VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2019

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER

SPECIALIST MATHEMATICS

Written examination 2

Wednesday 5 June 2019

Reading time: 10.00 am to 10.15 am (15 minutes)
Writing time: 10.15 am to 12.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 23 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1

The graph of which one of the following relations does **not** have a vertical asymptote?

A. $y = \dfrac{x^3 - 1}{x}$

B. $y = \dfrac{5x^2 + 2}{x^2 + 1}$

C. $y = \dfrac{x^4 - 3}{x^2}$

D. $y = \dfrac{1}{x^2 + 4x}$

E. $y = \dfrac{x - 1}{\sqrt{x + 2}}$

Question 2

The curve given by $x = 3\sec(t) + 1$ and $y = 2\tan(t) - 1$ can be expressed in cartesian form as

A. $\dfrac{(y+1)^2}{4} - \dfrac{(x-1)^2}{9} = 1$

B. $\dfrac{(x+1)^2}{3} - \dfrac{(y-1)^2}{2} = 1$

C. $\dfrac{(y+1)^2}{9} - \dfrac{(x-1)^2}{4} = 1$

D. $\dfrac{(x-1)^2}{3} + \dfrac{(y+1)^2}{2} = 1$

E. $\dfrac{(x-1)^2}{9} - \dfrac{(y+1)^2}{4} = 1$

Question 3

The maximal domain and range of the function $f(x) = a\cos^{-1}(bx) + c$, where a, b and c are real constants with $a > 0$, $b < 0$ and $c > 0$, are respectively

A. $[0, \pi]$ and $[-a, a]$

B. $[0, \pi]$ and $[-a + c, a + c]$

C. $\left[-\frac{1}{b}, \frac{1}{b}\right]$ and $[c, a\pi + c]$

D. $\left[\frac{1}{b}, -\frac{1}{b}\right]$ and $[c, a\pi + c]$

E. $\left[\frac{1}{b}, -\frac{1}{b}\right]$ and $[-a\pi + c, a\pi + c]$

Question 4

Which one of the following statements is **false** for $z_1, z_2 \in C$?

A. $z^{-1} = \frac{\overline{z}}{|z|^2}, \ z \neq 0$

B. $|z_1 + z_2| > |z_1| + |z_2|$

C. $\frac{z_1}{z_2} = \frac{z_1\overline{z_2}}{|z_2|^2}, \ z_2 \neq 0$

D. $|z_1 z_2| = |z_1||z_2|$

E. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, \ z_2 \neq 0$

Question 5

The circle defined by $|z + a| = 3|z + i|$, where $a \in R$, has a centre and radius respectively given by

A. $\left(\frac{a}{8}, -\frac{9}{8}\right), \ \frac{3}{8}\sqrt{a^2 + 1}$

B. $\left(\frac{a}{8}, -\frac{9}{8}\right), \ \frac{9a^2 + 9}{64}$

C. $\left(\frac{a}{8}, -\frac{9}{8}\right), \ \frac{1}{8}\sqrt{153 - 7a^2}$

D. $\left(-\frac{a}{8}, \frac{9}{8}\right), \ \frac{9a^2 + 9}{64}$

E. $\left(-\frac{a}{8}, \frac{9}{8}\right), \ \frac{3}{8}\sqrt{a^2 + 1}$

Question 6

$P(z)$ is a polynomial of degree n with real coefficients where $z \in C$. Three of the roots of the equation $P(z) = 0$ are $z = 3 - 2i$, $z = 4$ and $z = -5i$.

The smallest possible value of n is

A. 3

B. 4

C. 5

D. 6

E. 7

Question 7

The gradient of the line that is **perpendicular** to the graph of a relation at any point $P(x, y)$ is half the gradient of the line joining P and the point $Q(-1, 1)$.

The relation satisfies the differential equation

A. $\dfrac{dy}{dx} = \dfrac{y-1}{2(x+1)}$

B. $\dfrac{dy}{dx} = \dfrac{2(x+1)}{y+1}$

C. $\dfrac{dy}{dx} = \dfrac{2(x-1)}{y+1}$

D. $\dfrac{dy}{dx} = \dfrac{x+1}{2(1-y)}$

E. $\dfrac{dy}{dx} = \dfrac{2(x+1)}{1-y}$

Question 8

The total area enclosed between the x-axis and the graph of $f(x) = \left|x^3\right| - x^2 - |x|$ is closest to

A. –2.015

B. –1.008

C. 1.008

D. 2.015

E. 2.824

SECTION A – continued

DO NOT WRITE IN THIS AREA

Question 9

With a suitable substitution, $\int_1^2 \sqrt{5x-1}\,dx$ can be expressed as

A. $5\int_1^2 \sqrt{u}\,du$

B. $\frac{1}{5}\int_1^2 \sqrt{u}\,du$

C. $5\int_4^9 \sqrt{u}\,du$

D. $\frac{1}{5}\int_4^9 \sqrt{u}\,du$

E. $5\int_4^9 \sqrt{5u-1}\,du$

Question 10

Euler's method, with a step size of 0.1, is used to approximate the solution of the differential equation

$\frac{1}{y}\frac{dy}{dx} = \cos(x)$, with $y = 2$ when $x = 0$.

When $x = 0.2$, the value obtained for y, correct to four decimal places, is

A. 2.2000
B. 2.3089
C. 2.3098
D. 2.4189
E. 2.4199

Question 11

The vector resolute of $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$ that is **perpendicular** to $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ is

A. $-\frac{2}{3}\left(\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}\right)$

B. $-\frac{2}{3}\left(2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}\right)$

C. $\frac{1}{3}\left(8\underset{\sim}{i} - \underset{\sim}{j} + 7\underset{\sim}{k}\right)$

D. $\underset{\sim}{i} - 2\underset{\sim}{j} + 4\underset{\sim}{k}$

E. $\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$

Question 12

Given that θ is the acute angle between the vectors $\underset{\sim}{a} = -2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = -4\underset{\sim}{i} + 4\underset{\sim}{j} + 7\underset{\sim}{k}$, then $\sin(2\theta)$ is equal to

A. $2\sqrt{2}$

B. $\frac{4\sqrt{2}}{9}$

C. $\frac{2\sqrt{2}}{9}$

D. $\frac{2\sqrt{2}}{3}$

E. $\frac{4\sqrt{2}}{3}$

Question 13

For the vectors $\underset{\sim}{a} = \underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$, $\underset{\sim}{b} = -\underset{\sim}{i} - 4\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{c} = -\underset{\sim}{i} - 6\underset{\sim}{j} + \lambda\underset{\sim}{k}$ to be **linearly dependent**, the value of λ must be

A. 0

B. 1

C. 2

D. 3

E. 4

Question 14

The position vector $\underset{\sim}{r}(t)$ of a mass of 3 kg after t seconds, where $t \geq 0$, is given by $\underset{\sim}{r}(t) = 10t\,\underset{\sim}{i} + \left(16t^2 - \frac{4}{3}t^3\right)\underset{\sim}{j}$.
The force, in newtons, acting on the mass when $t = 2$ seconds is

A. $16\underset{\sim}{j}$

B. $32\underset{\sim}{j}$

C. $48\underset{\sim}{j}$

D. $30\underset{\sim}{i} + 144\underset{\sim}{j}$

E. 16

Question 15

A lift accelerates from rest at a constant rate until it reaches a speed of 3 ms^{-1}. It continues at this speed for 10 seconds and then decelerates at a constant rate before coming to rest. The total travel time for the lift is 30 seconds.

The total distance, in metres, travelled by the lift is

A. 30

B. 45

C. 60

D. 75

E. 90

Question 16

An object of mass 2 kg is travelling horizontally in a straight line at a constant velocity of magnitude 2 ms^{-1}. The object is hit in such a way that it deflects 30° from its original path, continuing at the same speed in a straight line.

The magnitude, correct to two decimal places, of the change of momentum, in kg ms^{-1}, of the object is

A. 0.00

B. 0.24

C. 1.04

D. 1.46

E. 2.07

Question 17

A ball is thrown vertically upwards with an initial velocity of $7\sqrt{6}$ ms^{-1}, and is subject to gravity and air resistance. The acceleration of the ball is given by $\ddot{x} = -(9.8 + 0.1v^2)$, where v ms^{-1} is its velocity when it is at a height of x metres above ground level.

The maximum height, in metres, reached by the ball is

A. $5\log_e(4)$

B. $\log_e\left(\sqrt{31}\right)$

C. $\frac{5\pi\sqrt{2}}{21}$

D. $5\log_e(2)$

E. $\frac{7\pi\sqrt{2}}{3}$

Question 18

Consider a random variable X with probability density function

$$f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & x < 0 \text{ or } x > 1 \end{cases}$$

If a large number of samples, each of size 100, is taken from this distribution, then the distribution of the sample means, $\bar{X}$, will be approximately normal with mean $\mathrm{E}\left(\bar{X}\right) = \frac{2}{3}$ and standard deviation $\mathrm{sd}\left(\bar{X}\right)$ equal to

A. $\frac{\sqrt{2}}{60}$

B. $\frac{\sqrt{2}}{6}$

C. $\frac{1}{180}$

D. $\frac{1}{18}$

E. $\frac{\sqrt{2}}{30}$

Question 19

Bags of peanuts are packed by a machine. The masses of the bags are normally distributed with a standard deviation of three grams.

The minimum size of a sample required to ensure that the manufacturer can be 98% confident that the sample mean is within one gram of the population mean is

A. 37

B. 38

C. 48

D. 49

E. 60

Question 20

Nitrogen oxide emissions for a certain type of car are known to be normally distributed with a mean of 0.875 g/km and a standard deviation of 0.188 g/km.

For two randomly selected cars, the probability that their nitrogen oxide emissions differ by more than 0.5 g/km is closest to

A. 0.030

B. 0.060

C. 0.960

D. 0.970

E. 0.977

END OF SECTION A

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SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (10 marks)

In the complex plane, L is the line with equation $|z+2| = |z-1-\sqrt{3}\,i|$.

a. Verify that the point (0, 0) lies on L. 1 mark

b. Show that the cartesian form of the equation of L is $y = -\sqrt{3}\,x$. 2 marks

c. The line L can also be expressed in the form $|z-1| = |z-z_1|$, where $z_1 \in C$.

Find z_1 in cartesian form. 2 marks

SECTION B – Question 1 – continued

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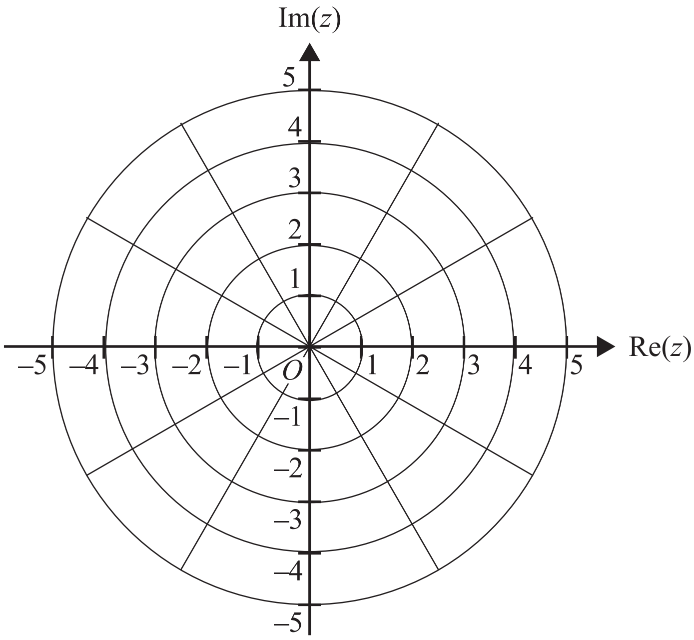
d. Find, in cartesian form, the point(s) of intersection of L and the graph of $|z| = 4$. 2 marks

e. Sketch L and the graph of $|z| = 4$ on the Argand diagram below. 2 marks



f. Find the area of the sector defined by the part of L where $\text{Re}(z) \geq 0$, the graph of $|z| = 4$ where $\text{Re}(z) \geq 0$, and the imaginary axis where $\text{Im}(z) > 0$. 1 mark

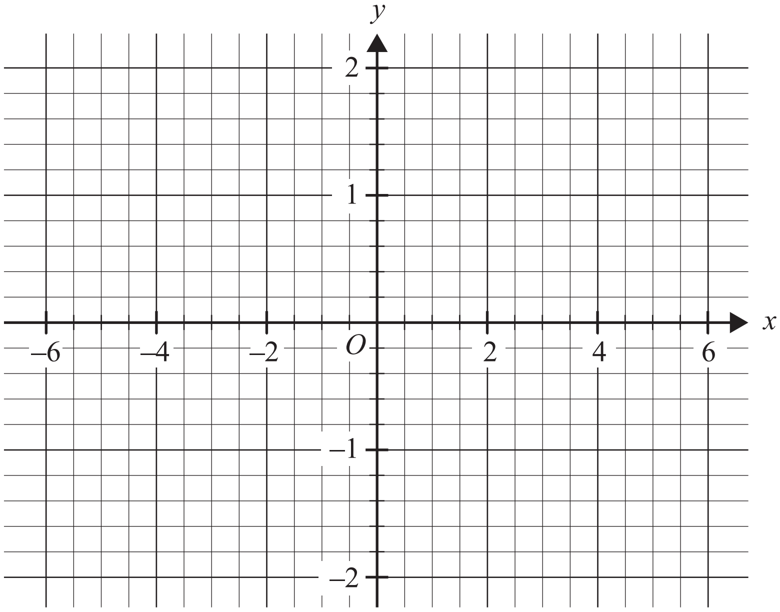
Question 2 (10 marks)

Consider the function f with rule $f(x) = \dfrac{x^2 + x + 1}{x^2 - 1}$.

a. **i.** State the equations of the asymptotes of the graph of f. 2 marks

ii. State the coordinates of the stationary points and the point of inflection. Give your answers correct to two decimal places. 2 marks

iii. Sketch the graph of f from $x = -6$ to $x = 6$ (endpoint coordinates are not required) on the set of axes below, labelling the turning points and the point of inflection with their coordinates correct to two decimal places. Label the asymptotes with their equations. 3 marks



Consider the function f_k with rule $f_k(x) = \dfrac{x^2 + x + k}{x^2 - 1}$, where $k \in R$.

b. For what values of k will f_k have **no stationary points**? 2 marks

c. For what value of k will the graph of f_k have a point of inflection located on the y-axis? 1 mark

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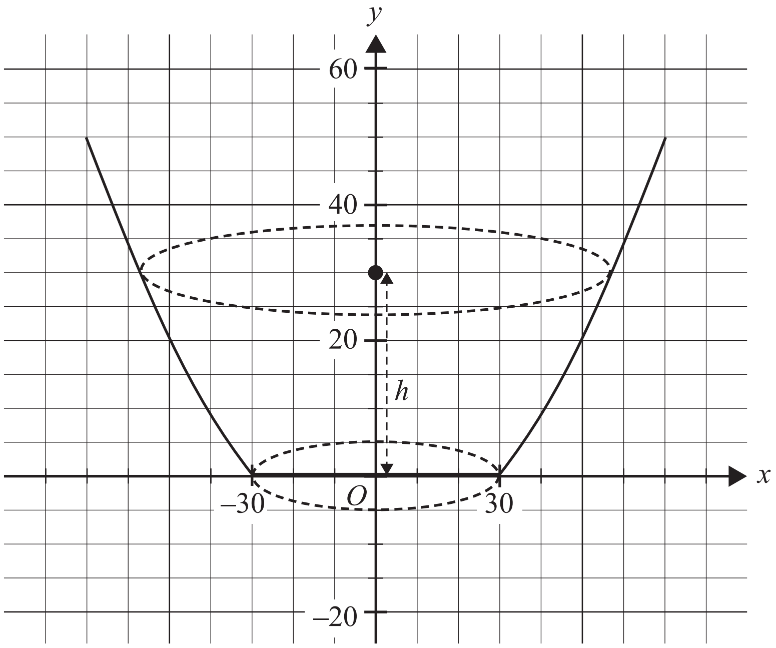
Question 3 (9 marks)



The vertical cross-section of a barrel is shown above. The radius of the circular base (along the x-axis) is 30 cm and the radius of the circular top is 70 cm. The curved sides of the cross-section shown are parts of the parabola with rule $y = \frac{x^2}{80} - \frac{45}{4}$. The height of the barrel is 50 cm.

a. **i.** Show that the volume of the barrel is given by $\pi \int_0^{50} (900 + 80y)\,dy$. 1 mark

ii. Find the volume of the barrel in cubic centimetres. 1 mark

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The barrel is initially full of water. Water begins to leak from the bottom of the barrel such that $\frac{dV}{dt} = \frac{-8000\pi\sqrt{h}}{A}$ cubic centimetres per second, where after t seconds the depth of the water is h centimetres, the volume of water remaining in the barrel is V cubic centimetres and the uppermost surface area of the water is A square centimetres.

b. Show that $\frac{dV}{dt} = \frac{-400\sqrt{h}}{4h+45}$. 2 marks

c. Find $\frac{dh}{dt}$ in terms of h. Express your answer in the form $\frac{-a\sqrt{h}}{\pi(b+ch)^2}$, where a, b and c are positive integers. 3 marks

d. Using a definite integral in terms of h, find the **time**, **in hours**, correct to one decimal place, taken for the barrel to empty. 2 marks

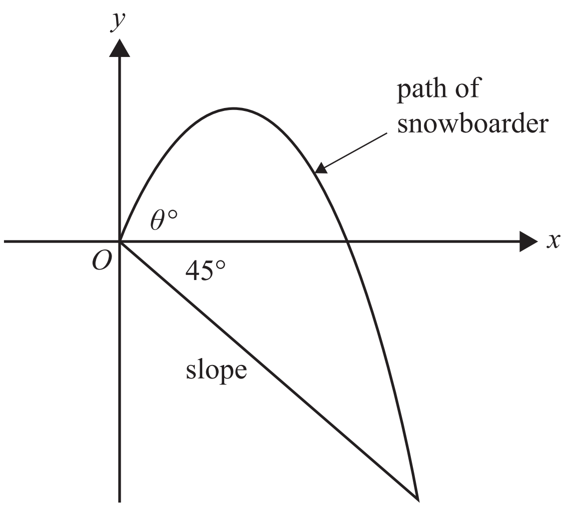
Question 4 (10 marks)

A snowboarder at the Winter Olympics leaves a ski jump at an angle of θ degrees to the horizontal, rises up in the air, performs various tricks and then lands at a distance down a straight slope that makes an angle of 45° to the horizontal, as shown below.

Let the origin O of a cartesian coordinate system be at the point where the snowboarder leaves the jump, with a unit vector in the positive x direction being represented by $\underset{\sim}{i}$ and a unit vector in the positive y direction being represented by $\underset{\sim}{j}$. Distances are measured in metres and time is measured in seconds.

The position vector of the snowboarder t seconds after leaving the jump is given by

$$\underset{\sim}{r}(t) = \left(6t - 0.01t^3\right)\underset{\sim}{i} + \left(6\sqrt{3}t - 4.9t^2 + 0.01t^3\right)\underset{\sim}{j},\ t \geq 0$$



a. Find the angle θ°. 2 marks

b. Find the speed, in metres per second, of the snowboarder when she leaves the jump at O. 1 mark

DO NOT WRITE IN THIS AREA

c. Find the maximum height above O reached by the snowboarder. Give your answer in metres, correct to one decimal place. 2 marks

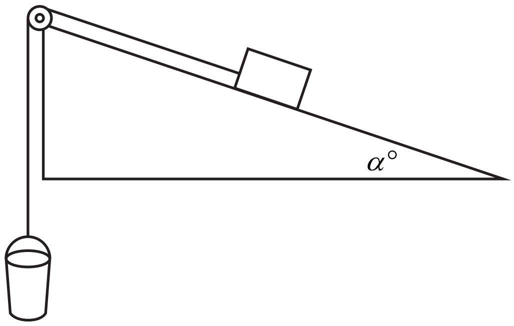
d. Show that the time spent in the air by the snowboarder is $\frac{60\left(\sqrt{3}+1\right)}{49}$ seconds. 3 marks

e. Find the total distance the snowboarder travels while airborne. Give your answer in metres, correct to two decimal places. 2 marks

DO NOT WRITE IN THIS AREA

Question 5 (12 marks)

A pallet of bricks weighing 500 kg sits on a rough plane inclined at an angle of $\alpha°$ to the horizontal, where $\tan(\alpha°) = \frac{7}{24}$. The pallet is connected by a light inextensible cable that passes over a smooth pulley to a hanging container of mass *m* kilograms in which there is 10 L of water. The pallet of bricks is held in equilibrium by the tension *T* newtons in the cable and a frictional resistance force of 50 *g* newtons acting up and parallel to the plane. Take the weight force exerted by 1 L of water to be *g* newtons.



a. Label all forces acting on both the pallet of bricks and the hanging container on the diagram above, when the pallet of bricks is in equilibrium as described. 1 mark

b. Show that the value of *m* is 80. 3 marks

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Suddenly the water is completely emptied from the container and the pallet of bricks begins to slide down the plane. The frictional resistance force of 50 g newtons acting up the plane continues to act on the pallet.

c. Find the distance, in metres, travelled by the pallet after 10 seconds. 3 marks

d. When the pallet reaches a velocity of 3 ms^{-1}, water is poured back into the container at a constant rate of 2 L per second, which in turn retards the motion of the pallet moving down the plane. Let *t* be the time, in seconds, after the container begins to fill.

i. Write down, in terms of *t*, an expression for the total mass of the hanging container and the water it contains after *t* seconds. Give your answer in kilograms. 1 mark

ii. Show that the acceleration of the pallet down the plane is given by $\frac{g(5-t)}{t+290}$ ms^{-2} for $t \in [0, 5)$. 2 marks

iii. Find the velocity of the pallet when $t = 4$. Give your answer in metres per second, correct to one decimal place. 2 marks

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Question 6 (9 marks)

A paint company claims that the mean time taken for its paint to dry when motor vehicles are repaired is 3.55 hours, with a standard deviation of 0.66 hours.

Assume that the drying time for the paint follows a normal distribution and that the claimed standard deviation value is accurate.

a. Let the random variable $\bar{X}$ represent the mean time taken for the paint to dry for a random sample of 36 motor vehicles.

Write down the mean and standard deviation of $\bar{X}$. 2 marks

At a crash repair centre, it was found that the mean time taken for the paint company's paint to dry on 36 randomly selected vehicles was 3.85 hours. The management of this crash repair centre was not happy and believed that the claim regarding the mean time taken for the paint to dry was too low. To test the paint company's claim, a statistical test was carried out.

b. Write down suitable null and alternative hypotheses H_0 and H_1 respectively to test whether the mean time taken for the paint to dry is longer than claimed. 1 mark

c. Write down an expression for the p value of the statistical test and evaluate it correct to three decimal places. 2 marks

d. Using a 1% level of significance, state with a reason whether the crash repair centre is justified in believing that the paint company's claim of a mean time taken for its paint to dry of 3.55 hours is too low. 1 mark

e. At the 1% level of significance, find the set of sample mean values that would support the conclusion that the mean time taken for the paint to dry exceeded 3.55 hours. Give your answer in hours, correct to three decimal places. 2 marks

f. If the true mean time taken for the paint to dry is 3.83 hours, find the probability that the paint company's claim is not rejected at the 1% level of significance, assuming the standard deviation for the paint to dry is still 0.66 hours. Give your answer correct to two decimal places. 1 mark

END OF QUESTION AND ANSWER BOOK

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2019

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\operatorname{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \operatorname{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$
$z^n = r^n \operatorname{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(X) + bE(Y)$ $\operatorname{var}(aX + b) = a^2\operatorname{var}(X)$
for independent random variables X and Y	$\operatorname{var}(aX + bY) = a^2\operatorname{var}(X) + b^2\operatorname{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $E\left(\bar{X}\right) = \mu$ variance $\operatorname{var}\left(\bar{X}\right) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e \lvert x \rvert + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a}\log_e \lvert ax+b \rvert + c$
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1 + \left(f'(x)\right)^2}\, dx$ or $\int_{t_1}^{t_2} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert \underset{\sim}{r} \rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

END OF FORMULA SHEET