Neap

Trial Examination 2022

VCE Specialist Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 22 pages

Formula sheet

Answer sheet for multiple-choice questions

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2022 VCE Specialist Mathematics Units 3&4 Written Examination 2.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

The graph of $f(x)=\dfrac{x^3+2x^2}{x^2+x}$ has

A. one non-vertical and two vertical asymptotes.

B. two non-vertical asymptotes.

C. two non-vertical and one vertical asymptote.

D. one non-vertical and one vertical asymptote.

E. two vertical asymptotes.

Question 2

The maximal domain of the function f with the rule $f(x)=a\cos^{-1}(x-2a)+3a$ is $[-3,-1]$.

The range of f is

A. $[0,\pi]$

B. $[-\pi,0]$

C. $[-\pi+3,0]$

D. $[0,\pi-3]$

E. $[-\pi-3,-3]$

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Question 3

The implied domain of the function with the rule $f(x) = 2 + \operatorname{cosec}\left(x + \frac{\pi}{3}\right)$ is

A. $\frac{(3n-1)\pi}{3}, n \in Z$

B. $R \setminus \left\{\frac{(n-3)\pi}{3}\right\}, n \in Z$

C. $R \setminus \left\{\frac{(3n+2)\pi}{3}\right\}, n \in Z$

D. $R \setminus \left\{\frac{(3n-2)\pi}{3}\right\}, n \in Z$

E. $\frac{(3n-2)\pi}{3}, n \in Z$

Question 4

If $\text{Re}(z+1) = \text{Im}(\overline{z})$, a possible value of z could be

A. $2-3i$

B. $1+i$

C. $3+4i$

D. $3-2i$

E. $1+2i$

Question 5

On an Argand diagram, a point that lies on the path defined by $|z-1-2i| = |z+4|$ is

A. $\left(0, \frac{11}{4}\right)$

B. $\left(0, \frac{7}{4}\right)$

C. $\left(\frac{-3}{2}, 1\right)$

D. $\left(\frac{3}{2}, -1\right)$

E. $\left(0, \frac{3}{2}\right)$

Question 6

Let $z = -1 + i$ and $w = i$.

The value of $\text{Arg}\left(\dfrac{z^{2k+1}}{w^k}\right)$, where k is an **odd** integer, is

A. $-\dfrac{\pi}{2}$

B. $-\dfrac{\pi}{4}$

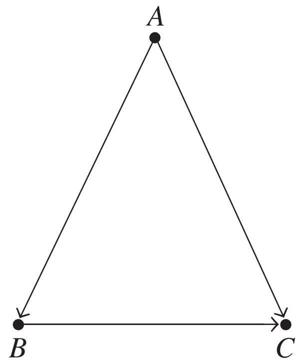
C. $\dfrac{\pi}{4}$

D. $\dfrac{\pi}{2}$

E. $\dfrac{3\pi}{4}$

Question 7

The following diagram shows an isosceles triangle such that $AB = AC$.



Let $\overrightarrow{AB} = \underset{\sim}{c}$, $\overrightarrow{AC} = \underset{\sim}{b}$ and $\overrightarrow{BC} = \underset{\sim}{a}$.

Based on the information above, which one of the following statements is correct?

A. $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} = \underset{\sim}{0}$

B. $\underset{\sim}{a} \cdot \left(\underset{\sim}{c} + \dfrac{1}{2}\underset{\sim}{a}\right) = 0$

C. $\underset{\sim}{a} - \underset{\sim}{b} - \underset{\sim}{c} = \underset{\sim}{0}$

D. $\underset{\sim}{b} = \underset{\sim}{c}$

E. $|2\underset{\sim}{b}| = |\underset{\sim}{b} + \underset{\sim}{c}|$

Question 8

The algebraic fraction $\frac{x}{2(x-b)^2}$, where b is a non-zero real number, can be written as a partial fraction, where A and B are real numbers.

The partial fraction is

A. $\frac{A}{2}+\frac{B}{(x-b)^2}$

B. $\frac{A}{x-b}+\frac{Bx}{2(x-b)^2}$

C. $\frac{A}{2(x-b)}+\frac{B}{(x-b)^2}$

D. $\frac{A}{x-b}+\frac{B}{2(x-b)}$

E. $\frac{2A}{x-b}+\frac{B}{x-b}$

Question 9

If $y=e^{2x}$, then

A. $3\frac{d^2y}{dx^2}+2\frac{dy}{dx}-14y=0$

B. $2\frac{d^2y}{dx^2}+3\frac{dy}{dx}-7y=0$

C. $2\frac{d^2y}{dx^2}+3\frac{dy}{dx}-14y=0$

D. $2\frac{d^2y}{dx^2}-3\frac{dy}{dx}-14y=0$

E. $2\frac{d^2y}{dx^2}+3\frac{dy}{dx}+7y=0$

Question 10

The scalar resolute of $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ in the direction of $\underset{\sim}{b} = \underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ is

A. $-\frac{2}{3}\underset{\sim}{i} + \frac{4}{3}\underset{\sim}{j} - \frac{2}{3}\underset{\sim}{k}$

B. $-\frac{2\sqrt{6}}{3}\underset{\sim}{i} + \frac{4\sqrt{6}}{3}\underset{\sim}{j} - \frac{2\sqrt{6}}{3}\underset{\sim}{k}$

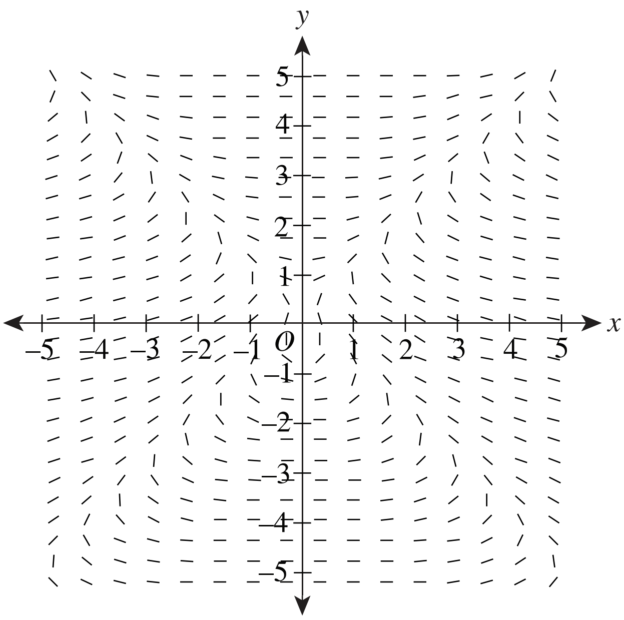
C. $-\frac{2\sqrt{6}}{3}$

D. $-\frac{4\sqrt{13}}{13}$

E. $\frac{2\sqrt{6}}{3}$

Question 11

Consider the following direction field.



The differential equation that has the direction field above is

A. $\frac{dy}{dx} = \frac{x}{y^2 + x^2}$

B. $\frac{dy}{dx} = \frac{x^2}{y^2 - x^2}$

C. $\frac{dy}{dx} = \frac{x}{x^2 - y^2}$

D. $\frac{dy}{dx} = \frac{x}{y^2 - x^2}$

E. $\frac{dy}{dx} = \frac{y}{y^2 - x^2}$

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Question 12

With a suitable substitution, $\int \frac{4x+1}{\sqrt{2x-1}}dx$ can be expressed as

A. $\int \frac{u+2}{u}du$

B. $\frac{1}{2}\int \frac{2u+3}{\sqrt{u}}du$

C. $2\int \frac{2u-2}{\sqrt{u}}du$

D. $\frac{1}{2}\int \frac{2u+1}{\sqrt{u}}du$

E. $\int \frac{u-2}{\sqrt{2u-1}}du$

Question 13

It is known that $\frac{dy}{dx} = \cos(e^x)$ and $y_0 = 1$ when $x_0 = 0$.

Using Euler's formula with step size 0.1, y_3 is equal to

A. $1 + 0.1\left(\cos(1) + \cos\left(e^{0.1}\right) + \cos\left(e^{0.2}\right)\right)$

B. $1 + 0.1\left(\cos(1) + \cos\left(e^{0.1}\right) + \cos\left(e^{0.2}\right) + \cos\left(e^{0.3}\right)\right)$

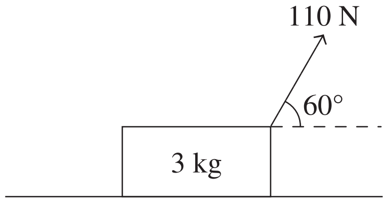
C. $1 + 0.1\left(\cos\left(e^{0.1}\right) + \cos\left(e^{0.2}\right) + \cos\left(e^{0.3}\right)\right)$

D. $1 + 0.1\cos\left(e^{0.3}\right)$

E. $0.1\left(\cos(1) + \cos\left(e^{0.1}\right) + \cos\left(e^{0.2}\right) + \cos\left(e^{0.3}\right)\right)$

Question 14

The following diagram shows an object of mass 3 kg, initially at rest, being pulled along a rough horizontal surface by a force of 110 N acting at an angle of 60° upwards from the horizontal. A friction force of 10 N opposes the motion.



After the pulling force has acted for 6 seconds, the magnitude of the momentum of the object is

A. 5 kg ms^{-1}

B. 25 kg ms^{-1}

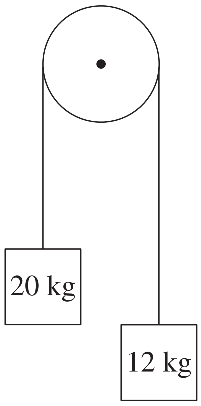
C. 75 kg ms^{-1}

D. 90 kg ms^{-1}

E. 270 kg ms^{-1}

Question 15

The following diagram shows two objects of mass 20 kg and 12 kg, respectively, attached to the ends of a light, inextensible string that passes over a smooth pulley.



Assuming that the system remains connected when it is released from rest, the magnitude of the acceleration of the system is

A. 2.45 ms^{-2}

B. 4.9 ms^{-2}

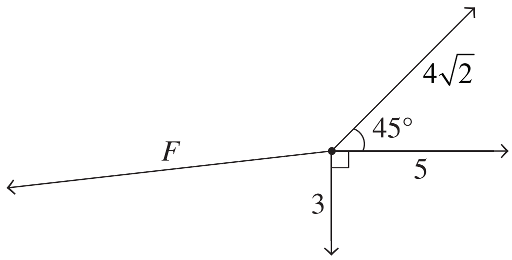
C. 7.2 ms^{-2}

D. 9.8 ms^{-2}

E. 19.6 ms^{-2}

Question 16

The following diagram shows a mass being acted on by four forces. The magnitudes of the forces are labelled. All the forces are measured in newtons, and the system is in equilibrium.



The value of F is

A. 9

B. $\sqrt{82}$

C. 10

D. $\sqrt{101}$

E. $9\sqrt{2}$

Question 17

A variable force acts on a particle of mass 2 kg, causing the mass to move in a straight line. At time t seconds, where $t \geq 0$, the mass's velocity, v metres per second, and position, x metres from the origin, are such that $v = \sin(2x) + \cos(2x)$.

The maximum force acting on the particle is

A. 2 N

B. 3 N

C. 4 N

D. 5 N

E. 6 N

Question 18

The time that participants take to complete a survey is normally distributed with a mean time of 7 minutes and a standard deviation of 1.5 minutes.

The probability that a sample of five people chosen at random has an average completion time less than 5 minutes is closest to

A. 0.0014

B. 0.0056

C. 0.0348

D. 0.0912

E. 0.9087

Question 19

A particular brand of kettle claims to boil water in 90 seconds. The brand's quality control department tests this claim using null and alternative hypotheses.

Which one of the following statements describes the quality control department making a type II error?

A. The quality control department states that the kettle boils water in less than 90 seconds when it actually boils water in 90 seconds.

B. The quality control department states that the kettle boils water in more than 90 seconds when it actually boils water in 90 seconds.

C. The quality control department states that the kettle does not boil water in 90 seconds when it actually boils water in 90 seconds.

D. The quality control department states that the kettle boils water in 90 seconds when it actually does not boil water in 90 seconds.

E. The quality control department states that the kettle boils water in 90 seconds when it actually boils water in more than 90 seconds.

Question 20

A 90% confidence interval for the mean height, h, in centimetres, of a random sample of 50 dwarf apple trees is $234.3 < h < 267.9$.

A 95% confidence interval for a random sample of the same size is closest to

A. (231.1, 271.1)

B. (109.5, 392.7)

C. (221.2, 261.2)

D. (233.3, 273.3)

E. (229.9, 269.9)

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

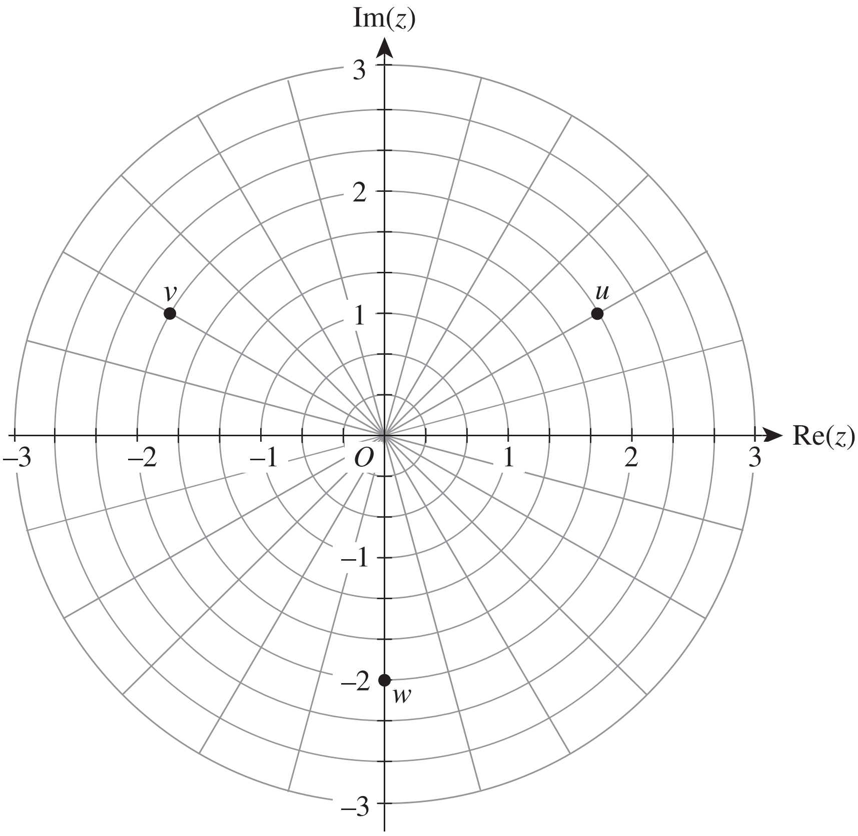
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (14 marks)

In the following Argand diagram, points u, v and w are solutions to the equation $z^3 = k$, where $u = \sqrt{3} + i$.



a. Express u, v and w in polar form. 3 marks

b. Find the value of k. 1 mark

c. **i.** Find the cartesian equation of the line represented by $\sqrt{3}\,\mathrm{Im}(z)-\mathrm{Re}(z)=0$ and sketch it on the Argand diagram on page 11. 2 marks

ii. The line from **part c.i.** can also be represented by $|z-z_1|=|z-z_2|$.

Find z_1 and z_2. Give your answers in the form $x+yi$, where $x, y \in R$. 2 marks

iii. Sketch the ray represented by $\mathrm{Arg}(z-2)=-\dfrac{\pi}{4}$ on the Argand diagram on page 11. 2 marks

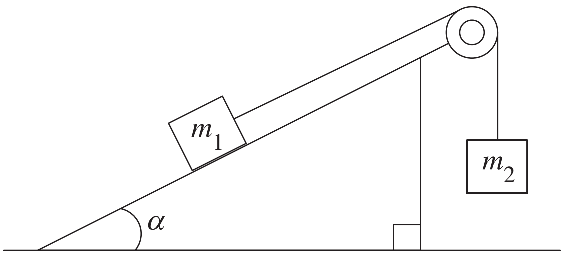
d. **i.** Find $\dfrac{2+2i}{u}$ in polar form. 2 marks

ii. Hence, express $\tan\left(\dfrac{\pi}{12}\right)$ in the form $m-\sqrt{n}$, where m and n are positive integers. 2 marks

Question 2 (15 marks)

Two objects of mass m_1 and m_2 kilograms, respectively, are initially held at rest. They are connected by a light, inextensible string that passes over a smooth pulley as shown in the diagram below. The tension in the string is T newtons. The string is long enough so that neither of the objects reach the pulley if the other object falls and hits the floor.

The object with mass m_1 is on a rough plane inclined at an angle of α to the horizontal. When it moves along the inclined plane, it experiences a force of magnitude F newtons opposing the direction of motion.



a. **i.** Once the system is released, the object with mass m_2 **falls to the floor**.
On the diagram above, mark and label all the forces that act on each object. 3 marks

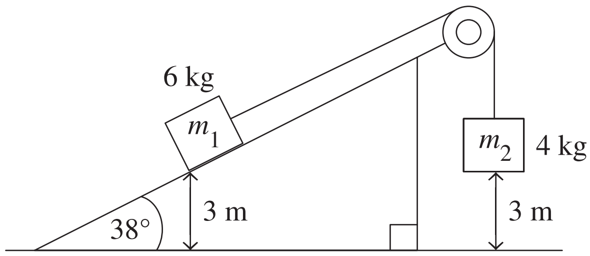
ii. Show that $F = g(m_2 - m_1 \sin\alpha) - (m_1 + m_2)a$, where a is the acceleration in ms^{-2}. 3 marks

b. Find an expression for $\sin\alpha$ if the system is in equilibrium once it has been released. 2 marks

c. The system is reset. Once the system is released, the object with mass m_1 **falls to the floor**.

Find all possible values of $\sin\alpha$ if $F = \dfrac{m_1}{5}$ and $m_1 = 2m_2$. 3 marks

d. The system is modified so that the object with mass m_1 is on a smooth plane.
Let $m_1 = 6$ kg, $m_2 = 4$ kg and $\alpha = 38°$. Both objects are 3 m above the ground and the object with mass m_1 is on a smooth plane, as shown in the following diagram.



i. Once the system is released, which object will fall to the floor? 2 marks

ii. How many seconds will it take the object identified in **part d.i.** to hit the floor? Give your answer correct to two decimal places. 2 marks

Question 3 (9 marks)

A water tank initially contains 400 grams of salt dissolved in 30 L of water. A solution containing 10 grams of salt per litre of water is poured into the tank at a rate of 2 L per minute and the mixture in the tank is kept well stirred. At the same time, the mixture flows out of the tank at a rate of 4 L per minute.

a. Show that the differential equation representing the mass, x grams, of salt in the tank at time t minutes is given by $\frac{dx}{dt} = \frac{2x}{t-15} + 20$. 1 mark

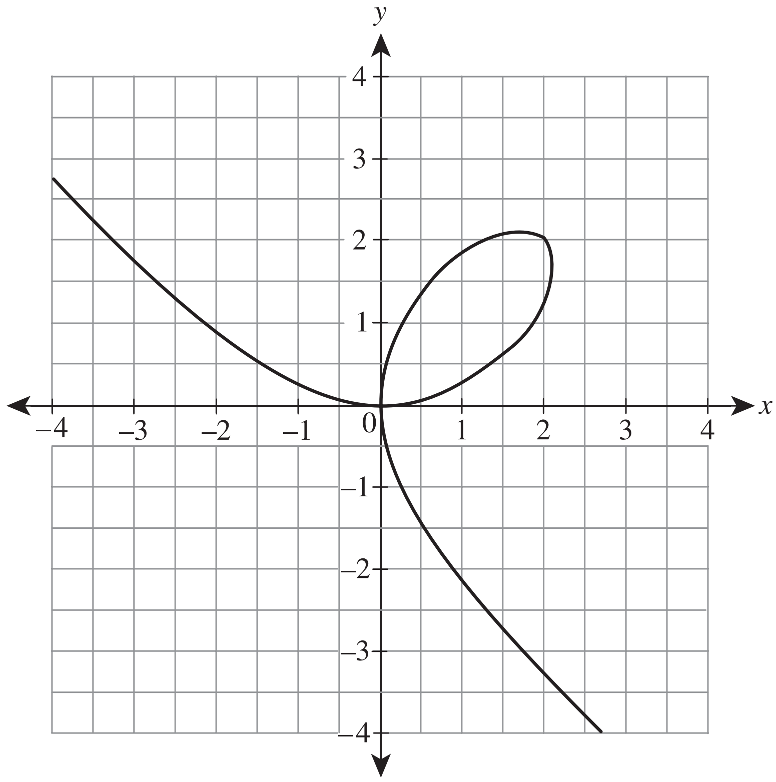
b. **i.** Using differentiation and substitution, prove that $x = \frac{4}{9}(t-60)(t-15)$ satisfies the differential equation from **part a.** Verify that the given solution for x also satisfies the initial condition. 3 marks

ii. State the domain for which the solution is valid. 1 mark

c. Using the information found in **part a.** and **part b.**, show that $\frac{d^2x}{dt^2}=\frac{8t-120}{9t-135}$. 4 marks

Question 4 (13 marks)

The graph of the curve defined by the equation $x^3 + y^3 = 4xy$ is as follows.



a. Find an expression for $\frac{dy}{dx}$ in terms of x and y. 2 marks

b. Write the equation of the tangent to the curve at the point (2, 2). 2 marks

c. Verify that the curve can be represented by the parametric equations $x = \dfrac{4t}{1+t^3}$ and $y = \dfrac{4t^2}{1+t^3}$. 2 marks

d. Find the coordinates of the points where the gradient of the curve is 1. Give your answers correct to two decimal places. 3 marks

e. The area enclosed by the curve can be expressed as $\int_{\infty}^{a} y(t)\frac{d}{dt}\big(x(t)\big)dt - \int_{0}^{a} y(t)\frac{d}{dt}\big(x(t)\big)dt$.

i. State the value of a. 1 mark

ii. Hence, find the enclosed area. 1 mark

f. Express the arc length of the boundary for the enclosed area as a definite integral and find this length, correct to two decimal places. 2 marks

Question 5 (9 marks)

A factory produces cans of energy drink with an advertised volume of 250 mL per can. The machine that fills the cans with the energy drink dispenses volumes that are normally distributed with a mean of 252 mL and a standard deviation of 6 mL.

a. Find the probability that the volume of a randomly selected can is less than 250 mL. Give your answer correct to three decimal places. 1 mark

The cans are placed in boxes before they leave the factory. Each box contains 24 cans.

b. Find the probability that the mean volume of the cans in one box is less than 250 mL. Give your answer correct to three decimal places. 1 mark

c. Find the 95% confidence interval for the mean volume of the cans in one box. Give your answer correct to one decimal place. 1 mark

d. A product inspector visits the factory and claims that the machine dispensing the energy drink is faulty, which means that the volume of drink in each can is less than the advertised volume. She inspects a random box of 24 cans and finds the mean volume to be 249.5 mL. A single-tailed statistical test at the 5% significance is to be carried out.

i. Write the suitable hypotheses, H_0 and H_1, for the single-tailed statistical test. 1 mark

ii. State whether the sample supports the claim that the machine is faulty. Give a reason for your answer. 2 marks

e. The factory's quality assurance policy requires that at least 99% of boxes have a mean volume greater than 250 mL.

Assuming that the mean volume dispensed by the machine remains 252 mL, find the maximum allowable standard deviation needed to achieve the quality assurance requirement. Give your answer in millilitres, correct to one decimal place. 3 marks

END OF QUESTION AND ANSWER BOOKLET

Neap

Trial Examination 2022

VCE Specialist Mathematics Units 3&4

Written Examination 2

Multiple-choice Answer Sheet

Student's Name: ______________________________

Teacher's Name: ______________________________

Instructions

Use a **pencil** for **all** entries. If you make a mistake, **erase** the incorrect answer – **do not** cross it out.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than **one** answer is completed for any question.

All answers must be completed like this example: A B C D E

Use pencil only

1	A	B	C	D	E	11	A	B	C	D	E
2	A	B	C	D	E	12	A	B	C	D	E
3	A	B	C	D	E	13	A	B	C	D	E
4	A	B	C	D	E	14	A	B	C	D	E
5	A	B	C	D	E	15	A	B	C	D	E
6	A	B	C	D	E	16	A	B	C	D	E
7	A	B	C	D	E	17	A	B	C	D	E
8	A	B	C	D	E	18	A	B	C	D	E
9	A	B	C	D	E	19	A	B	C	D	E
10	A	B	C	D	E	20	A	B	C	D	E

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Trial Examination 2022

VCE Specialist Mathematics Units 3&4

Written Examinations 1 & 2

Formula Sheet

Instructions

This formula sheet is provided for your reference. A question and answer booklet is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SPECIALIST MATHEMATICS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\sin(y) - \sin(x)\cos(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\text{cis}(\theta)$	
$\lvert z \rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) < \pi$
$z_1 z_2 = r_1 r_2 \text{cis}\left(\theta_1 + \theta_2\right)$	$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}\left(\theta_1 - \theta_2\right)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\text{E}(aX + b) = a\text{E}(X) + b$ $\text{E}(aX + bY) = a\text{E}(X) + b\text{E}(Y)$ $\text{var}(aX + b) = a^2\text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\dfrac{s}{\sqrt{n}}, \bar{x} + z\dfrac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $\quad \text{E}\left(\bar{X}\right) = \mu$ variance $\quad \text{var}\left(\bar{X}\right) = \dfrac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}(x^n)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}(e^{ax})=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}(\log_e(x))=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}(\sin(ax))=a\cos(ax)$	$\int \sin(ax)dx=-\frac{1}{a}\cos(-ax)+c$
$\frac{d}{dx}(\cos(ax))=-a\sin(ax)$	$\int \cos(ax)dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}(\tan(ax))=a\sec^2(ax)$	$\frac{d}{dx}(\tan(ax))=a\sec^2(ax)$
$\frac{d}{dx}(\sin^{-1}(x))=\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}(\cos^{-1}(x))=\frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}(\tan^{-1}(x))=\frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
	$\int (ax+b)^{-1}dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x)$, $x_0=a$ and $y_0=b$, then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x_n)$.
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+(f'(x))^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

END OF FORMULA SHEET