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SPECIALIST MATHEMATICS UNITS 3 & 4

TRIAL EXAMINATION 2

2023

Reading Time: 15 minutes
Writing time: 2 hours

Instructions to students

This exam consists of Section A and Section B.
Section A consists of 20 multiple-choice questions and should be answered on the detachable answer sheet which can be found on page 25 of this exam.
Section B consists of 6 extended-answer questions.
Section A begins on page 2 of this exam and is worth 20 marks.
Section B begins on page 10 of this exam and is worth 60 marks.
There is a total of 80 marks available.
All questions in Section A and B should be answered.
In Section B, where more than one mark is allocated to a question, appropriate working must be shown.
An exact value is required to a question unless otherwise directed.
Unless otherwise stated, diagrams in this exam are not drawn to scale.
Students may bring one bound reference into the exam.
Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.
A formula sheet can be found at the end of this exam.

SECTION A – Multiple-choice questions

Question 1

Consider the following statement.

‘If a number is a multiple of 6, then it is also a multiple of 3.’

Which one of the following is the contrapositive of this statement?

A. If a number is a multiple of 3, then it is also a multiple of 6.
B. There exists a number that is a multiple of 6 but not a multiple of 3.
C. If a number is not a multiple of 6, then it is not a multiple of 3.
D. If a number is not a multiple of 3, then it is not a multiple of 6.
E. There exists a number that is a multiple of 3 but not a multiple of 6.

Question 2

The procedure below has been written in pseudocode.

```
declare integer a
declare integer b
declare integer c
declare integer n
set n to 1
set a to 3
set b to 2
set c to 0
for n from 1 to 3
      c = a×b
      b = c
      a = a + 1
      print c
end for
```

The output of the pseudocode is which of the following lists of numbers?

A. 6, 24, 120
B. 0, 6, 24
C. 0, 6, 24, 120
D. 120
E. 6, 18, 54

Question 3

The maximal domain and the range of the function $f(x)=\cos^{-1}\left(\frac{2}{|x|}\right)$ are respectively

A. $R\backslash(-2,2)$ and $\left[0,\frac{\pi}{2}\right)$

B. $R\backslash[-2,2]$ and $\left(0,\frac{\pi}{2}\right)$

C. $[-2,2]$ and $\left[0,\frac{\pi}{2}\right)$

D. $[-2,2]$ and $[0,\pi]$

E. $[-1,1]$ and $[0,\pi]$

Question 4

Let $\cos(\alpha)=a$, where $\alpha\in\left(\frac{\pi}{2},\pi\right)$ and $a\in R$.

If $\cos(x)=-a$, where $x\in[0,2\pi]$, then the value(s) of x in terms of α, is/are

A. $x=\pi-\alpha$ only
B. $x=\pi-\alpha$ and $x=\pi+\alpha$
C. $x=2\pi-\alpha$ only
D. $x=2\pi-\alpha$ and $x=\alpha$
E. $x=\alpha$ only

Question 5

Consider the vector $\underset{\sim}{b}=2\underset{\sim}{i}-\underset{\sim}{j}+2\underset{\sim}{k}$. If the vector resolute of vector $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$ is equal to $2\underset{\sim}{b}$, then vector $\underset{\sim}{a}$ could be

A. $\underset{\sim}{a}=5\underset{\sim}{i}-4\underset{\sim}{j}+6\underset{\sim}{k}$

B. $\underset{\sim}{a}=5\underset{\sim}{i}+4\underset{\sim}{j}+6\underset{\sim}{k}$

C. $\underset{\sim}{a}=\underset{\sim}{i}+\frac{1}{2}\underset{\sim}{j}+\frac{\sqrt{3}}{2}\underset{\sim}{k}$

D. $\underset{\sim}{a}=\underset{\sim}{i}+\underset{\sim}{j}+2\underset{\sim}{k}$

E. $\underset{\sim}{a}=3\underset{\sim}{i}+4\underset{\sim}{j}+6\underset{\sim}{k}$

Question 6

Consider the triangle *OAB*, where $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$.

If *P* is the point on *AB* such that $\overrightarrow{AP} = m\,\overrightarrow{PB}$, where $m \in R$, then the value of *m* such that the position vector of *P* is $\frac{1}{5}\left(\underset{\sim}{a} + 4\underset{\sim}{b}\right)$ is

A. $m = -5$
B. $m = -4$
C. $m = 1$
D. $m = 4$
E. $m = 5$

Question 7

Let $z = a + bi$ be a complex number, where $a, b \in R$. Which one of the following is not real?

A. $\text{Re}(z) \times \text{Im}(z)$
B. $z + \overline{z}$
C. $z\overline{z}$
D. $\text{Im}(z) \times i^8$
E. $\text{Re}(\overline{z}) \times \overline{z}$

Question 8

If $z = \text{cis}(\theta)$, where $\theta \neq \pi$, then $\text{Arg}(z+1)$ is

A. $\frac{\theta}{3}$
B. $\frac{\theta}{2}$
C. θ
D. 2θ
E. 3θ

Question 9

The graph given by $|z-3+i|=|z+2+2i|,\ z\in C,$ is

A. a circle with centre $(3,-1)$ and radius of 2.

B. a straight line through the points $(3,-1)$ and $(-2,-2)$.

C. a straight line which is the perpendicular bisector of the points $(-3,1)$ and $(2,2)$.

D. a straight line which is the perpendicular bisector of the points $(3,-1)$ and $(-2,-2)$.

E. a straight line through the points $(-3,1)$ and $(2,2)$.

Question 10

Consider the following two planes,

$$\Pi_1 \text{ given by } 2x+3y+z=8$$
$$\text{and } \Pi_2 \text{ given by } x+5y-3z=4.$$

The angle between the planes Π_1 and Π_2, correct to two decimal places, is

A. 25.38°
B. 39.23°
C. 50.77°
D. 64.62°
E. 74.97°

Question 11

A plane is perpendicular to the line $\underset{\sim}{r}(t)=\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}+t(2\underset{\sim}{i}+3\underset{\sim}{j}+\underset{\sim}{k})$ and contains the point $(-2,1,4)$. A Cartesian equation of this plane could be

A. $2x+3y+z=3$
B. $2x+3y+z=10$
C. $x+y-z=3$
D. $x+y-z=10$
E. $-2x+y+4z=0$

Question 12

The graph of $y = f(x)$, where $f(x) = \dfrac{x^2 - 3x + 2}{x - a}$ and $a \in R$, will have **no** points of inflection when

A. $a = 1$ only.
B. $a = 2$ only.
C. $a = 1$ or $a = 2$ only.
D. $a = -1$ or $a = -2$ only.
E. $a \in R$.

Question 13

The gradient of the tangent to a curve at the point *A*(*x*, *y*) is half the gradient of the line joining point *A* and the point *B*(2, 3). The coordinates of the point *A* satisfy the differential equation

A. $\dfrac{dy}{dx} - \dfrac{y-3}{2(x-2)} = 0$

B. $\dfrac{dy}{dx} - \dfrac{2(y-3)}{x-2} = 0$

C. $\dfrac{dy}{dx} - \dfrac{2(x-2)}{y-3} = 0$

D. $\dfrac{dy}{dx} - \dfrac{x-2}{2(y-3)} = 0$

E. $\dfrac{dy}{dx} = \dfrac{y-3}{x-2}$

Question 14

The acceleration of a particle moving in a straight line, in ms^{-2}, is given by $a = \sin^{-1}\left(\dfrac{x}{2}\right)$ where *x*, in metres, is the position of the particle from a fixed origin *O*. If the particle has a velocity of $2\ \text{ms}^{-1}$ at the origin, then its speed when it is 1 metre to the right of the origin, correct to two decimal places, is

A. $2.12\ \text{ms}^{-1}$
B. $2.36\ \text{ms}^{-1}$
C. $2.78\ \text{ms}^{-1}$
D. $4.51\ \text{ms}^{-1}$
E. $5.56\ \text{ms}^{-1}$

Question 15

Using a suitable substitution, $\int_1^3 (x-2)\sqrt{2x+1}\ dx$ can be expressed as

A. $\frac{1}{2}\int_3^7 (u-5)\sqrt{u}\ du$

B. $\frac{1}{4}\int_1^3 (u-5)\sqrt{u}\ du$

C. $\frac{1}{4}\int_3^7 (u-5)\sqrt{u}\ du$

D. $\frac{1}{4}\int_3^7 (u-2)\sqrt{u}\ du$

E. $\frac{1}{4}\int_1^3 (u-2)\sqrt{u}\ du$

Question 16

$\int \sin\left(\frac{7x}{2}\right)\sin\left(\frac{3x}{2}\right)\ dx$ is equivalent to

A. $\frac{1}{2}\int \sin(5x)-\sin(2x)\ dx$

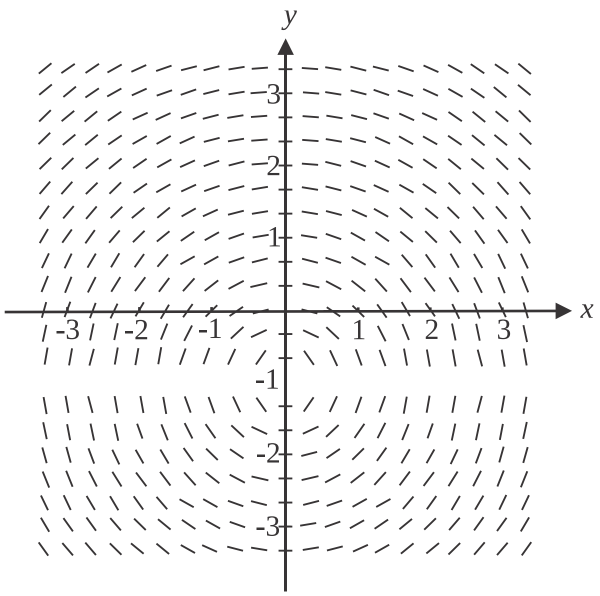
B. $\frac{1}{2}\int \cos(5x)-\cos(2x)\ dx$

C. $\int \cos(2x)-\cos(5x)\ dx$

D. $\frac{1}{2}\int \sin(2x)-\sin(5x)\ dx$

E. $\frac{1}{2}\int \cos(2x)-\cos(5x)\ dx$

Question 17



The differential equation that best represents the slope field shown above is

A. $\frac{dy}{dx} = \frac{x}{y+1}$

B. $\frac{dy}{dx} = \frac{y}{x+1}$

C. $\frac{dy}{dx} = \frac{-y}{x+1}$

D. $\frac{dy}{dx} = \frac{-x}{y+1}$

E. $\frac{dy}{dx} = \frac{x}{y}$

Question 18

A continuous random variable *X* has the probability density function *f* given by

$$f(x)=\begin{cases}\dfrac{12}{x^2} & 4\le x\le 6\\ 0 & \text{otherwise.}\end{cases}$$

If $Y=2X-1$, then $\Pr(Y<8)$ is closest to

A. 0.18
B. 0.33
C. 0.5
D. 0.6
E. 0.67

Question 19

The daily number of airline passengers who travel from Melbourne to Sydney is normally distributed with a standard deviation of 3000.
The daily number of passengers is collected for a random sample of 30 days and a *C*% confidence interval for the actual mean daily number is calculated.
If the sample mean of the daily number of passengers is within 1000 of the actual mean, the value of *C* is closest to

A. 1.8%
B. 46.7%
C. 93.2%
D. 95%
E. 98.2%

Question 20

A battery company claims that their Type A battery has an average lifetime of 1000 hours and a standard deviation of 100 hours.
Unconvinced of the company's claim, a consumer rights organisation conducts a one-tailed statistical test at the 1% level of significance and obtains a random sample of 50 Type A batteries.
If the actual average lifetime of a Type A battery is 950 hours, the probability of making a type II error is closest to

A. 0.01
B. 0.1133
C. 0.4321
D. 0.5
E. 0.8867

SECTION B

Question 1 (9 marks)

Consider the function $f: R \to R,\ f(x) = \dfrac{6x^2 + 4}{x^2 + 2}$.

a. State the equation(s) of any asymptotes of $f(x)$. 1 mark

b. State the coordinates of the stationary point of f. 1 mark

c. Determine the coordinates of any point(s) of inflection of f. 2 marks

d. The part of the graph of f for $x \in [0, a]$, where $a > 0$, is rotated about the x-axis to form a solid of revolution.
The volume of the solid formed is equal to 50π cubic units.
Find the value of a, correct to three decimal places. 2 marks

Consider the function $g : R \to R,\ g(x) = \dfrac{6x^2 + 4}{x^2 + b}$, where $b \in R \setminus \{0\}$.

e. Find the value(s) of b, such that

i. the stationary point of g is a local minimum. 1 mark

ii. the two points of inflection are a distance of three units apart. 2 marks

Question 2 (11 marks)

Let the complex number $z_1 = \sqrt{3} - i$.

a. If z_1 is one of the solutions to the equation $P(z) = 0$, where $P(z) = z^3 + az^2 + bz + 12$ and $a, b \in R$, find the values of a and b. 3 marks

b. Express z_1 in polar form. 1 mark

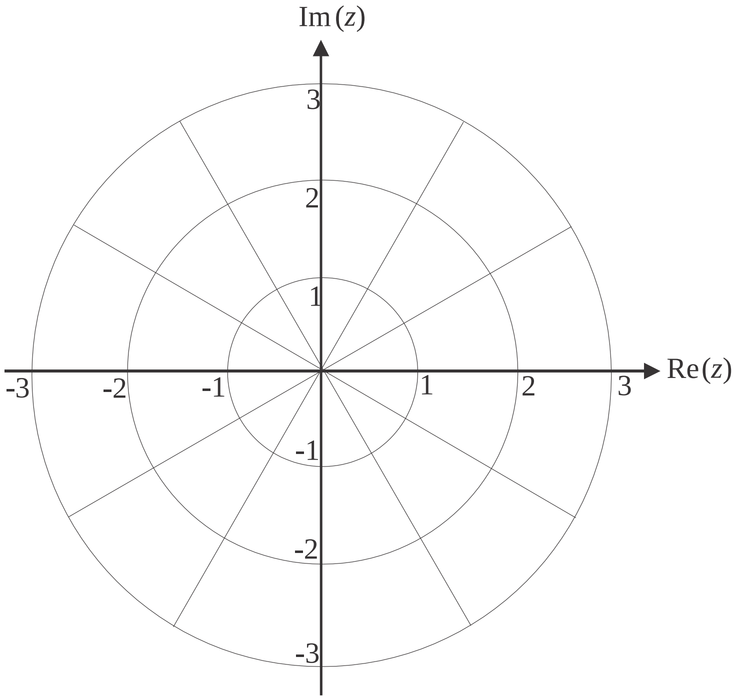
z_1 is also a solution to the equation $z^6 + w = 0$, $w \in R$.

c. Show that w = 64. 1 mark

d. The point representing z_1 lies on the circle $\left|z-\sqrt{3}\right|=1$.

Graph the circle $\left|z-\sqrt{3}\right|=1$ and the point z_1 on the Argand diagram below.

It is not necessary to label the axis intercepts of the graph. 2 marks



e. The line L passes through the point z_1 and through the solution to $z^6+w=0$ in the second quadrant.

Draw the line L on the Argand diagram in part **d.** above. 1 mark

f. The line L divides the circle into two segments.

Determine the area of the minor segment. 3 marks

Question 3 (9 marks)

A plane, Π_1, is described by the parametric equations

$$x = 2 + 3s + t$$
$$y = -1 + 2s + 3t$$
$$z = 3 - 4s - 2t$$

where $s, t \in R$.

a. Find a vector equation of Π_1 in the form $\underset{\sim}{r}(s,t) = \underset{\sim}{a} + s\underset{\sim}{u} + t\underset{\sim}{v}$. 1 mark

b. Hence show that a Cartesian equation of Π_1 is $8x + 2y + 7z = 35$. 2 marks

c. Find the coordinates of the point where the line L with equation $\underset{\sim}{r}(\lambda) = 2\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k} + \lambda(\underset{\sim}{i} - 5\underset{\sim}{j} + \underset{\sim}{k})$, $\lambda \in R$, intersects the plane Π_1. 2 marks

d. Find the angle between the line L and the plane Π_1, in degrees, correct to two decimal places. 2 marks

e. A second plane, Π_2, is parallel to Π_1. The two planes are a distance of 2 units apart. Find an equation of the plane Π_2. 2 marks

Question 4 (10 marks)

In a small forest, a native animal population is being studied. The number of animals in the population, P, t days after the study begins, is modelled by the logistic differential equation

$$\frac{dP}{dt} = 0.05P\left(1 - \frac{P}{2000}\right) \quad \text{where } P(0) = 20.$$

a. If $P = 20$ when $t = 0$, use Euler's method with a step size of 1 to estimate the value of P when $t = 2$. Give your answer correct to the nearest whole number. 2 marks

b. Show that the rule for the population of animals after t days is given by

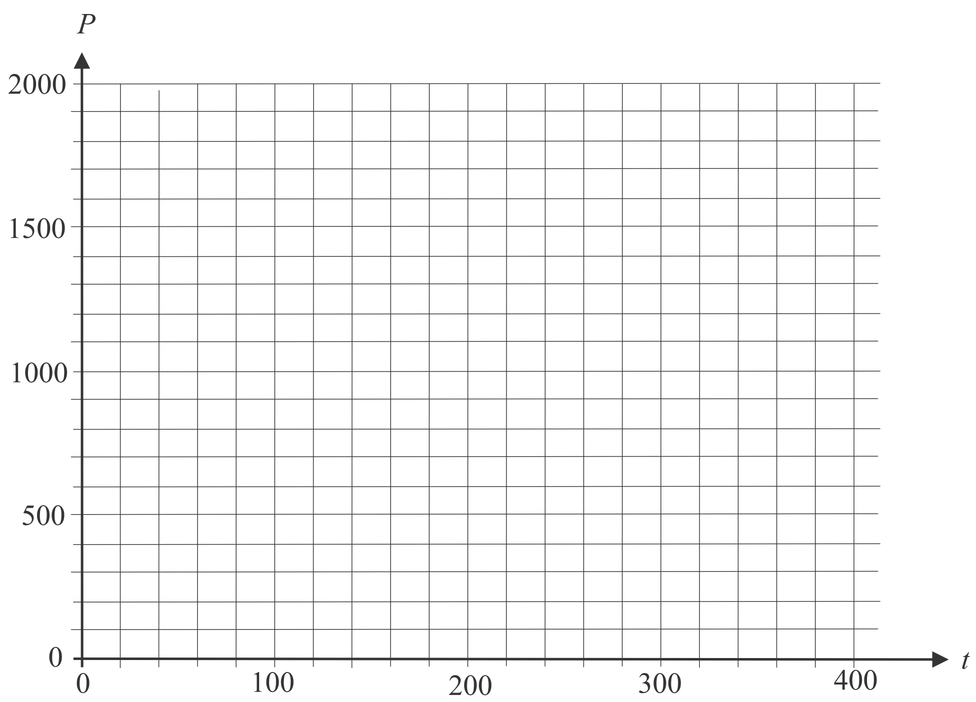
$$P = \frac{2000e^{\frac{t}{20}}}{99 + e^{\frac{t}{20}}}.$$

3 marks

c. How many animals are in the forest after 100 days? Give your answer correct to the nearest whole number. 1 mark

d. On the set of axes below, sketch the graph of P against t, labelling any axis intercepts with coordinates and any asymptotes with their equation.

2 marks



e. After how many days is the population increasing most rapidly? Give your answer correct to the nearest whole number. 2 marks

Question 5 (10 marks)

A rabbit in a forest is tracked over a ten second time period. Its position, in metres, relative to a fixed origin *O*, is given by

$$\underset{\sim}{r}(t) = (10 - 2t)\underset{\sim}{i} + (-t^2 + 6t - 8)\underset{\sim}{j} \qquad \text{for } 0 \le t \le 10$$

a. **i.** Find the Cartesian equation of the path of the rabbit. 1 mark

ii. State the domain of the relation found in part **a.i.** 1 mark

b. Find the distance the rabbit travels in the 10 second period. Give your answer in metres and correct to two decimal places. 2 marks

c. Find the speed of the rabbit at the point when it is closest to the origin. Give your answer in metres per second and correct to two decimal places. 2 marks

A fox is also being tracked in the forest over the same 10 second period. Its position relative to the origin *O* is given by

$$\underset{\sim}{f}(t) = (10 - at^2)\underset{\sim}{i} + (bt - 10)\underset{\sim}{j} \qquad 0 \le t \le 10 \text{ and } a, b \in R.$$

d. State the values of *a* and *b*, such that the rabbit and the fox collide after four seconds. 1 mark

e. If $b = 0$, state the possible values of *a* such that the paths of the rabbit and the fox intersect. 3 marks

Question 6 (11 marks)

A company manufactures plastic pop-up sprinkler sprays used in irrigation systems.
The height of these sprays is normally distributed with a mean of 75 mm and a standard deviation of 4 mm.
A random sample of 20 sprays is taken.

a. Find the probability that the mean height of this random sample of 20 sprays is between 73 mm and 77 mm. Give your answer correct to three decimal places. 2 marks

b. Four independent random samples of 20 sprays are now taken. Find the probability that more than two of these four samples have a mean height between 73 mm and 77 mm. Give your answer correct to three decimal places. 1 mark

To protect these sprays during transport, a cap is attached to the top and to the bottom of each spray.
The height of a cap is independent of the height of a spray and is normally distributed with a mean of 10 mm and a standard deviation of 3 mm.

c. State the mean and the standard deviation, in millimetres, of the total height of a spray with two caps attached. 2 marks

d. Each spray with two caps attached is packaged in a cardboard box with an internal height of 110 mm.
Find the probability that a spray with two caps attached will **not** fit into one of these cardboard boxes. Give your answer correct to three decimal places. 1 mark

The company uses a new contractor to service the machine that produces the sprays.
To check whether the machine has been properly serviced, that is, to check whether the machine is still producing sprays with a mean height of 75 mm, a random sample of 60 sprays is taken.
The mean height of this sample of sprays is found to be 76 mm.
The company conducts a two-tailed test at the 5% level of significance.
Assume that the standard deviation of the height of the sprays remains at 4 mm.

e. Write down suitable hypotheses H_0 and H_1 for this test. 1 mark

f. Find the p value for this test, correct to three decimal places. 1 mark

g. State, with a reason, whether or not this sample of 60 sprays suggests that the machine has **not** been properly serviced. 1 mark

h. What is the largest value of the mean height of the sample of 60 sprays for H_0 not to be rejected? Give your answer correct to three decimal places. 1 mark

i. Using this sample of 60 sprays which has a mean height of 76 mm, find an approximate 98% confidence interval for the mean height of all the sprays produced by the machine after it was serviced. Express the endpoints of the interval in millimetres correct to one decimal place. 1 mark

Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(\theta)$	$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \text{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \text{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2 \ldots X_n$	$\text{E}(aX_1 + b) = a\text{E}(X_1) + b$ $\text{E}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1\text{E}(X_1) + a_2\text{E}(X_2) + \ldots + a_n\text{E}(X_n)$	
	$\text{Var}(aX_1 + b) = a^2\text{Var}(X_1)$ $\text{Var}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1^2\text{Var}(X_1) + a_2^2\text{Var}(X_2) + \ldots + a_n^2\text{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2 \ldots X_n$	$\text{E}(X_1 + X_2 + \ldots + X_n) = n\mu$	
	$\text{Var}(X_1 + X_2 + \ldots + X_n) = n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean	$\text{E}(\bar{X}) = \mu$
	variance	$\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \ne -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x}\,dx = \log_e\lvert x\rvert + c$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$	$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}(\cot(ax)) = -a\text{cosec}^2(ax)$	$\int \text{cosec}^2(ax)\,dx = -\frac{1}{a}\cot(ax) + c$
$\frac{d}{dx}(\sec(ax)) = a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax)\,dx = \frac{1}{a}\sec(ax) + c$
$\frac{d}{dx}(\text{cosec}(ax)) = -a\text{cosec}(ax)\cot(ax)$	$\int \text{cosec}(ax)\cot(ax)\,dx = -\frac{1}{a}\text{cosec}(ax) + c$
$\frac{d}{dx}(\sin^{-1}(ax)) = \frac{a}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}(\cos^{-1}(ax)) = \frac{-a}{\sqrt{1-(ax)^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}(\tan^{-1}(ax)) = \frac{a}{1+(ax)^2}$	$\int \frac{a}{a^2 + x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \ne -1$
	$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\log_e\lvert ax+b\rvert + c$

Calculus - continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x, y), x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h \times f(x_n, y_n)$
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{r}}(t) = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 . \underset{\sim}{r}_2 = \lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$
parametric equation of a line	$x(t) = x_1 + x_2t \qquad y(t) = y_1 + y_2t \qquad z(t) = z_1 + z_2t$
vector equation of a plane	$\underset{\sim}{r}(s,t) = \underset{\sim}{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2$ $= (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$
parametric equation of a plane	$x(s,t) = x_0 + x_1s + x_2t,\ \ y(s,t) = y_0 + y_1s + y_2t,\ \ z(s,t) = z_0 + z_1s + z_2t$
Cartesian equation of a plane	$ax + by + cz = d$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \frac{1}{2}(1 - \cos(2ax))$	$\cos^2(ax) = \frac{1}{2}(1 + \cos(2ax))$

SPECIALIST MATHS

TRIAL EXAMINATION 2

MULTIPLE - CHOICE ANSWER SHEET

STUDENT NAME:..

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: A ■ C D E

The answer selected is B. Only one answer should be selected.

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E
16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E