2019 VCE Specialist Mathematics 1 examination report

General comments

In the 2019 VCE Specialist Mathematics examination 1 students were required to answer 10 questions worth a total of 40 marks. Students were not permitted to bring technology or notes into the examination.

Among the highest-scoring questions were Questions 3a., 5ai., 6 and 7a. Questions 7c., 7d., 9b. and 10 did not score as highly. High scoring students presented their answers clearly and their working followed a logical sequence. Students are reminded that in questions requiring a given result to be shown, it is important that sufficient evidence is presented in order to be eligible for the marks.

Students should ensure that they read questions carefully and that their answers are reasonable. In Question 2, for example, answers could be verified as correct by substitution.

Areas of strength included:

- recognising a separable differentiation equation and applying separation of variables in order to solve a differential equation (Question 1)
- correct us of integral notation (Questions 1 and 8)
- finding the time when two particles are in the same position (Question 4)
- demonstrating linear dependence of a set of vectors (Question 6)
- use of de Moivre’s theorem (Question 7b.)
- finding the volume of the solid of revolution (Question 8)
- using the product and chain rules in implicit differentiation (Question 10).

Areas of weakness included:

- arithmetic (Question 3)
- algebra (Questions 2, 4 and 10)
- not reading questions carefully. In Question 9b. students were told that a mass hangs from a smooth ring on a string and that the tension in the string has a constant magnitude; a large number of students did not use this information and consequently had difficulty making progress with the question
- not recognising and writing down the anti-derivative of standard functions leading to unnecessary use of substitutions in integration problems (Questions 1 and 8).

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Question 1

Marks	0	1	2	3	4	Average
%	21	9	15	13	42	**2.5**

$y=\frac{\pi}{2}\left(1+e^{2x}\right)$

Most students recognised that they needed to separate and integrate in order to solve the differential equation although not all were then able to obtain the correct equation. Common errors were $\int 2y\,dy=\int\frac{e^{2x}}{1+e^{2x}}dx$ and $\int 2ye^{2x}\,dx=\int\frac{1}{1+e^{2x}}dx$. Students who failed to recognise that $\frac{d}{dx}\left(1+e^{2x}\right)=2e^{2x}$ did not score highly. Some students spent time using a substitution to determine $\int\frac{2e^{2x}}{1+e^{2x}}dx$, which was not necessary. Some students who managed to correctly find the value of the constant of integration did not use log or index laws correctly, presenting incorrect solutions such as $y=e^{2x}+1+\frac{\pi}{2}$.

An alternative approach was to solve $\int_{\pi}^{y}\frac{1}{t}dt=\int_{0}^{x}\frac{2e^{2t}}{1+e^{2t}}dt$. Note that a different variable of integration must be used. Students using this method typically retained x and y as the variables of integration and thus did not obtain full marks.

Question 2

Marks	0	1	2	3	Average
%	15	5	20	60	**2.3**

$x=-2$ or $x=22$

Common methods were to solve two linear equations: $x-4=\frac{x}{2}+7$ and $4-x=\frac{x}{2}+7$ or to solve a quadratic equation $(x-4)^2=\left(\frac{x}{2}+7\right)^2$. Some students drew a graph to support their reasoning. Students who solved linear equations were generally more successful than those who solved a quadratic equation. In the latter case, some students wrote $(x-4)^2=x^2\pm 16$ or $\left(\frac{x}{2}+7\right)^2=\frac{x^2}{4}+49$. Many had difficulty solving the quadratic equation.

Question 3a.

Marks	0	1	Average
%	11	89	**0.9**

$\frac{3\pi}{4}=0.75\pi$

This question was well done. Occasionally the π was missing from the answer.

Question 3b.

Marks	0	1	Average
%	70	30	**0.3**

$\frac{\pi^2}{1600} = 0.000625\pi^2$

Students could use fractions to find $\mathrm{Var}(V) = \mathrm{Var}\left(\pi r^2 h\right) = \frac{\pi^2}{16} \times \frac{1}{100} = \frac{\pi^2}{1600}$. Students who used this approach tended to score more highly than those using decimals, who sometimes were not able to evaluate $\left(\pi \times 0.25\right)^2 \times (0.1)^2$ correctly. A number of students omitted the π^2 from their answer.

Question 3c.

Marks	0	1	Average
%	50	50	**0.5**

$\frac{7\pi}{2} = 3.5\pi$

Some students were unable to evaluate $\frac{\pi}{2} + 3\pi$ correctly.

Question 4

Marks	0	1	2	3	Average
%	15	11	9	64	**2.2**

$a = \frac{\pi - 1}{3}$

Most students attempted to equate the $\underset{\sim}{i}$ components of $\underset{\sim}{r}_a(t)$ and $\underset{\sim}{r}_b(t)$ in order to determine the value of t when the particles collided. Various algebraic and transcription errors were made by students, which meant they could not be awarded full marks for the question.

Question 5ai.

Marks	0	1	Average
%	14	86	**0.9**

$f'(x) = -2\cos(x)\sin(x) - \sin(x)$ or $f'(x) = -\sin 2x - \sin x$

This question was well done. A small number of students had difficulty finding the derivative and some students who correctly differentiated attempted to factorise their answer with mixed success.

Some students used a double angle formula to write the answer in an alternative form. This was not always done correctly nor was it helpful for the next part of the question.

Question 5aii.

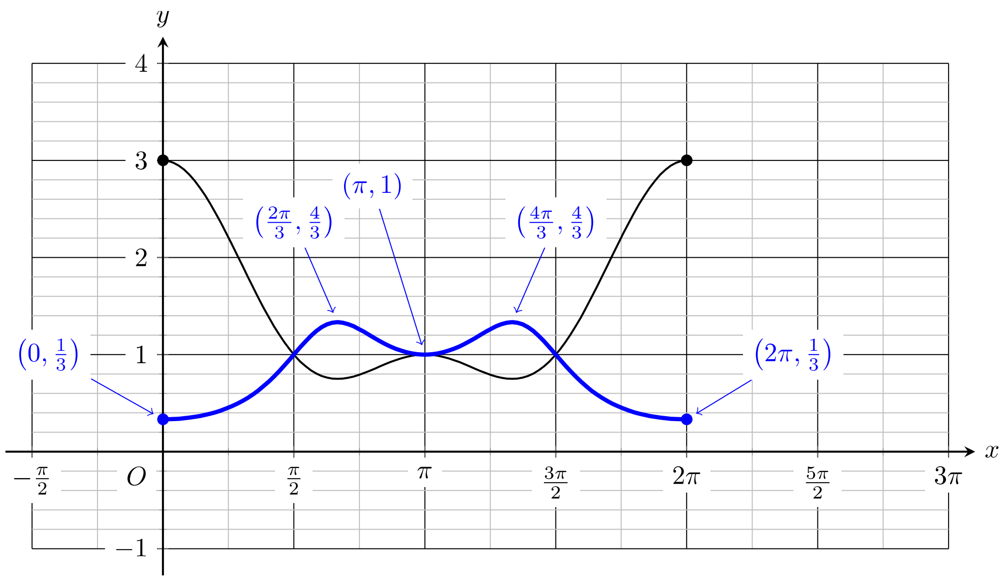
Marks	0	1	2	Average
%	6	52	42	1.4

$\left(\frac{2\pi}{3},\frac{3}{4}\right)$, $(\pi,1)$, $\left(\frac{4\pi}{3},\frac{3}{4}\right)$

This question was generally well done. A common mistake was to include the endpoints at $x=0$ and $x=2\pi$ even though the question specifically asked students to find the coordinates of the turning points in the interval $(0,2\pi)$.

Question 5b.

Marks	0	1	2	3	Average
%	23	19	31	27	1.7



Students' graph-sketching abilities were reasonable, with most drawing a single, smooth curve with the correct shape. Common errors included neglecting to label the turning point at $(\pi, 1)$, their graph not passing through the intersection points $\left(\frac{\pi}{2},1\right)$ and $\left(\frac{4\pi}{3},\frac{4}{3}\right)$, and poor estimation of the location of the heights $\frac{1}{3}$ and $\frac{4}{3}$ with respect to the given scale. Some students drew their graphs with an open circle at the endpoints.

Question 6

Marks	0	1	2	3	Average
%	16	4	17	63	2.3

$d=16$

This question was handled well with most students being able to write down correct simultaneous equations to solve. Occasional arithmetic and transcription errors were noted, but a large number were successful in finding the value of d. A number of students successfully evaluated a 3×3 determinant in order to find the value of d.

Question 7a.

Marks	0	1	Average
%	19	81	0.8

Students were required to show that $3-\sqrt{3}i=2\sqrt{3}\operatorname{cis}\left(-\frac{\pi}{6}\right)$ and students generally did this quite well. Some particular errors were noted. Some students wrote such things as

$\tan\left(\frac{\sqrt{3}}{3}\right)$ or $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)=\frac{\pi}{6}=-\frac{\pi}{6}$.

Question 7b.

Marks	0	1	2	Average
%	6	23	72	1.7

$-12\sqrt{12}i=-24\sqrt{3}i$

The efficient method was to use de Moivre's theorem although some students attempted to expand $\left(3-\sqrt{3}i\right)^3$. Students who chose the latter approach generally did not score as well.

Question 7c.

Marks	0	1	Average
%	61	39	0.4

$n=6k,\ k\in Z$

There were several ways to answer this question. Some students realised that if n was a positive or negative multiple of 6 then $\left(3-\sqrt{3}i\right)^n$ was real, but were unable to express this mathematically. Some students did not indicate that k was a member of Z, the set of integers.

Question 7d.

Marks	0	1	Average
%	74	26	0.3

$n=6k+3,\ k\in Z$

This question was answered poorly. There were a number of equivalent correct answers but many students were unable to find a general solution.

Question 8

Marks	0	1	2	3	4	Average
%	13	16	10	9	52	**2.7**

$\pi\left(\frac{\pi}{4}+\log_e(2)\right)$ or $\frac{\pi^2}{4}+\ \pi\log(2)$

Most students were able to write down the correct integral to find the volume of the solid of revolution. Some students did not recognise the way in which the integrand split naturally and had difficulty proceeding further with the question. Some attempted solutions using partial fractions were seen. Many students who were able to successfully split the integrand used a substitution method to integrate $\int_0^1 \frac{2x}{1+x^2}dx$. This was unnecessary and resulted in a loss of marks if not done correctly.

Question 9a.

Marks	0	1	Average
%	48	52	**0.5**

$T = \frac{mg}{2\sin(\alpha)}$

An efficient method was to resolve forces vertically giving $2T\sin\alpha = mg$, from which a correct answer follows. Various other correct answers were seen, often resulting from the use of a triangle of forces method. Some students did not give T in terms of m, g and α.

Question 9b.

Marks	0	1	2	3	Average
%	49	31	10	10	**0.8**

Students were required to show that

$$F = mg\left(\frac{1-\cos(\beta)}{\sin(\beta)}\right)$$

Students needed to resolve forces both vertically and horizontally. By considering the vertical forces, they found that

$$T = \frac{mg}{\sin(\beta)+\sin(2\beta)}$$

From consideration of the horizontal forces, students then found that

$$F = mg\left(\frac{\cos(\beta)-\cos(2\beta)}{\sin(\beta)+\sin(2\beta)}\right)$$

Use of a double angle formula and factorisation were required in order to obtain the given result. Students who went directly from their first expression for F to the answer were not able to receive full marks for the question.

Many students made the assumption that the tension in the string was different on either side of the ring, contradicting the statement of the question. Students who made this error had difficulty making further progress.

Question 10

Marks	**0**	**1**	**2**	**3**	**4**	**5**	**Average**
%	12	4	8	21	36	18	**3.2**

$$\frac{\pi - 2\sqrt{3}}{\sqrt{2}\left(\pi + \sqrt{3}\right)}$$

Most students were able to differentiate implicitly correctly. It made little difference if students substituted the values of x and y into their equation before or after obtaining an expression for $\frac{dy}{dx}$. Although various arithmetic and algebraic errors were seen, many students knew the exact values for $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{3}\right)$. Some students were unable to express the answer in the required form.

