Neap

Trial Examination 2022

VCE Specialist Mathematics Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (3 marks)

$$a = \frac{|\underset{\sim}{F}|}{m}$$

$$= \frac{\sqrt{8^2 + 6^2}}{2}$$

$$= 5$$ A1

$$s = ut + \frac{1}{2}at^2$$

$$90 = 0 + \frac{1}{2} \times 5t^2$$

$$t = \sqrt{\frac{180}{5}}$$

$t = 6$ seconds A1

$$\underset{\sim}{v} = \underset{\sim}{a}t$$

$$= \frac{8\underset{\sim}{i} + 6\underset{\sim}{j}}{2} \times 6$$

$= 24\underset{\sim}{i} + 18\underset{\sim}{j}$ ms^{-1} A1

Question 2 (3 marks)

Since $x \in \left(\pi, \frac{3\pi}{2}\right)$:

$$\sin(x) = -\sqrt{1-\cos^2 x}$$
$$= -\sqrt{1-\left(-\frac{2}{5}\right)^2}$$
$$= -\sqrt{\frac{21}{25}}$$
$$= -\frac{\sqrt{21}}{5}$$ A1

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$
$$= \frac{-\frac{\sqrt{21}}{5}}{-\frac{2}{5}}$$
$$= \frac{\sqrt{21}}{2}$$ A1

$$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$$
$$= \frac{\sqrt{21}}{1-\frac{21}{4}}$$
$$= -\frac{4\sqrt{21}}{17}$$ A1

Note: Accept consequential result from incorrect values obtained for tan(x).

Question 3 (2 marks)

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos(\alpha)$$

$$\cos(\alpha) = \frac{1-1+\sqrt{6}}{\sqrt{3}\times\sqrt{8}}$$ M1

$$\cos(\alpha) = \frac{\sqrt{6}}{\sqrt{24}}$$

$$\cos(\alpha) = \frac{1}{2}$$

$$\alpha = \frac{\pi}{3} \textbf{ OR } 60°$$ A1

Question 4 (3 marks)

$I = \int_0^{\frac{\pi}{2}} \sin(2x)\cos^3(x)dx = \int_0^{\frac{\pi}{2}} 2\sin(x)\cos^4(x)dx$ M1

$u = \cos(x)$

$du = -\sin(x)dx$

$I = -\int_{\cos(0)}^{\cos\left(\frac{\pi}{2}\right)} 2u^4 du = -\int_1^0 2u^4 du = \int_0^1 2u^4 du$ A1

Note: Accept any of the expressions shown above for I.

$= \left[\frac{2u^5}{5}\right]_0^1$

$= \frac{2}{5}$ A1

Question 5 (4 marks)

Let D be the time it takes in hours to build a deck: $D \sim N(12,\ 4^2)$.

Let P be the time it takes in hours to build a pergola: $P \sim N(10,\ 3^2)$.

$E(D - P) = 12 - 10 = 2$ A1

$\text{var}(D - P) = 16 + 9 = 25$ A1

$D - P \sim N(2, 5^2)$

Let Z be the standard normal variable for $D - P$.

$\Pr(D - P < 0) = \Pr\left(Z < \frac{0-2}{5}\right)$ M1

$\Pr(Z < -0.4) = \Pr(Z > 0.4)$

$= 1 - 0.66$

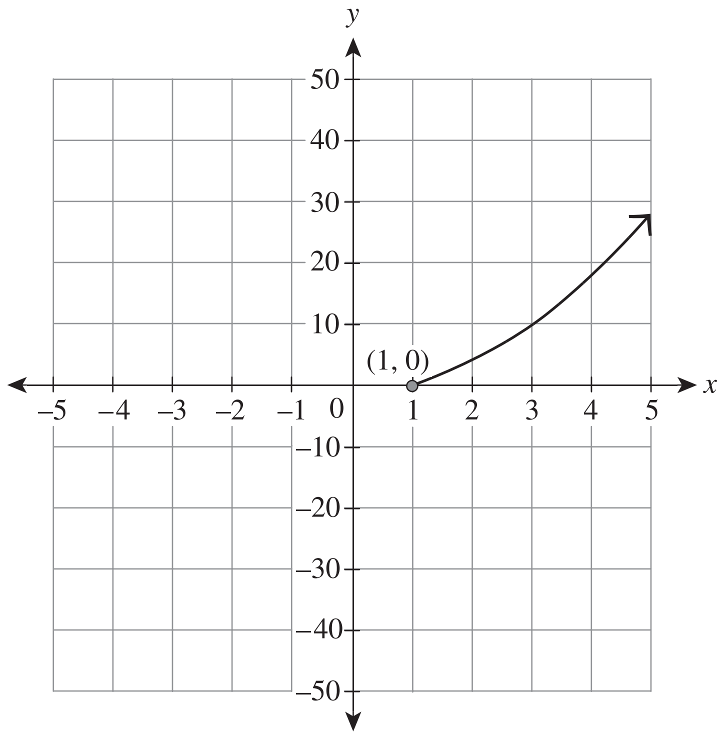
$= 0.34$ A1

Question 6 (6 marks)

a. $x = t + 1 \Rightarrow t = x - 1$

$$
\begin{aligned}
y &= t^2 + 3t \\
&= (x-1)^2 + 3(x-1) \\
&= x^2 - 2x + 1 + 3x - 3 \\
&= x^2 + x - 2
\end{aligned}
$$
M1

b.



correct shape A1
correct endpoint and direction A1

c. speed $= |\dot{\underset{\sim}{r}}|$

$\dot{\underset{\sim}{r}}(t) = \underset{\sim}{i} + (2t+3)\underset{\sim}{j}$ M1

$$
\begin{aligned}
|\dot{\underset{\sim}{r}}| &= \sqrt{(1)^2 + (2t+3)^2} \\
&= \sqrt{4t^2 + 12t + 10}
\end{aligned}
$$
A1

$|\dot{\underset{\sim}{r}}| = 5\sqrt{2} \Rightarrow \sqrt{4t^2 + 12t + 10} = 5\sqrt{2}$

$4t^2 + 12t + 10 = 50$

$t^2 + 3t - 10 = 0$

$t = -5$ or $t = 2$

$t = 2 \quad (\text{since } t \geq 0)$ A1

Question 7 (5 marks)

a. Considering the range of $\arctan\left(\sqrt{3}t\right)$ to find the domain gives:

$$\text{domain} = \left[\arctan\left(-\sqrt{3}\right), \arctan\left(\sqrt{3}\right)\right] \quad \left(\text{since } t \in [-1, 1]\right)$$

$$= \left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$$ A1

Considering the range of $\arcsin(t)$ to find the range gives:

$$\text{range} = [\arcsin(-1),\ \arcsin(1)]$$

$$= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad \left(\text{since } t \in [-1,\ 1]\right)$$ A1

b.

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}}$$

$$\frac{dx}{dt} = \frac{\sqrt{3}}{1+3t^2}$$

$$x = \frac{\pi}{4} \Rightarrow \sqrt{3}t = \tan\left(\frac{\pi}{4}\right) \Rightarrow t = \frac{1}{\sqrt{3}}$$ A1

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{1}{\sqrt{1-t^2}}}{\frac{\sqrt{3}}{1+3t^2}}$$ M1

$$t = \frac{1}{\sqrt{3}} \Rightarrow \text{gradient} = \frac{\frac{1}{\sqrt{1-\frac{1}{3}}}}{\frac{\sqrt{3}}{1+3\times\frac{1}{3}}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}} \times \frac{2}{\sqrt{3}}$$

$$= \sqrt{2}$$ A1

Question 8 (5 marks)

a. **Method 1:**

$$\frac{1}{T-10}dT = kdt$$

$$\int_{100}^{70} \frac{1}{T-10}dT = \int_0^5 kdt$$ M1

$$\left[\log_e\left|T-10\right|\right]_{100}^{70} = \left[kt\right]_0^5$$ A1

$$\log_e(60) - \log_e(90) = 5k$$

$$k = \frac{1}{5}\log_e\left(\frac{60}{90}\right)$$ M1

$$= \frac{1}{5}\log_e\left(\frac{2}{3}\right)$$

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Method 2:

$\int \frac{1}{T-10}dT = \int k\,dt$

$\log_e|T-10| = kt + c$ M1

$\log_e(T-10) = kt + c \quad (\text{since } T \geq 10)$

$\begin{cases} t = 0 \\ T = 100 \end{cases} \Rightarrow \log_e(100-10) = 0 + c$

$c = \log_e(90)$

$\log_e(T-10) = kt + \log_e(90)$

$T = e^{kt+\log_e(90)} + 10$

$= 90e^{kt} + 10$ A1

$\begin{cases} t = 5 \\ T = 70 \end{cases} \Rightarrow 70 = 90e^{5k} + 10$

$e^{5k} = \frac{70-10}{90}$

$5k = \log_e\left(\frac{60}{90}\right)$ M1

$k = \frac{1}{5}\log_e\left(\frac{2}{3}\right)$

b. **Method 1:**

$\int_{70}^{T} \frac{1}{T-10}dT = \int_0^5 \frac{1}{5}\log_e\left(\frac{2}{3}\right)dt$ M1

$\left[\log_e|T-10|\right]_{70}^{T} = \left[\frac{1}{5}\log_e\left(\frac{2}{3}\right)\cdot t\right]_0^5$

$\log_e(T-10) - \log_e(60) = \log_e\left(\frac{2}{3}\right)$

$\frac{T-10}{60} = \frac{2}{3}$

$T = 50°\text{C}$ A1

Method 2:

$T = 90e^{kt} + 10$

$t = 10 \Rightarrow T = 90e^{\frac{1}{5}\log_e\left(\frac{2}{3}\right)\times 10} + 10$ M1

$= 90e^{\log_e\left(\frac{4}{9}\right)} + 10$

$= 90 \times \frac{4}{9} + 10$

$= 50°\text{C}$ A1

Question 9 (5 marks)

a. Let $z = a + bi$, where $a, b \in R$.

If $z^2 = -12 + 16i$, then:

$a^2 + 2abi + b^2 i^2 = -12 + 16i$ M1

Comparing real and imaginary parts:

$$\begin{cases} a^2 - b^2 = -12 \\ 2ab = 16 \end{cases}$$

$$b = \frac{8}{a} \Rightarrow a^2 - \left(\frac{8}{a}\right)^2 = -12$$

$a^4 + 12a^2 - 64 = 0$ M1

$(a^2 - 4)(a^2 + 16) = 0$

$a = -2$ or $a = 2$ $\quad (\text{since } a \in R)$

Therefore, the square roots are $-2 - 4i$ and $2 + 4i$. A1

b. $z_{1,2} = \dfrac{-2 \pm \sqrt{2^2 - 4(4 - 4i)}}{2}$ M1

$= \dfrac{-2 \pm \sqrt{-12 + 16i}}{2}$ $\quad$ (from **part a.**, $\sqrt{-12 + 16i} = 2 + 4i$ or $-2 - 4i$)

$= \dfrac{-2 \pm (2 + 4i)}{2} = -1 \pm (1 + 2i)$

$= 2i$ or $-2 - 2i$ A1

*Note: Consequential on answer to **Question 9a**.*

Question 10 (4 marks)

The region bounded is between two semi-circles for $x \in [1 - 2, 1 + 2] = [-1, 3]$. M1

$(y - 3)^2 = 4 - (x - 1)^2$

$y = 3 \pm \sqrt{4 - (x - 1)^2} = 3 \pm \sqrt{3 + 2x - x^2}$ M1

$\text{volume} = \pi \int_{-1}^{3} \left[\left(3 + \sqrt{3 + 2x - x^2}\right)^2 - \left(3 - \sqrt{3 + 2x - x^2}\right)^2\right] dx$ A1

Using the identity $a^2 - b^2 = (a + b)(a - b)$ gives:

$$\text{volume} = \pi \int_{-1}^{3} \left(3 + \sqrt{3 + 2x - x^2} + 3 - \sqrt{3 + 2x - x^2}\right)\left(3 + \sqrt{3 + 2x - x^2} - 3 + \sqrt{3 + 2x - x^2}\right) dx$$

$= \pi \int_{-1}^{3} 6 \times 2\sqrt{3 + 2x - x^2}\, dx$ M1

$= 12\pi \int_{-1}^{3} \sqrt{3 + 2x - x^2}\, dx$