Neap

Trial Examination 2022

VCE Specialist Mathematics Units 1&2

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
7	7	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

No calculator is allowed in this examination.

Materials supplied

Question and answer booklet of 11 pages

Formula sheet

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (4 marks)

The average height of secondary school students across Australia is measured. A sample of 200 students is drawn randomly; the total sum of their heights is 33 500 cm.

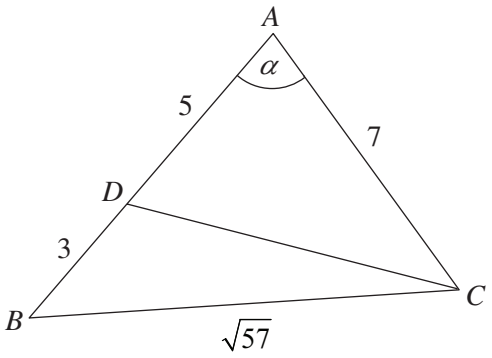
a. Calculate the sample mean. 1 mark

It is later found that 60% of secondary school students in Australia are taller than 167 cm.

b. How many students from the sample would be expected to be taller than 167 cm? 1 mark

c. Write an expression that represents the probability that at least one student from the sample is taller than 167 cm. 2 marks

Question 2 (4 marks)
Consider the following triangle.



a. Find the magnitude of α. 2 marks

b. Find the length of DC. 2 marks

Question 3 (6 marks)

a. The third term of a positive geometric sequence is 45, and the seventh term is 3645.
Find the ninth term. 2 marks

b. A sequence is defined recursively by the rule $t_{n+1} = 2t_n + 3,$ where $t_1 = 1.$

i. Find the first four terms of the sequence. 1 mark

ii. Find the value of S_1, S_2, S_3 and S_4, where S_n represents the sum of the first n terms. 1 mark

iii. It is found that the sequence's series can also be defined recursively.
Find the rule for the series between S_n and S_{n-1}. 2 marks

Question 4 (9 marks)

a. If $z = 6 + 8i$ and $w = 5 + 12i$, express the following in cartesian form.

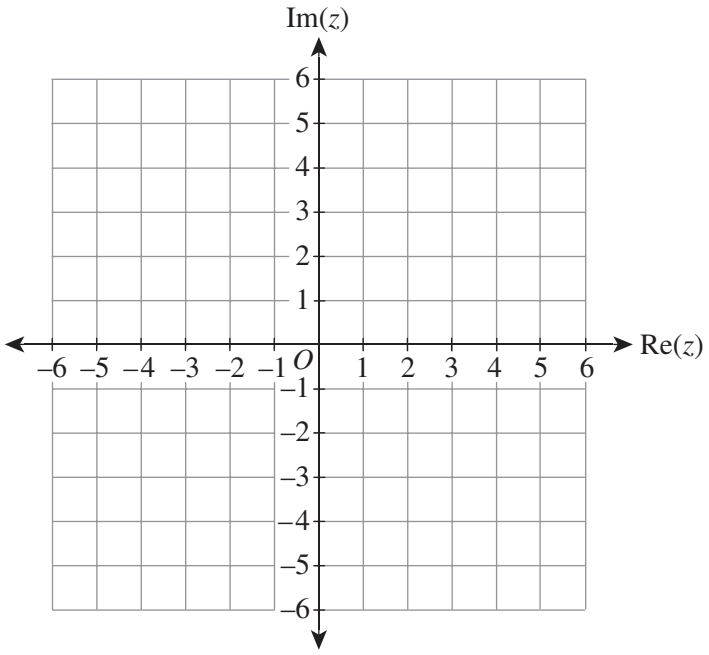
i. $z + w$ 1 mark

ii. zw 1 mark

iii. $\dfrac{\overline{z}}{w}$ 1 mark

b. Let $z = x + yi$.

On the Argand diagram below, shade the area defined by the intersection of the expressions $|z - 1 + 2i| \leq 1$ and $\text{Re}(z) > 1$. 3 marks



c. Express $(-2 - 2i)^{15}$ in polar form. 3 marks

Question 5 (4 marks)

The position vectors of points A and B are given by $2\underset{\sim}{i}+13\underset{\sim}{j}$ and $12\underset{\sim}{i}+8\underset{\sim}{j}$ respectively.

a. Find the vector $\overrightarrow{AB}$. 1 mark

C is a point on the line AB. The line OC has a length of $\frac{28}{\sqrt{5}}$ and is perpendicular to line AB.

b. Find the coordinates of point C. 2 marks

c. Find the area of the triangle OAB. 1 mark

Question 6 (9 marks)

a. Find the value of k if the graph of the function $f(x) = \dfrac{1}{x^2 + 7x + 2k}$ has two vertical asymptotes. 2 marks

b. What shape is described by the equation $5x^2 - 20x + 6y^2 + 72y + 206 = 0$? 3 marks

c. Consider the points $A(0, 5)$ and $B(0, -5)$.
Find the equation of the locus of point $P(x, y)$ satisfying $|AP - BP| = 2$. Give your answer in the form $\frac{x^2}{m} + \frac{y^2}{n}$, where $m, n \in R$. 4 marks

Question 7 (4 marks)

a. Write $\frac{3}{2}\sin(4x)+\frac{\sqrt{3}}{2}\cos(4x)$ in the form $r\sin(4x+\alpha)$. 3 marks

b. Prove that $\frac{3}{2}\sin(4x)+\frac{\sqrt{3}}{2}\cos(4x)=2\sqrt{3}\sin\left(2x+\frac{\pi}{12}\right)\cos\left(2x+\frac{\pi}{12}\right)$. 1 mark

END OF QUESTION AND ANSWER BOOKLET

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VCE Specialist Mathematics Units 1&2

Written Examinations 1 & 2

Formula Sheet

Instructions

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SPECIALIST MATHEMATICS FORMULAS

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	

Vectors in two dimensions

$\underset{\sim}{r}=x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert=\sqrt{x^2+y^2+z^2}=r$
$\underset{\sim}{r}_1\cdot\underset{\sim}{r}_2=r_1r_2\cos(\theta)=x_1x_2+y_1y_2+z_1z_2$

Polar coordinates

$x=r\cos\theta$
$y=r\sin\theta$

END OF FORMULA SHEET