6 (5 marks)

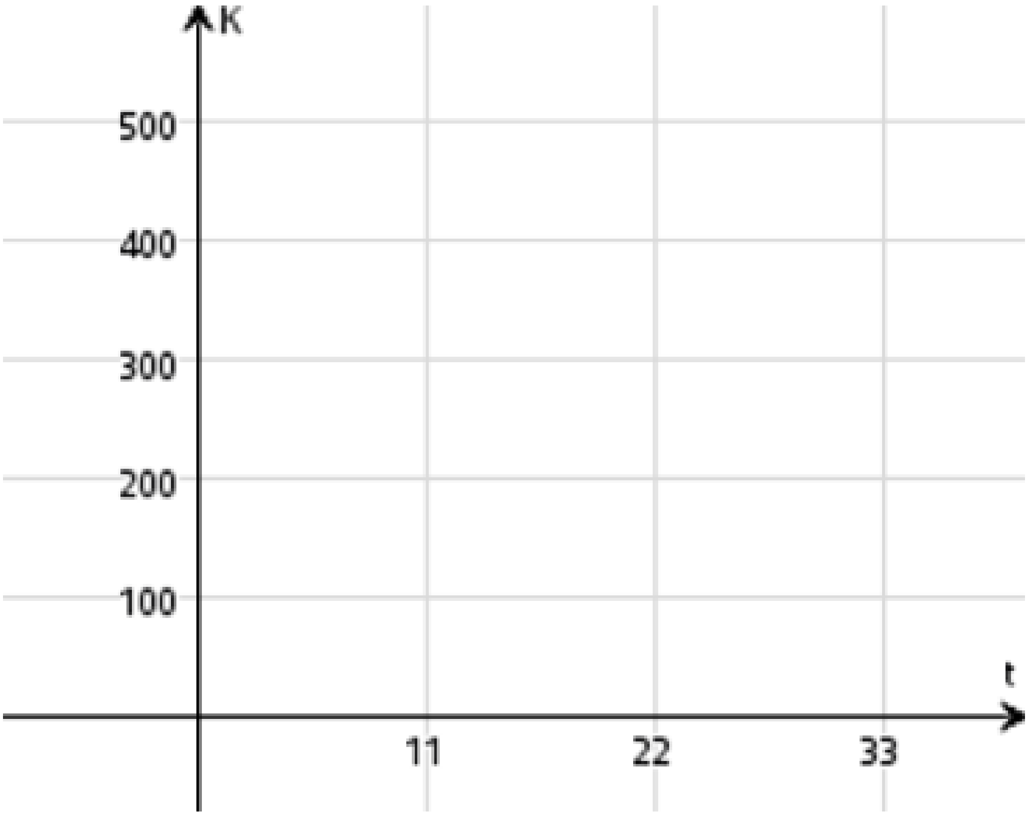
On a plantation in the year 2000 there were initially 50 kangaroos. The population growth of the number of kangaroos K on the plantation is given by $\frac{dK}{dt} = \frac{K}{5}\left(1 - \frac{K}{500}\right)$, where t is the time in years after 2000.

a. Solve the differential equation to find K in terms of t.

4 marks

b. It is known that the number of kangaroos on the plantation was increasing most rapidly around the year 2011. Sketch the graph of K versus t on the axes below, clearly labelling the point of inflexion, the equations of any asymptotes and coordinates of any axial intercepts.

1 mark



Question 7 (4 marks)

When the area bounded by the curve the curve $y=\sqrt{3x+4}$ the x-axis $x=0$ and $x=2$ is rotated about the x-axis it forms a volume of revolution, find the surface area of this volume of revolution.

Question 8 (4 marks)

Prove by induction that $5^{2n}+3n-1$ is divisible by 9 for $n \in N$.

Question 9 (6 marks)

A plane has the vector equation $\underset{\sim}{r}(s,t)=(3+4t+3s)\underset{\sim}{i}+(-2+t+2s)\underset{\sim}{j}+(1+2t-s)\underset{\sim}{k}$

a. Show that the equation of the plane can be written in the form $x-2y-z=6$.

2 marks

b. One line has the vector equation $\underset{\sim}{r}(t)=2\underset{\sim}{i}-\underset{\sim}{j}+z_0\underset{\sim}{k}+t\left(\underset{\sim}{i}-2\underset{\sim}{j}+c\underset{\sim}{k}\right)$, find the values of z_0 and c for which the line and the plane do not intersect.

2 marks

c. Another line has the equation $\frac{x-3}{2}=\frac{y+1}{b}=z-2$, this line intersects the plane making an angle of 30^0 with the plane. Determine the value(s) of *b*.

2 marks

Question 10 (5 marks)

Let $I_n = \int_0^1 x^{2n+1} e^{x^2}\, dx$ for integers $n \ge 0$.

a. Show that $I_0 = \frac{1}{2}(e-1)$

1 mark

b. Using integration by parts on I_n, show that $I_{n+1} = \frac{e}{2} - (n+1) I_n$

3 marks

c. Hence evaluate I_3.

1 mark

EXTRA WORKING SPACE

End of question and answer book for the
2023 Kilbaha VCE Specialist Mathematics Trial Examination 1

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SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics formulas

Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\,\text{cis}(\theta)$	$\|z\| = \sqrt{x^2 + y^2} = r$	
$-\pi < \text{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \text{cis}(n\theta)$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \text{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \frac{1}{2}(1 - \cos(2ax))$	$\cos^2(ax) = \frac{1}{2}(1 + \cos(2ax))$

Data analysis, probability and statistics

for independent random variables $X_1, X_2 .... X_n$	$E(aX_1+b)=aE(X_1)+b$ $E(a_1X_1+a_2X_2+...a_nX_n)$ $=a_1E(X_1)+a_2E(X_2)+...+a_nE(X_n)$	
	$\text{Var}(aX_1+b)=a^2\text{Var}(X_1)$ $\text{Var}(a_1X_1+a_2X_2+...a_nX_n)$ $=a_1^2\text{Var}(X_1)+a_2^2\text{Var}(X_2)+...+a_n^2\text{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2 .... X_n$	$E(X_1+X_2+...+X_n)=n\mu$	
	$\text{Var}(X_1+X_2+...+X_n)=n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x}-z\dfrac{s}{\sqrt{n}}, \bar{x}+z\dfrac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean	$E(\bar{X})=\mu$
	variance	$\text{Var}(\bar{X})=\dfrac{\sigma^2}{n}$

Vectors in two and three dimensions

$\underset{\sim}{r}(t)=x(t)\underset{\sim}{i}+y(t)\underset{\sim}{j}+z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert=\sqrt{x(t)^2+y(t)^2+z(t)^2}$
	$\dot{\underset{\sim}{r}}(t)=\dfrac{d\underset{\sim}{r}}{dt}=\dfrac{dx}{dt}\underset{\sim}{i}+\dfrac{dy}{dt}\underset{\sim}{j}+\dfrac{dz}{dt}\underset{\sim}{k}$
for $\underset{\sim}{r_1}=x_1\underset{\sim}{i}+y_1\underset{\sim}{j}+z_1\underset{\sim}{k}$ and $\underset{\sim}{r_2}=x_2\underset{\sim}{i}+y_2\underset{\sim}{j}+z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r_1}.\underset{\sim}{r_2}=\lvert r_1\rvert\lvert r_2\rvert\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$
	vector cross product $\underset{\sim}{r_1}\times\underset{\sim}{r_2}=\begin{vmatrix}\underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k}\\ x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\end{vmatrix}=(y_1z_2-y_2z_1)\underset{\sim}{i}+(x_2z_1-x_1z_2)\underset{\sim}{j}+(x_1y_2-x_2y_1)\underset{\sim}{k}$
vector equation of a line	$\underset{\sim}{r}(t)=\underset{\sim}{r_1}+t\underset{\sim}{r_2}=(x_1+x_2t)\underset{\sim}{i}+(y_1+y_2t)\underset{\sim}{j}+(z_1+z_2t)\underset{\sim}{k}$
parametric equation of line	$x(t)=x_1+x_2t \quad y(t)=y_1+y_2t \quad z(t)=z_1+z_2t$
vector equation of a plane	$\underset{\sim}{r}(s,t)=\underset{\sim}{r_0}+s\underset{\sim}{r_1}+t\underset{\sim}{r_2}$ $=(x_0+x_1s+x_2t)\underset{\sim}{i}+(y_0+y_1s+y_2t)\underset{\sim}{j}+(z_0+z_1s+z_2t)\underset{\sim}{k}$
parametric equation of a plane	$x(s,t)=x_0+x_1s+x_2t \quad y(s,t)=y_0+y_1s+y_2t \quad z(s,t)=z_0+z_1s+z_2t$
Cartesian equation of a plane	$ax+by+cz=d$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\left(\lvert x\rvert\right)+c$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$	$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$
$\frac{d}{dx}\left(\cot(ax)\right)=-a\,\text{cosec}^2(ax)$	$\int \text{cosec}^2(ax)\,dx=-\frac{1}{a}\cot(ax)+c$
$\frac{d}{dx}\left(\sec(ax)\right)=a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax)\,dx=\frac{1}{a}\sec(ax)+c$
$\frac{d}{dx}\left(\text{cosec}(ax)\right)=-a\,\text{cosec}(ax)\cot(ax)$	$\int \text{cosec}(ax)\cot(ax)\,dx=-\frac{1}{a}\text{cosec}(ax)+c$
$\frac{d}{dx}\left(\sin^{-1}(ax)\right)=\frac{1}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}(ax)\right)=\frac{-1}{\sqrt{1-(ax)^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}(ax)\right)=\frac{a}{1+(ax)^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1}\,dx=\frac{1}{a}\log_e\left(\lvert ax+b\rvert\right)+c$

Calculus- continued

product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$
Euler's method	If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf\left(x_n,y_n\right)$
arc length parametric	$\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about the x-axis	$\int_{x_1}^{x_2}2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about the y-axis	$\int_{y_1}^{y_2}2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about the x-axis	$\int_{t_1}^{t_2}2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about the y-axis	$\int_{t_1}^{t_2}2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics

acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v=u+at$	$s=ut+\frac{1}{2}t^2$
	$v^2=u^2+2as$	$s=\frac{1}{2}(u+v)t$

END OF FORMULA SHEET