

STUDENT NUMBER

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# SPECIALIST MATHEMATICS

## Written examination 2

Wednesday 1 June 2022

Reading time: 10.00 am to 10.15 am (15 minutes)

Writing time: 10.15 am to 12.15 pm (2 hours)

### QUESTION AND ANSWER BOOK

#### Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

#### Materials supplied

- Question and answer book of 25 pages
- Formula sheet
- Answer sheet for multiple-choice questions

#### Instructions

- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

#### At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

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**SECTION A – Multiple-choice questions****Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude  $g \text{ ms}^{-2}$ , where  $g = 9.8$

**Question 1**

The graph of  $f$  with rule  $f(x) = \frac{x^2 + 1}{x^2 + bx + 4}$ , where  $b \in R$ , has two asymptotes when

- A.  $\{b : b < -4\} \cup \{b : b > 4\}$
- B.  $\{b : -4 < b < 4\}$
- C.  $\{b : -16 < b < 16\}$
- D.  $\{b : b < -16\} \cup \{b : b > 16\}$
- E.  $\{b : b = \pm 4\}$

**Question 2**

If the implied domain of  $y = \sin(\cos^{-1}(ax - 1))$ , where  $a \in R \setminus \{0\}$ , is the same as the range, then the value of  $a$  is

- A.  $-2$
- B.  $-1$
- C.  $1$
- D.  $2$
- E.  $3$

**Question 3**

The area of the region, in square units, enclosed by the graph of the relation with rule  $x + |2y| = 4$  and the  $y$ -axis is

- A.  $2$
- B.  $4$
- C.  $8$
- D.  $16$
- E.  $32$

**Question 4**

The relation  $|z - 2 + 2\sqrt{3}i| = 4$  represents a circle with centre and radius respectively given by

- A.  $(-2, 2\sqrt{3}), 4$
- B.  $(-2\sqrt{3}, 2), 4$
- C.  $(2, -2\sqrt{3}), 2$
- D.  $(2, -2\sqrt{3}), 4$
- E.  $(-2, 2\sqrt{3}), 2$

**Question 5**

Which one of the following, where  $A, B, C$  and  $D$  are non-zero real numbers, is a partial fraction form for the expression  $\frac{x-1}{(x^2+1)(x-4)^2}$ ?

- A.  $\frac{A}{x^2+1} - \frac{B}{(x-4)^2}$
- B.  $\frac{Ax+B}{x^2+1} + \frac{C}{(x-4)^2} + \frac{D}{x-4}$
- C.  $\frac{Ax+B}{x^2+1} + \frac{C}{(x-4)^2} + \frac{Dx}{x-4}$
- D.  $\frac{A}{x^2+1} + \frac{C}{(x-4)^2} + \frac{D}{x-4}$
- E.  $\frac{Ax+B}{x^2+1} + \frac{C}{(x-4)^2}$

**Question 6**

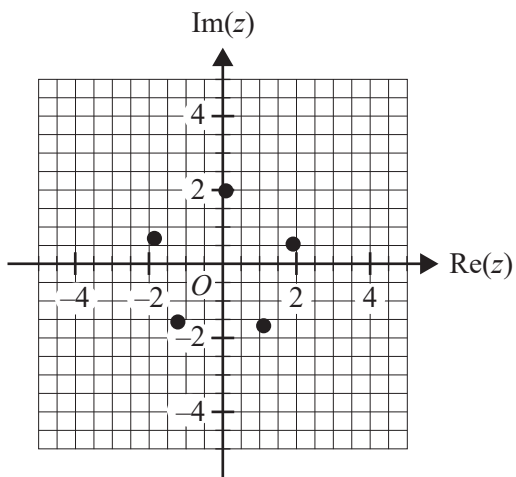
The equation  $2z^3 - 3z^2 - 12z + b = 0$ , where  $z \in C$  and  $b$  is a real constant, will have one real and two complex solutions for

- A.  $\{b : b > 20\}$  only
- B.  $\{b : b < -7\}$  only
- C.  $\{b : -7 < b < 20\}$
- D.  $\{b : b \leq -7\} \cup \{b : b \geq 20\}$
- E.  $\{b : b < -7\} \cup \{b : b > 20\}$

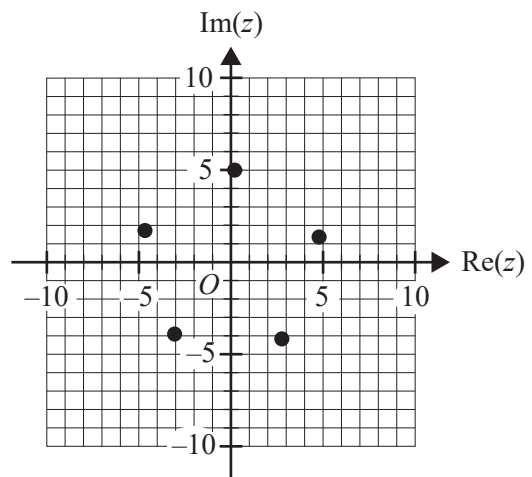
**Question 7**

Which one of the following could represent the solutions to  $z^5 = 6 + 30i$  on an Argand diagram?

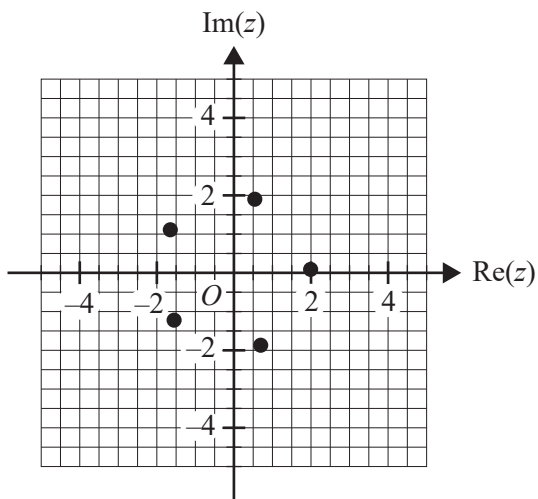
A.



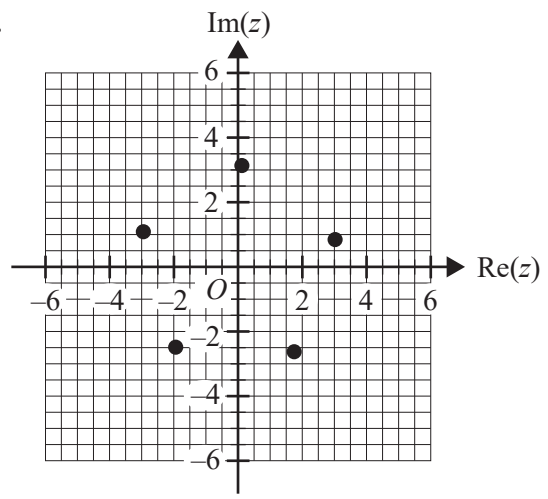
B.



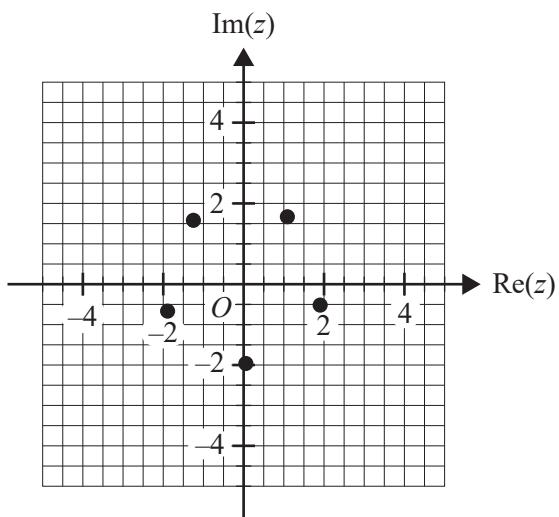
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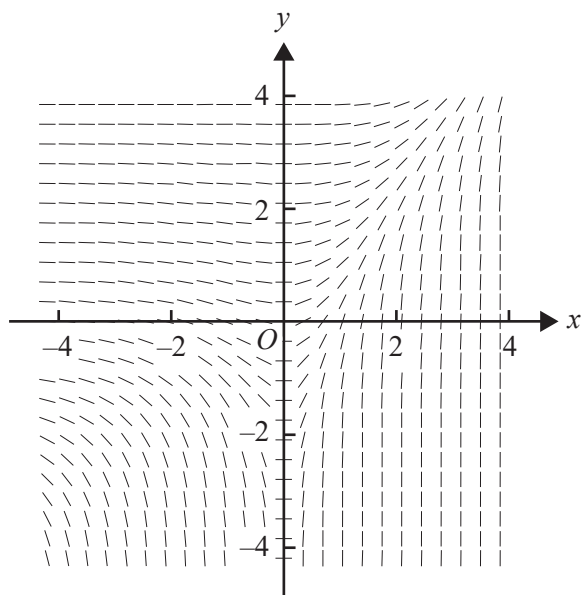


D.



E.



**Question 8**

The slope field above would best represent the solutions of the differential equation

- A.  $\frac{dy}{dx} = e^{x-y}$
- B.  $\frac{dy}{dx} = xe^{x-y}$
- C.  $\frac{dy}{dx} = \frac{y}{e^{x-y}}$
- D.  $\frac{dy}{dx} = xe^y$
- E.  $\frac{dy}{dx} = xe^x$

**Question 9**

The arc length of the curve given by  $y = \cos(x)$  from  $x = 0$  to  $x = \pi$  is closest to

- A. 2.00
- B. 3.14
- C. 3.82
- D. 4.00
- E. 4.44

**Question 10**

Using the substitution  $x = e^u - e^{-u}$ ,  $\int_0^{\frac{3}{2}} \sqrt{4 + x^2} dx$  can be expressed as

- A.  $\int_0^{\log_e 2} e^{2u} - e^{-2u} du$
- B.  $\int_0^{\log_e \frac{1}{2}} e^{2u} + e^{-2u} du$
- C.  $\int_0^{\log_e 2} (e^u + e^{-u}) \sqrt{e^{2u} + e^{-2u} + 4} du$
- D.  $\int_0^{\frac{3}{2}} e^u + 2 + e^{-u} du$
- E.  $\int_0^{\log_e 2} e^{2u} + 2 + e^{-2u} du$

**Question 11**

Let  $\underline{a} = n\underline{i} + \underline{j} - \underline{k}$  and  $\underline{b} = \underline{i} - \sqrt{3}\underline{j} + n\underline{k}$ , where  $n$  is a positive real number. The vector resolute of  $\underline{a}$  in the direction of  $\underline{b}$  is  $\frac{1}{6}(-\sqrt{3}\underline{i} + 3\underline{j} + p\underline{k})$ , where  $p$  is a real number.

The value of  $p$  is

- A.  $-\sqrt{6}$
- B.  $-\frac{1}{2}$
- C.  $-3$
- D.  $-\frac{\sqrt{6}}{6}$
- E.  $\sqrt{6}$

**Question 12**

The positions of two particles are given by  $\underline{r}_A(t) = \left(7 - \frac{25}{2}t\right)\underline{i} + (5t - 6)\underline{j}$  and  $\underline{r}_B(t) = \left(t^3 - \frac{13}{2}t^2\right)\underline{i} + (t^2)\underline{j}$  at time  $t$  seconds, where  $t \geq 0$ .

The time, in seconds, at which the particles collide is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 3.5

**Question 13**

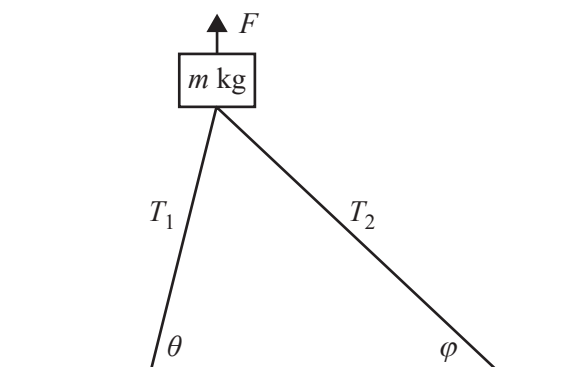
Given that  $\theta$  is the angle between the vectors  $\underline{a} = \underline{i} - 4\underline{j} + 8\underline{k}$  and  $\underline{b} = -2\underline{i} - 10\underline{j} + 11\underline{k}$ ,  $\cos(2\theta)$  is equal to

- A.  $\frac{46}{135}$
- B.  $\frac{126}{299}$
- C.  $\frac{167}{225}$
- D.  $\frac{623}{729}$
- E.  $\frac{14}{15}$



**Question 14**

A hot air balloon of mass  $m$  kilograms is hovering above the ground, held in place by two ropes fixed to the ground with tensions of magnitudes  $T_1$  newtons and  $T_2$  newtons, as shown in the diagram below. There is an upward force of  $F$  newtons due to the upward thrust of the hot air.



Which one of the following statements is true?

- A.  $T_1 \cos(\varphi) = T_2 \cos(\theta)$
- B.  $T_1 \sin(\theta) + T_2 \sin(\varphi) + F + mg = 0$
- C.  $T_1 \sin(\theta) + T_2 \sin(\varphi) + F - mg = 0$
- D.  $T_1 \sin(\theta) + T_2 \sin(\varphi) - F + mg = 0$
- E.  $T_1 \sin(\theta) - T_2 \sin(\varphi) = 0$

**Question 15**

A car of mass 2 tonnes accelerates from 0 to  $100 \text{ km h}^{-1}$  in 12 seconds under constant acceleration along a straight horizontal path.

The magnitude of the net force, in newtons, that acts on the car is closest to

- A. 1700
- B. 4630
- C. 16670
- D. 45370
- E. 55560

**Question 16**

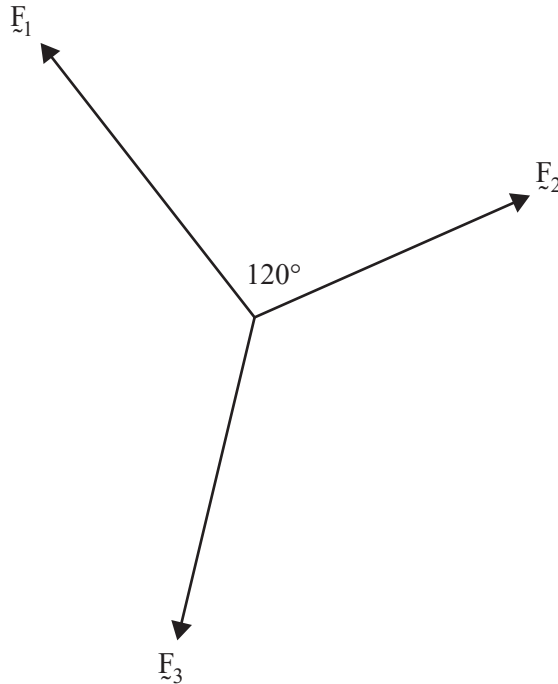
The velocity of a particle moving in a straight line at time  $t$  seconds when it is at a displacement of  $x$  metres from an origin  $O$  is given by  $v = te^{-x}$ .

Given that  $x(0) = 0$ , the velocity, in  $\text{ms}^{-1}$ , of the particle in terms of  $t$  is given by

- A.  $\frac{2}{t}$
- B.  $\frac{2t}{t^2 + 2}$
- C.  $\frac{1}{2}t^3 + t$
- D.  $\frac{t}{t^2 + 1}$
- E.  $\frac{1}{2}t^3$

**Question 17**

A particle acted on by three coplanar forces,  $\vec{F}_1$ ,  $\vec{F}_2$  and  $\vec{F}_3$ , is in equilibrium, as shown in the diagram below. The angle between  $\vec{F}_1$  and  $\vec{F}_2$  is  $120^\circ$ , where  $|\vec{F}_1| = 10$  N and  $|\vec{F}_2| = 8$  N.



The magnitude of  $\vec{F}_3$ , in newtons, is

- A.  $2\sqrt{21}$
- B.  $2\sqrt{41}$
- C.  $2\sqrt{61}$
- D.  $4\sqrt{3}$
- E.  $4\sqrt{21}$

**Question 18**

A student travels to and from college by bus each day. The time that the student spends waiting for the bus in the morning is normally distributed with a mean of 8 minutes and a standard deviation of 2 minutes.

The time that the student spends waiting for the bus in the afternoon is normally distributed with a mean of 10 minutes and a standard deviation of 3 minutes. Morning and afternoon waiting times are independent.

The probability, correct to three decimal places, that the student spends more than 15 minutes in total waiting for buses on a particular day is

- A. 0.048
- B. 0.591
- C. 0.726
- D. 0.797
- E. 0.910

**Question 19**

A random sample of 100 oranges is selected from an orange orchard and their diameters are recorded. It is found, for the sample, the mean is 6 cm and the standard deviation is 0.8 cm.

Which one of the following statements is **necessarily** correct?

- A. The means of random samples of 100 oranges would be normally distributed with a standard deviation of  $\frac{0.8}{100}$  cm.
- B. To obtain a 90% confidence interval for  $\mu$ , a standardised  $z$  value of 1.96 is used.
- C. The mean diameter of all oranges grown at the orchard is 6 cm.
- D. The standard deviation of the diameters of all oranges grown is  $\frac{0.8}{\sqrt{100}}$  cm.
- E. The smallest value in the 99% confidence interval for  $\mu$  would not be less than 5.79 cm.

**Question 20**

A 90% confidence interval for the mean sodium content, in milligrams, of a particular type of biscuit is calculated to be (80.4, 81.7), based on a sample of 25 of these biscuits.

The standard deviation, correct to two decimal places, used in this calculation is

- A. 0.40
- B. 0.65
- C. 1.66
- D. 1.97
- E. 1.98

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**SECTION B****Instructions for Section B**

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude  $g \text{ ms}^{-2}$ , where  $g = 9.8$

**Question 1** (10 marks)

a. Consider the function  $f$  with rule  $f(x) = \frac{x^3}{x^2 - 1}$ .

i. Express  $f(x)$  in the form  $Ax + \frac{B}{x-1} + \frac{C}{x+1}$ , where  $A, B$  and  $C$  are real constants. 1 mark

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ii. Write down the equations of the asymptotes of the graph of  $f$ . 1 mark

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b. i. Write down  $f'(x)$ . 1 mark

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ii. Find the stationary points of the graph of  $f$  and state their nature. 2 marks

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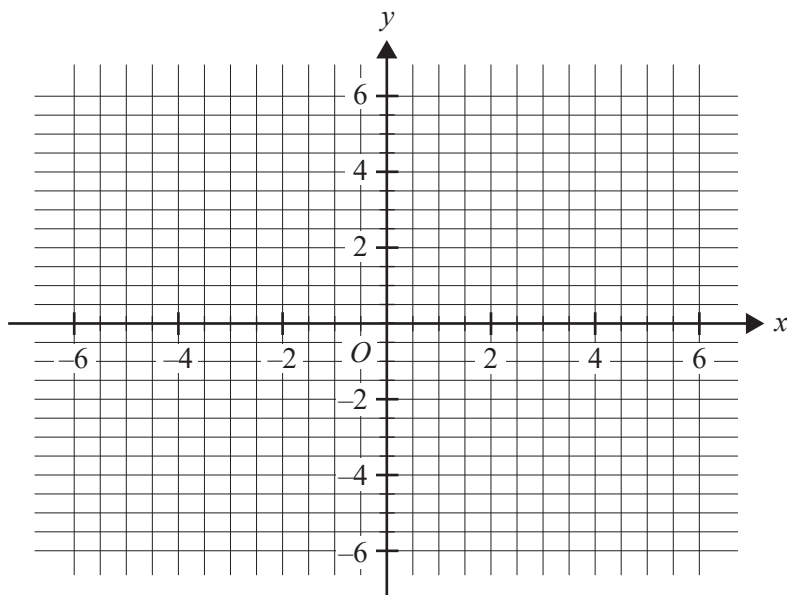


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- c. Sketch the graph of  $f$  on the set of axes below. Label the asymptotes with their equations and label the stationary points with their coordinates. 3 marks



- d. The part of the graph of  $f$  from  $x = 2$  to  $x = 4$  is rotated about the  $x$ -axis to form a solid of revolution.
- i. Write down a definite integral that gives the volume of the solid formed. 1 mark
- ii. Find the volume of the solid, correct to one decimal place. 1 mark

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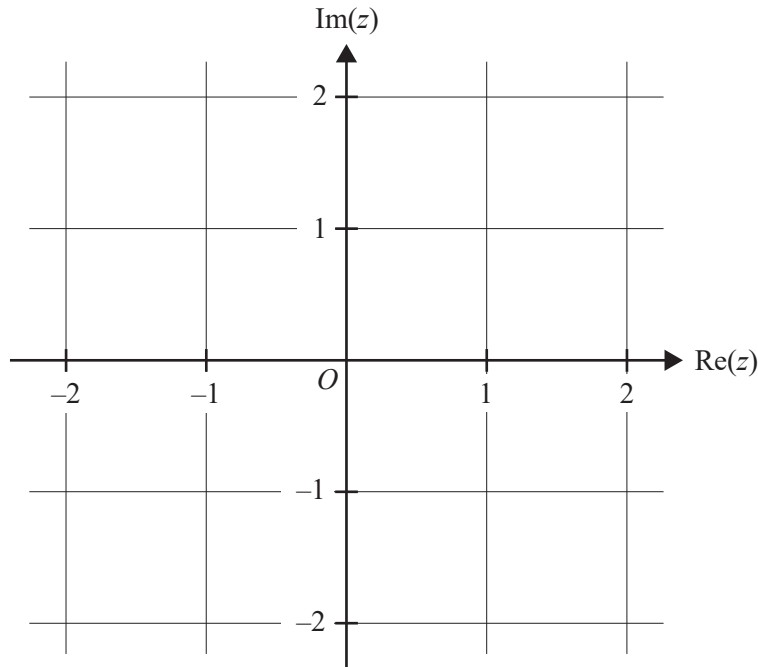
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**Question 2** (9 marks)

$\text{Arg}(z - z_0) = \theta$  defines a ray in the complex plane originating at point  $z_0$ , where  $z_0 = \sqrt{3} + i$ .

- a. Sketch  $\text{Arg}(z - z_0) = \theta$  on the Argand plane below, given that the ray passes through the origin.

1 mark



- b. Find the angle  $\theta$ .

1 mark

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- c. Determine, in terms of  $z$ , the equation of the circle with centre at the origin  $O$  that passes through  $z_0$ .

1 mark

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- d. If  $z_0$  is one of three points equally spaced on the circle described in **part c.**, find the other two points,  $z_1$  and  $z_2$ .

2 marks

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- e. The points  $z_0, z_1$  and  $z_2$  are the roots of the equation  $f(z) = 0$ .

Find  $f(z)$  in polynomial form.

2 marks

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- f. The line that passes through the origin and the point  $z_0$  can be expressed in the form

$$|z - i| = |z - z_3|.$$

Find  $z_3$  in the form  $a + bi$ , where  $a, b \in R$ .

2 marks

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**Question 3** (9 marks)

A tank initially contains 15 L of water with 20 grams of a dissolved chemical. Water that

contains a variable concentration of the chemical,  $\frac{4}{(1+t)^2}$  grams per litre, where  $t \geq 0$ , flows into the tank at the rate of  $30 \text{ L min}^{-1}$ .

The solution of water and chemical, which is kept uniform by stirring, flows out of the tank at the rate of  $15 \text{ L min}^{-1}$ .

- a. If  $x$  grams of the chemical are present in the tank at time  $t$  minutes, write down, in terms of  $x$  and  $t$ , an expression for the concentration of chemical in the tank, in grams per litre, at time  $t$ .

1 mark

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- b. Show that the differential equation relating  $x$  to  $t$  is  $\frac{dx}{dt} + \frac{x}{1+t} = \frac{120}{(1+t)^2}$ .

2 marks

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It can be shown that  $x = \frac{120 \log_e(1+t)}{(1+t)} + \frac{20}{(1+t)}$ .

- c. i. Find  $\frac{dx}{dt}$ . 1 mark

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- ii. Verify by substitution that  $x = \frac{120 \log_e(1+t)}{(1+t)} + \frac{20}{(1+t)}$  satisfies **both** the differential equation and the initial condition. 3 marks

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- d. Find, in grams per minute, the maximum rate at which the chemical flows from the tank and the time, in minutes, at which this occurs. Give your answers correct to one decimal place. 2 marks

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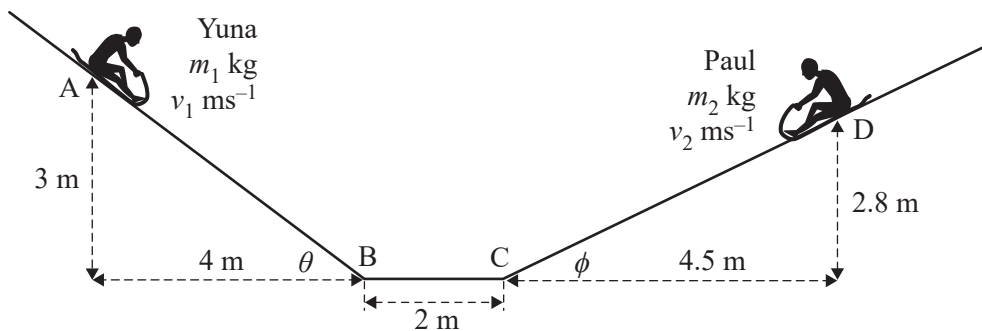


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**Question 4** (11 marks)

Yuna, whose mass is  $m_1$  kilograms, and Paul, whose mass is  $m_2$  kilograms, are using sliding mats to move in the same vertical plane along different connected sections of a slide, as shown below. Yuna’s speed at point A is  $v_1 \text{ ms}^{-1}$  and Paul’s speed at point D is  $v_2 \text{ ms}^{-1}$ . The mass of each sliding mat can be considered to be negligible.

The three sections of the slide, AB, BC and CD, are rough and the mats are subject to frictional resistance forces as they slide over these sections.



- a. Given that the frictional resistance, in newtons, acting on Yuna’s mat is given by  $k_1 m_1 g \cos(\theta)$ , where  $k_1 \in R^+$ , and that Yuna’s speed over the section AB is constant, find the value of  $k_1$ . 2 marks

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Paul’s acceleration over the section CD is  $0.2g \text{ ms}^{-2}$  and his speed at point C is twice his speed at point D.

- b. i. If the frictional resistance, in newtons, acting on Paul’s mat is given by  $k_2 m_2 g \cos(\phi)$ , where  $k_2 \in R^+$ , calculate the value of  $k_2$ . 2 marks

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- ii. Determine the value of  $v_2^2$  in terms of  $g$ . 2 marks

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When Yuna slides in the section BC, the frictional resistance acting on her mat is  $k_3 m_1 g$ , where  $k_3 \in R^+$ .

- c. Show that the distance  $s_1$  that Yuna slides in the section BC before coming to rest is

$$\frac{v_1^2}{2gk_3} \text{ metres.}$$

2 marks

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- d. When Paul slides in the section BC, the frictional resistance acting on his mat is  $k_3 m_2 g$ , where  $k_3 \in R^+$ .

Given that  $v_1 = v_2$ , find the condition on  $v_1^2$ , in terms of  $k_3$  and  $g$ , for Yuna not to collide with Paul in the section BC.

3 marks

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**Question 5** (10 marks)

A machine is calibrated to produce ball bearings such that their diameters are normally distributed with a mean of 0.5 cm and a standard deviation of 0.005 cm. To determine whether the machine is working correctly, random samples of 10 ball bearings are selected at various times and the mean diameter of the 10 ball bearings in each sample is calculated.

For a particular sample of 10 ball bearings, the mean diameter is 0.5003 cm. It is proposed to carry out a statistical test at the 5% level of significance to determine whether the machine is now producing ball bearings with a mean diameter greater than 0.5 cm and possibly needs resetting. Assume that the standard deviation is still 0.005 cm.

- a. Write down the mean and the standard deviation of the sampling distribution for samples of 10 ball bearings. Give the value of the standard deviation correct to four decimal places. 2 marks

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- b. Write down suitable null and alternative hypotheses for the statistical test. 1 mark

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- c. Find, correct to three decimal places, the  $p$  value for the statistical test. 1 mark

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- d. Draw an appropriate conclusion about the hypotheses in **part b.** from the  $p$  value found in **part c.** Give a reason involving  $p$  for your conclusion. 1 mark

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- e. Find the largest sample mean of 10 ball bearings that could be observed for the null hypothesis not to be rejected at the 5% level of significance. Assume that the machine is still producing ball bearings with a standard deviation of 0.005 cm. Give your answer correct to four decimal places. 1 mark

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The settings of the machine will be checked if a sample mean  $\bar{x}$  of 10 ball bearings falls outside the interval  $(a, b)$ , where  $\Pr(\bar{x} < a) = 0.01$  and  $\Pr(\bar{x} > b) = 0.01$

- f. Find the values of  $a$  and  $b$ , correct to three decimal places, assuming that the machine is producing ball bearings with a mean diameter of 0.5 cm and a standard deviation of 0.005 cm. 2 marks

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- g. On a certain day, five random samples of 10 ball bearings are selected.

Find the probability that only one of the five sample means falls outside the interval found in **part f**. Give your answer correct to three decimal places.

2 marks

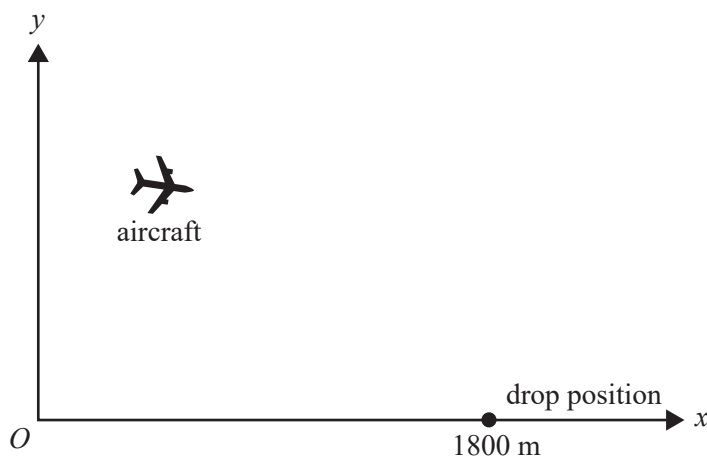
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**Question 6** (11 marks)

A supply aircraft has the position vector  $\mathbf{r}(t) = 60t\mathbf{i} + (500 - 2t)\mathbf{j}$  for  $0 \leq t \leq 100$  seconds relative to a fixed observation point  $O$ , where  $\mathbf{i}$  is a horizontal unit vector in the direction of the drop position and  $\mathbf{j}$  is a unit vector vertically up. Displacement components are measured in metres.



- a. Find the cartesian equation of the path of the aircraft. 1 mark

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- b. Find the speed of the aircraft, in  $\text{ms}^{-1}$ , correct to two decimal places. 2 marks

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- c. Assuming that the aircraft maintains its course, how close does it get to the drop position, which is located 1800 m horizontally from  $O$ ? Give your answer in metres, correct to two decimal places. 2 marks

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- d. When the aircraft is at an altitude of 460 m, it releases a package to try to land it at the drop position, which is located 1800 m horizontally from  $O$ .

Assuming negligible air resistance acting on the package, find how far short of the drop position the package lands. Give your answer in metres, correct to one decimal place.

3 marks

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- e. At what altitude should the package be released so that it lands at the drop position? Give your answer in metres, correct to two decimal places.

3 marks

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**Victorian Certificate of Education  
2022**

**SPECIALIST MATHEMATICS**

**Written examination 2**

**FORMULA SHEET**

**Instructions**

This formula sheet is provided for your reference.  
A question and answer book is provided with this formula sheet.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

## Specialist Mathematics formulas

### Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc \sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab \cos(C)$

### Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

**Circular functions – continued**

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	$R$
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

**Algebra (complex numbers)**

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r \operatorname{cis}(\theta)$	
$ z  = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \leq \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

**Probability and statistics**

for random variables $X$ and $Y$	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(X) + bE(Y)$ $\operatorname{var}(aX + b) = a^2 \operatorname{var}(X)$
for independent random variables $X$ and $Y$	$\operatorname{var}(aX + bY) = a^2 \operatorname{var}(X) + b^2 \operatorname{var}(Y)$
approximate confidence interval for $\mu$	$\left(\bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $E(\bar{X}) = \mu$ variance $\operatorname{var}(\bar{X}) = \frac{\sigma^2}{n}$

**Calculus**

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x  + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$	$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a} \log_e ax+b  + c$
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$ , $x_0 = a$ and $y_0 = b$ , then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1+(f'(x))^2} dx$ or $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$

**Vectors in two and three dimensions**

$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$
$ \underline{r}  = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$
$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

**Mechanics**

momentum	$\underline{p} = m\underline{v}$
equation of motion	$\underline{R} = m\underline{a}$