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YEAR 12 *Trial Exam Paper*

2023

SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (3 marks)

Evaluate $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \left(\sin(x) + \cos(x)\right)^2 dx$

Question 2 (6 marks)

a. Two position vectors relative to a fixed origin O (0, 0, 0) are given by $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k}$.

Find a unit vector perpendicular to both $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

2 marks

b. Find the Cartesian equation of the plane containing vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and O (0,0,0).

2 marks

c. Find the distance between the plane containing vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ and the point P (–2, 2, 1).

2 marks

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Question 3 (4 marks)

The rate of change of the area, A, of an oil spill measured in square metres t seconds after the spill was formed is described by $\frac{dA}{dt} = \frac{A^{\frac{3}{2}}}{t^2}, t > 0$.

a. Find a solution to the above differential equation giving your answer in the form $A = f(t)$ given that $A = \frac{1}{4}\text{m}^2$ one second after the spill was formed.

3 marks

b. Find the limiting value of A as t becomes very large.

1 mark

Question 4 (4 marks)

Solve the equation $2z^2 + \bar{z}^2 = 27 - 2\sqrt{10}i$ for $z \in \mathbb{C}$.
Give your answers in the form $z = a + bi,\ a,\ b \in \mathbb{R}.$.

Question 5 (4 marks)

Find the equation of the line normal to the curve described by $x^2 + (y - x)^3 = 9$ at the point where $x = 1$.

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Question 6 (4 marks)

The curve given by $y = x^3$ is rotated about the x-axis to form a solid of revolution. Find the surface area of the part of this solid where $y \in \left[0, \frac{\sqrt{3}}{9}\right]$.

Question 7 (3 marks)

A machine makes a serving of lime cordial by combining 7 units of water and 4 units of syrup. A unit of water is normally distributed with a mean of 25 mL and standard deviation of 1 mL. A unit of syrup is normally distributed with a mean of 5 mL and a standard deviation of 1.5 mL.

a. Find the mean and standard deviation for the volume of one serving of lime cordial.

2 marks

b. Find the probability that the machine dispenses a serving of cordial with a volume more than 202.84 mL. Give your answer correct to 3 decimal places.

1 mark

Question 8 (4 marks)

Two jets, A and B, are observed from a control tower at O (0, 0, 0). Relative to O, their position vectors at time t hours after midday are given by

$\underset{\sim}{r}_A(t) = (2+2t)\underset{\sim}{i} + (9+3t)\underset{\sim}{j} + (15+6t)\underset{\sim}{k}$ and $\underset{\sim}{r}_B(t) = (1+t)\underset{\sim}{i} + (2+4t)\underset{\sim}{j} + (2+8t)\underset{\sim}{k}$ where displacements are measured in kilometres.

a. Find the distance between jets A and B at 3 pm.

2 marks

b. Find the time after midday when the distance between jet B and an observation tower located at T (20, 20, 0) is a minimum.

2 marks

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Question 9 (3 marks)

Prove by mathematical induction that the number $4^n + 6n - 1$ is divisible by 3 for all $n \in \mathbb{N}$.

Question 10 (5 marks)

Find the area bound by $f(x) = e^x \sin(x)$ and the x-axis for $x \in [0, \pi]$.

END OF QUESTION AND ANSWER BOOKLET