Neap

Trial Examination 2022

VCE Specialist Mathematics Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 11 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on the answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (3 marks)

A particle of mass 2 kg is initially at rest. It travels 90 m under a constant force of $8\underset{\sim}{i} + 6\underset{\sim}{j}$ newtons.

Find the total time of travel, in seconds, and the final velocity, in ms^{-1}.

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Question 2 (3 marks)

Given that $\cos(x) = -\frac{2}{5}$, where $x \in \left(\pi, \frac{3\pi}{2}\right)$, find the exact value of $\tan(2x)$.

Question 3 (2 marks)

Find the angle between vectors $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j} + \sqrt{6}\underset{\sim}{k}$.

Question 4 (3 marks)

Evaluate $\int_0^{\frac{\pi}{2}} \sin(2x)\cos^3(x)\,dx$.

Question 5 (4 marks)

A builder is building a deck and a pergola. The time it takes to build a deck is known to be normally distributed with a mean of 12 hours and a standard deviation of 4 hours. The time it takes to build a pergola is known to be normally distributed with a mean of 10 hours and a standard deviation of 3 hours.

The time taken to build the pergola is independent of the time taken to build the deck.

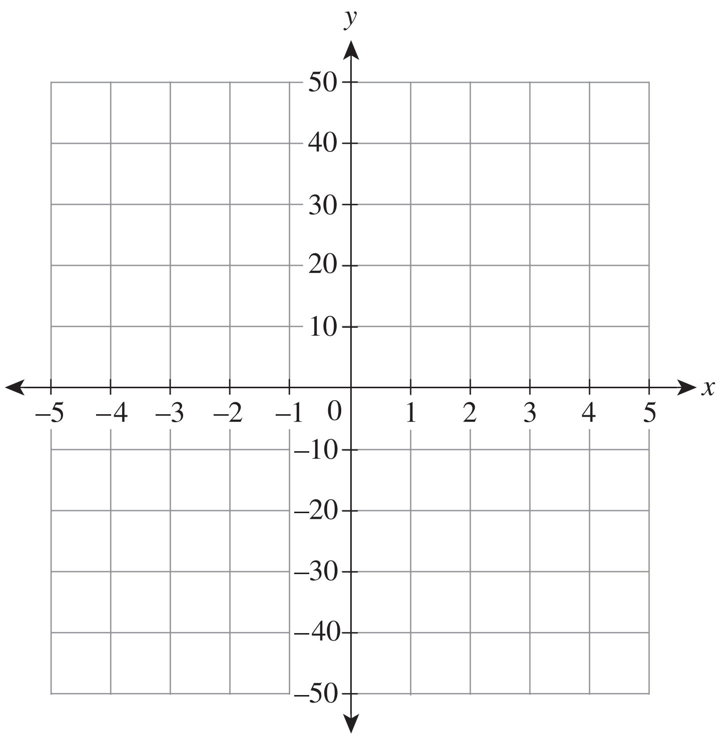
What is the probability that it will take the builder less time to build the deck than to build the pergola? Give your answer correct to two decimal places using $\Pr(Z < 0.4) = 0.66$ where Z is the standard normal variable.

Question 6 (6 marks)

The position vector of a particle at time $t \geq 0$ is given by $\underset{\sim}{r}(t) = (t+1)\underset{\sim}{i} + \left(t^2 + 3t\right)\underset{\sim}{j}$.

a. Show that the cartesian equation of the path followed by the particle is $y = x^2 + x - 2$. 1 mark

b. Sketch the path followed by the particle on the axes below. Label the particle's initial position and direction of motion. 2 marks



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c. When will the particle have a speed of $5\sqrt{2}$? 3 marks

Question 7 (5 marks)

Consider the curve defined parametrically by the equations $x = \arctan\left(\sqrt{3}t\right)$ and $y = \arcsin(t)$ for $t \in [-1, 1]$.

a. State the maximal domain and range of the curve. 2 marks

b. Find the gradient of the tangent to the curve at $x = \frac{\pi}{4}$. 3 marks

Question 8 (5 marks)

A container of water is heated to boiling point (100°C) and then placed in a room that has a constant temperature of 10°C. After 5 minutes, the temperature of the water is 70°C.

a. Using Newton's law of cooling, $\frac{dT}{dt} = k(T - 10)$, where T is the temperature of the water, in degrees Celsius, at time t minutes after the water is placed in the room, show that $k = \frac{1}{5}\log_e\left(\frac{2}{3}\right)$. 3 marks

b. Find the temperature of the water, in degrees Celsius, after **another** 5 minutes has passed. 2 marks

Question 9 (5 marks)

a. Find the square roots of $-12+16i$. Express your answer in the form $a+bi$, where a and b are real numbers. 3 marks

b. Hence, solve $z^2+2z+4-4i=0$. Express your answer in the form $x+yi$, where x and y are real numbers. 2 marks

Question 10 (4 marks)

The region bounded by the circle $(x-1)^2+(y-3)^2=4$ is rotated around the x-axis to form a solid of revolution.

Show that volume of the revolution is equal to $12\pi\int_{-1}^{3}\sqrt{3+2x-x^2}\,dx$.

END OF QUESTION AND ANSWER BOOKLET

Neap

Trial Examination 2022

VCE Specialist Mathematics Units 3&4

Written Examinations 1 & 2

Formula Sheet

Instructions

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SPECIALIST MATHEMATICS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\sin(y)-\sin(x)\cos(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\text{cis}(\theta)$	
$\lvert z \rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) < \pi$
$z_1 z_2 = r_1 r_2 \text{cis}\left(\theta_1 + \theta_2\right)$	$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}\left(\theta_1 - \theta_2\right)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\text{E}(aX + b) = a\text{E}(X) + b$ $\text{E}(aX + bY) = a\text{E}(X) + b\text{E}(Y)$ $\text{var}(aX + b) = a^2\text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\dfrac{s}{\sqrt{n}}, \bar{x} + z\dfrac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $\quad \text{E}\left(\bar{X}\right) = \mu$ variance $\quad \text{var}\left(\bar{X}\right) = \dfrac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e\lvert x\rvert + c$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(-ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$	$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$
$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2}dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a}\log_e\lvert ax+b\rvert + c$
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$.
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1 + (f'(x))^2}\,dx$ or $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

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Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

END OF FORMULA SHEET