The Mathematical Association of Victoria

Trial Exam 2020

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers,
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 17 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Note: This examination was written for the Adjusted 2020 VCE Mathematics Study Design and accordingly does not include the Specialist Mathematics Area of Study 6 (Probability and Statistics).

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Instructions

Answer **all** questions in the space provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2} where $g = 9.8$.

Question 1 (2 marks)

Find $\frac{dy}{dx}$ at the point where $x = 0$ for the curve defined by the relation $y^3 - xy + e^{-x} = -7$.

Question 2 (6 marks)

Let $g : D \to R$, $g(x) = x - 2 - \dfrac{2}{|x-1|-2}$ where D is the maximal domain of g.

a. Find the maximal domain D of g. 2 marks

b. Solve $g(x) = 0$. 3 marks

c. Express g as a piecewise (hybrid) function. 1 mark

TURN OVER

Question 3 (5 marks)

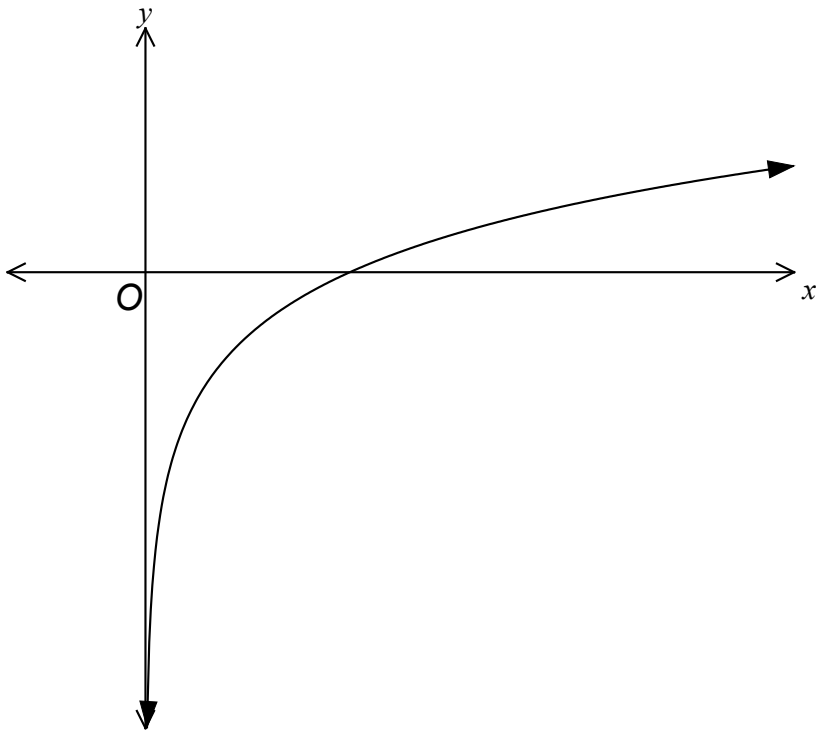
a. Express $\frac{4}{4-x^2}$ in partial fraction form. 1 mark

b. Solve the differential equation $\frac{dy}{dx} = \frac{4y\cos(x)}{3+\cos^2(x)}$ given that $y(\pi) = 1$. Express your answer in the form $y = \frac{a}{b-\sin(x)} + c$ where a, b and c are integers. 4 marks

TURN OVER

Question 4 (3 marks)

Part of the graph of $y = \log_2(x)$ is shown below.



Sketch the graph of $y = \dfrac{1}{\log_2(x)}$ on the set of axes above, given that it has a point of inflection at $x = \dfrac{1}{e^2}$. Clearly label its asymptotes with their equation and any endpoints and points of intersection with $y = \log_2(x)$ with their coordinates.

Working space

Question 5 (3 marks)

The position vectors of two particles *A* and *B* at time *t* seconds after they have started moving are given by $\underset{\sim A}{\mathrm{r}}(t) = \sin(at)\,\underset{\sim}{\mathrm{i}} + \cos(t)\,\underset{\sim}{\mathrm{j}}$ and $\underset{\sim B}{\mathrm{r}}(t) = \sin(t)\,\underset{\sim}{\mathrm{i}} - \cos(at)\,\underset{\sim}{\mathrm{j}}$ respectively, where *a* is a positive real constant and $t \geq 0$.

Find the two smallest possible values of *a* if the particles are moving perpendicular to each other at $t = 2$.

TURN OVER

Question 6 (5 marks)

After being given an initial push, an object of mass 4 kg slides down a rough plane inclined at 30^0 to the horizontal with an acceleration of $3\,\text{ms}^{-2}$.
After four seconds the velocity of the object is $14\,\text{ms}^{-1}$.

a. Find, in newtons, the size of the friction force acting on the object. 2 marks

Working space

b. Use calculus to find, in ms^{-1}, the speed of the object when it has travelled 4 metres. 3 marks

TURN OVER

Question 7 (5 marks)

a. Write the function $f(x) = \dfrac{x}{\sin\left(2\arcsin\left(\sqrt{x}\right)\right)}$, $0 < x < 1$, in the form $f(x) = \dfrac{a\sqrt{x}}{\sqrt{b-x}}$ where where a and b are positive real constants. 2 marks

b. Find the volume of the solid of revolution formed when the graph of $y = \dfrac{x}{\sin\left(2\arcsin\left(\sqrt{x}\right)\right)}$ is rotated about the x-axis over the interval $\left[\dfrac{1}{4}, \dfrac{1}{2}\right]$. 3 marks

TURN OVER

Question 8 (6 marks)

Let $z = r\text{cis}(\theta)$ be a solution to $z^n - 3i\bar{z} = 0$, $z \in C$, where $n \in Z^+$ and $n \geq 2$.

a. Find all possible values of $|z|$. 2 marks

b. Show that either $\theta = \frac{(1+4k)}{2(n+1)}\pi$, $k \in Z$, or θ is undefined. 3 marks

c. State **with justification** the number of solutions to $z^5 - 3i\bar{z} = 0$. 1 mark

TURN OVER

Question 9 (5 marks)

a. Find the derivative of $x^2 \arccos\left(\frac{1}{\sqrt{x}}\right)$. 2 marks

b. Hence find an anti-derivative of $x \arccos\left(\frac{1}{\sqrt{x}}\right)$. 3 marks

END OF QUESTION AND ANSWER BOOK