2021 VCE Specialist Mathematics 1 (NHT) examination report

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Question 1a.

$f'(x) = x + \cos(x)$

$f''(x) = 1 - \sin(x)$

$f''(x) = 0 \Rightarrow \sin(x) = 1$

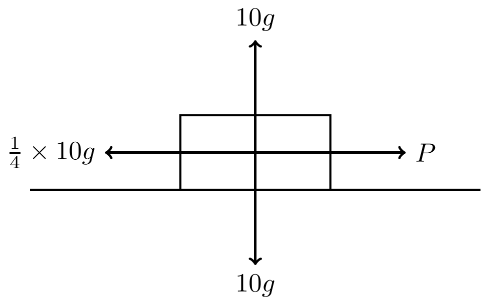
$x = \frac{\pi}{2} + 2n\pi\ ,\ n \in Z$

Question 1b.

$f''(x) = 1 - \sin(x) \geq 0$ therefore, there are no points of inflection

Question 2a.

A diagram is useful but not required.



$10 \times 2 = P - \frac{5}{2}g$

$P = 20 + \frac{5}{2}g$

$= \frac{40 + 5g}{2}$

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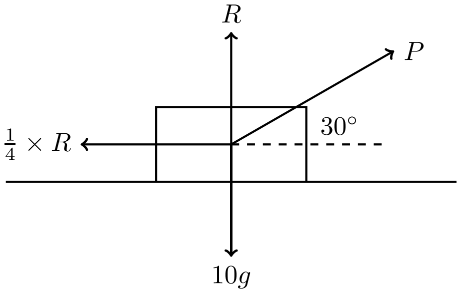
Question 2b.

A diagram is useful but not required.



$$R + P\sin(30^{\circ}) = 10g$$

$$R = 10g - \frac{1}{2}P$$

Therefore

$$P = \frac{4(40+5g)}{4\sqrt{3}+1}$$

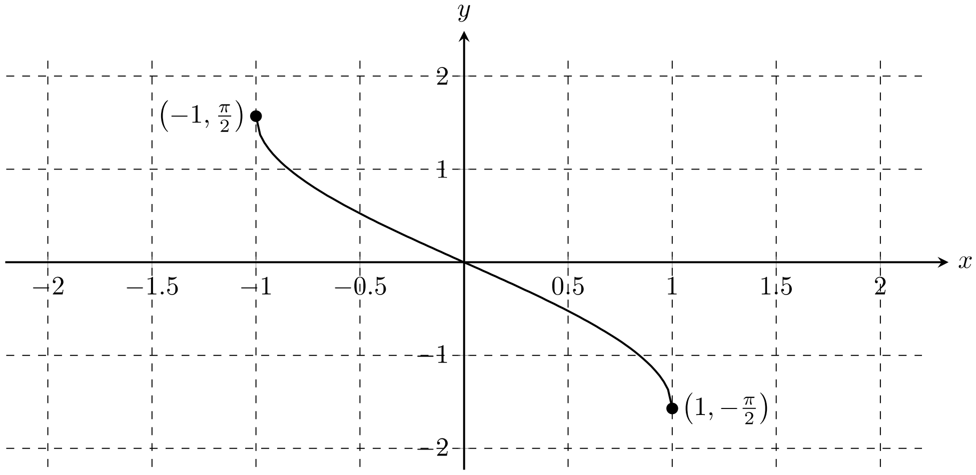
Question 3a.

$$E(Z) = 2E(X) + 3E(Y)$$
$$= 2\times 4\times \frac{1}{2} + 3\times 6\times \frac{1}{2}$$
$$= 4 + 9 = 13$$

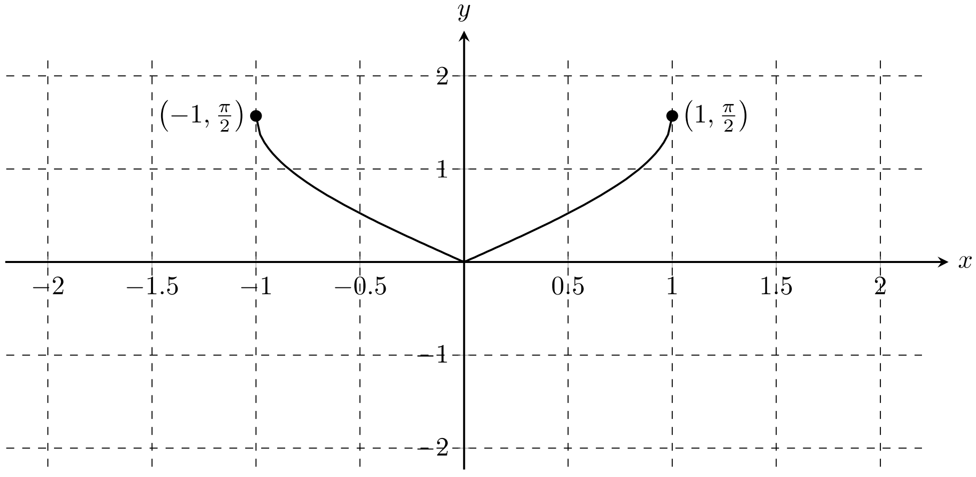
Question 3b.

$$\text{Var}(Z) = 4\,\text{Var}(X) + 9\,\text{Var}(Y)$$
$$= 4\times 4\times \frac{1}{2}\times \frac{1}{2} + 9\times 6\times \frac{1}{2}\times \frac{1}{2}$$
$$= 4 + \frac{27}{2}$$
$$= \frac{35}{2}$$
$$\text{sd}(Z) = \sqrt{\frac{35}{2}}$$

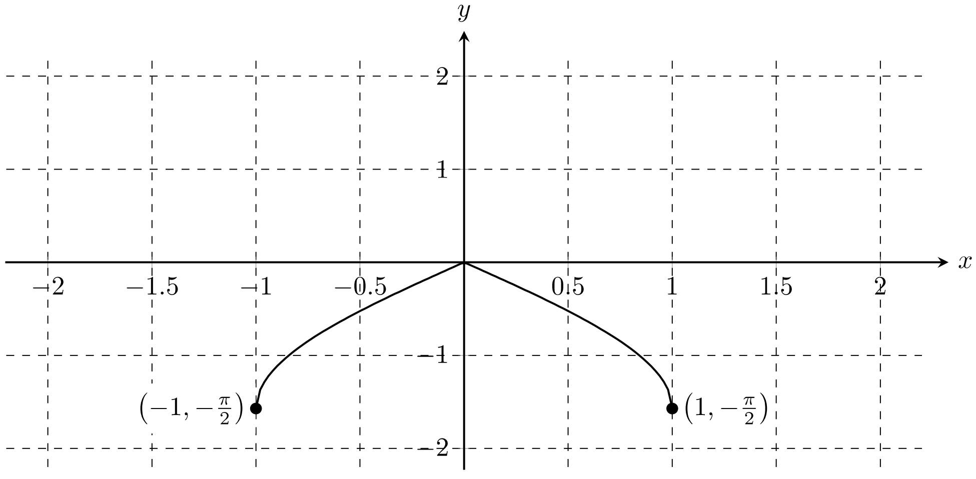
Question 4a.



Question 4b.



Question 4c.



Question 5a.

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\arctan(y) = \arctan(x) + c$$

When $x = 0$, $y = 1$ and so $c = \frac{\pi}{4}$. Therefore

$$y = \tan\left(\arctan(x) + \frac{\pi}{4}\right)$$

$$= \frac{x + \tan\left(\frac{\pi}{4}\right)}{1 - x\tan\left(\frac{\pi}{4}\right)}$$

$$= \frac{x+1}{1-x}$$

Question 5b.

$$y\left(\frac{1}{\sqrt{3}}\right) = \frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}}$$

$$= \frac{1+\sqrt{3}}{\sqrt{3}-1}$$

$$= \frac{\left(1+\sqrt{3}\right)\left(\sqrt{3}+1\right)}{3-1}$$

$$= \frac{4+2\sqrt{3}}{2}$$

$$= 2+\sqrt{3}$$

Question 6

Differentiate implicitly to obtain

$$3x^2y + x^3\frac{dy}{dx} + 2ay\frac{dy}{dx} = 0$$

The gradient of the line $4x+5y=9$ is $-\frac{4}{5}$. When $x=1$, $y=1$, $\frac{dy}{dx} = -\frac{4}{5}$:

$$3-\frac{4}{5}-\frac{8a}{5}=0$$

$$15-4-8a=0$$

$$8a=11$$

$$a=\frac{11}{8}$$

The curve passes through $(1,1)$:

$$1+\frac{11}{8}=b$$

$$b=\frac{19}{8}$$

$$a=\frac{11}{8},\ b=\frac{19}{8}$$

Question 7

Use either $v\frac{dv}{dx}=5+6x$ or $\frac{d}{dx}\left(\frac{1}{2}v^2\right)=5+6x$ to obtain

$$\frac{1}{2}v^2=5x+3x^2+c$$

When $x=0$, $v=4$. Therefore $c=8$ and $v^2=10x+6x^2+16$

When $x=2$, $v^2=20+24+16=60$ and since $v>0$, $v=\sqrt{60}=2\sqrt{15}$

Question 8

$$V=\pi\int_0^{\frac{1}{6}}\frac{36}{1-9x^2}dx=36\pi\int_0^{\frac{1}{6}}\frac{1}{1-9x^2}dx$$

Use partial fractions to obtain

$$V=18\pi\int_0^{\frac{1}{6}}\left(\frac{1}{1-3x}+\frac{1}{1+3x}\right)dx$$

$$=18\pi\left[-\frac{1}{3}\log_e|1-3x|+\frac{1}{3}\log_e|1+3x|\right]_0^{\frac{1}{6}}$$

$$=18\pi\left(-\frac{1}{3}\log_e\left(\frac{1}{2}\right)+\frac{1}{3}\log_e\left(\frac{3}{2}\right)\right)$$

$$=6\pi\log_e(3)$$

Question 9a.

$$\frac{d}{dx}(\sec(x)) = \frac{d}{dx}\left(\frac{1}{\cos(x)}\right)$$
$$= \frac{\sin(x)}{\cos^2(x)}$$
$$= \frac{1}{\cos(x)} \cdot \frac{\sin(x)}{\cos(x)}$$
$$= \sec(x)\tan(x)$$

Question 9b.

$$\frac{d}{dx}\left(\log_e(\sec(x)+\tan(x))\right) = \frac{\sec(x)\tan(x)+\sec^2(x)}{\sec(x)+\tan(x)}$$
$$= \frac{\sec(x)\left(\tan(x)+\sec(x)\right)}{\sec(x)+\tan(x)}$$
$$= \sec(x)$$

Question 9c.

Note that

$$y = \log_e(\sec(x))$$
$$= \log_e\left(\frac{1}{\cos(x)}\right)$$
$$= -\log_e(\cos(x))$$
$$\frac{dy}{dx} = -\frac{-\sin(x)}{\cos(x)}$$
$$= \tan(x)$$

The length of the curve is

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sqrt{1+\tan^2(x)}\,dx = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec(x)\,dx$$
$$= \left[\log_e(\sec(x)+\tan(x))\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$
$$= \log_e\left(2+\sqrt{3}\right) - \log_e\left(2-\sqrt{3}\right)$$
$$= \log_e\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}\right)$$
$$= \log_e\left(7+4\sqrt{3}\right)$$

Question 10

Squaring both sides gives:

$$1+i\sqrt{a}+2\sqrt{1+i\sqrt{a}}\sqrt{1-i\sqrt{a}}+1-ia=2a$$

Therefore

$$\sqrt{1-i^2a}=a-1$$
$$1+a=(a-1)^2=a^2-2a+1$$
$$a^2-3a=0$$
$$a(a-3)=0$$

and so $a=0$ or $a=3$.

Since $a=0$ does not satisfy the original equation, $a=3$.