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Specialist Mathematics

2020

Trial Examination 1
(1 hour)

Instructions

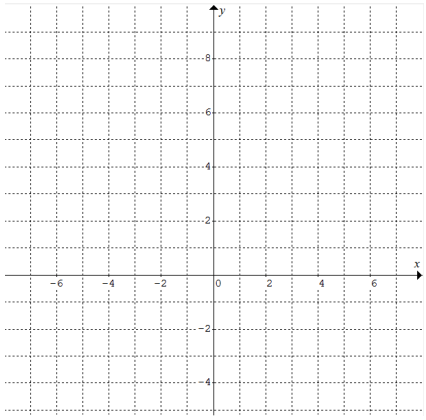
Answer **all** questions. Do **not** use calculators.
Unless otherwise specified, an **exact** answer is required to a question.
Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.
In questions where more than one mark is available, show appropriate working or explanation.
Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1 Consider $f(x) = \dfrac{1}{x+2} + x + 2$.

a. Determine the equations of asymptotes, axis intercepts, nature and coordinates of stationary points of $f(x)$.

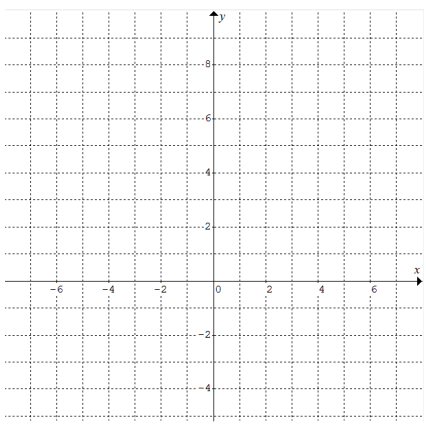
3 marks

b. Sketch the graph of $f(x) = \dfrac{1}{x+2} + x + 2$. Include features found in part a. 2 marks



c. Sketch the graph of $g(x) = \dfrac{1}{|x|+2} + |x| + 2$. Include features found in part a if they exist.

2 marks



Question 2 Evaluate $\int_0^1 \left(\frac{x^2 + x + 1}{2x^3 - x^2 - x - 3} \right) dx$. 3 marks

Question 3 Vector $\tilde{c}$ bisects the angle between non-parallel vectors $\tilde{a}$ and $\tilde{b}$.
Let $|\tilde{a}|$, $|\tilde{b}|$ and $|\tilde{c}|$ be a, b and c respectively, and $c = \sqrt{2ab}$.

a. Express $\tilde{c}$ in terms of a, b, $\tilde{a}$ and $\tilde{b}$. 2 marks

b. Given $\tilde{a} = 4\tilde{i} - 3\tilde{j}$ and $\tilde{b} = 3\tilde{j} + 4\tilde{k}$, find $\tilde{c}$ in terms of $\tilde{i}$ and $\tilde{k}$. 1 mark

Question 4 Given $y^2 = xy\left(x^2 - 1\right) + 1$, find the value/s of $\frac{dy}{dx}$ when $y = 1$. 3 marks

Question 5 Consider $\frac{z+1}{z+3}$ where $z = x + iy$ and $x \in R^-$ and $y \in R^+$.

a. $\text{Arg}\left(\frac{z+1}{z+3}\right)$ can be expressed in the form $\tan^{-1}\left(\frac{\alpha}{\beta}\right)$. Find α and β in terms of x and y.

2 marks

b i. Write a Cartesian equation for $\text{Arg}\left(\frac{z+1}{z+3}\right) = \frac{\pi}{6}$. 1 mark

b ii. Show that $\left\{z : \text{Arg}\left(\frac{z+1}{z+3}\right) = \frac{\pi}{6}\right\} \subset \left\{z : \left|z + 2 - i\sqrt{3}\right| = 2\right\}$. 3 marks

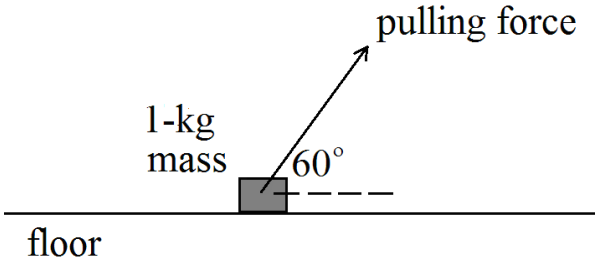
Question 6 The positions of particle A and particle B at time $t \geq 0$ is given by $x = \sin^{-1}(t)$ and $x = \cos^{-1}\left(\frac{t}{2}\right)$ respectively. Time *t* is in seconds and position *x* from origin *O* is in metres.

a. Determine the time when the particles collide. 2 marks

b. Just before collision the velocity of particle B is $\frac{\sqrt{a}}{b}$ where $a, b \in R \setminus \{0\}$.

Find the values of a and b. 2 marks

Question 7 A 1-kg mass is placed on a rough floor. It is then pulled by a force at $60°$ angle to the horizontal. The mass starts from rest and moves in a straight line horizontally.



a. Determine the maximum acceleration of the mass. 2 marks

b. Determine the average speed of the mass moving at maximum acceleration in the first $\sqrt{3}$ s.

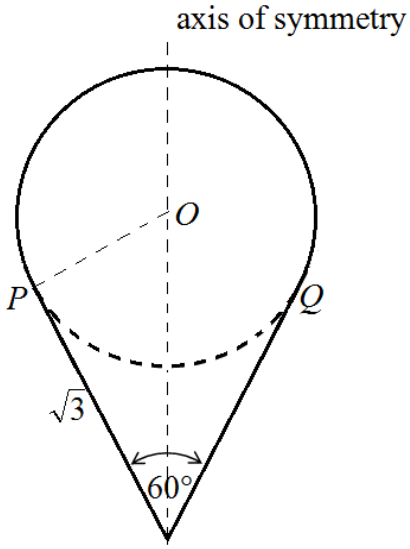
1 mark

Question 8 Oranges grown in an orchard have mean mass of 0.18 kg and standard deviation of 0.01 kg. The oranges are packaged in 6 or 18 in a bag. The mass of a bag is negligible.
Compare the mean and standard deviation of the mass of *a bag of 18 oranges* with that of *three bags of 6 oranges*. Show relevant calculations to substantiate your answer.

3 marks

Question 9 Refer to the shape shown in the diagram below.
A solid of revolution is formed by rotating the shape about its axis of symmetry.
The two solid straight lines, each of length $\sqrt{3}$, are tangents to the circle with centre at O.
The solid formed consists of a cone (defined by the 2 tangents) and part of a sphere (defined by the major arc of the circle).

a. Calculate the volume of the cone. 1 mark



b. Calculate the distance of O above the line joining P and Q, points where the tangent lines touch the circle.

1 mark

c. Determine the volume of the solid of revolution of the shape about its axis of symmetry.

3 marks

Question 10 In a large population the height X of an adult has a normal distribution with a mean of 175 cm. A random sample of 100 adults from the population is taken.

a. If the sample mean height is 174.4 cm and the mean height of the 80 tallest in the sample is 175.5 cm, calculate the mean height of the remaining adults in the sample.

1 mark

b. Determine the standard deviation of adult height in the population if $\Pr(\overline{X} > 178.0) \approx 0.025$. Express your answer to the nearest cm.

2 marks

End of Exam 1