TSSM
Creating VCE Success

**2019 Trial Examination**

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**STUDENT NUMBER** | | | | | | | | | Letter | |

# SPECIALIST MATHEMATICS

## Units 3&4 - Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

## QUESTION AND ANSWER BOOK

**Structure of book**

| *Section* | *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|---|
| A | 20 | 20 | 20 |
| B | 4 | 4 | 60 |
| | | | Total 80 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

**Materials supplied**

- Question and answer book of 29 pages.

**Instructions**

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.**

## SECTION A – Multiple-choice questions

| **Instructions for Section A** |
| --- |
| Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.<br>Choose the response that is **correct** for the question.<br>A correct answer scores 1, an incorrect answer scores zero.<br>Marks will **not** be deducted for incorrect answers.<br>No marks will be given if more than one answer is completed for any question. |

**Question 1**

Given $\cot\theta = \frac{3}{2}$, $\sec\theta =$

**A.** $\frac{\sqrt{13}}{3}$ only

**B.** $-\frac{\sqrt{13}}{3}$ only

**C.** $\frac{\sqrt{13}}{3}\ or\ -\frac{\sqrt{13}}{3}$

**D.** $-\frac{3}{\sqrt{13}}$ only

**E.** $\frac{3}{\sqrt{13}}\ or -\frac{3}{\sqrt{13}}$

**Question 2**

$\frac{\cos 2\alpha}{cos^4\alpha - sin^4\alpha}$ simplifies to

**A.** 1

**B.** $-1$

**C.** 2

**D.** $-\frac{1}{2}$

**E.** $\frac{1}{2}$

**SECTION A -** continued

**Question 3**
The number of x-intercepts for the function $f(x) = \tan(20x - \pi), x \in (0, 5\pi]$ is:

**A.** 4
**B.** 19
**C.** 20
**D.** 99
**E.** 100

**Question 4**
The set of solutions over C to $z^5 = i$, where $i = \sqrt{-1}$ can be generated from the rule:

**A.** $cis\left(\frac{\pi \pm 4n\pi}{10}\right)$, where n is an integer
**B.** $cis\left(\frac{\pi + 4n\pi}{5}\right)$, where n is an integer
**C.** $cis\left(\frac{\pi - 4n\pi}{5}\right)$, where n is an integer
**D.** $cis\left(\frac{\pi \pm 2n\pi}{10}\right)$, where n is an integer
**E.** $cis\left(\frac{\pi + 5n\pi}{10}\right)$, where n is an integer

**Question 5**
Given $\boldsymbol{a} = 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$ , a vector parallel to vector *a* with a length of 12 units is:

**A.** $12\boldsymbol{a}$
**B.** $12\hat{\boldsymbol{a}}$
**C.** $4\boldsymbol{i} + 2\boldsymbol{j} - 4\boldsymbol{k}$
**D.** $-8\boldsymbol{i} + 4\boldsymbol{j} - 8\boldsymbol{k}$
**E.** $8\boldsymbol{i} - 4\boldsymbol{j} - 8\boldsymbol{k}$

**SECTION A -** continued
**TURN OVER**

**Question 6**
Consider the following vectors:

$\overrightarrow{AB} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\overrightarrow{BC} = 6\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$

Let M be the midpoint of line AC. The acute angle between line AM and line MB is closest to:

**A.** 55°
**B.** 57°
**C.** 59°
**D.** 61°
**E.** 63°

**Question 7**
The cubic polynomial $P(z)$ has coefficients that are **NOT** all real.
Consider the solutions to $P(z) = 0$ over the complex field.
Which statement is true?

**A.** P(z) **cannot** have a complex conjugate pair of solutions.
**B.** P(z) **cannot** have 3 different complex solutions.
**C.** P(z) **must** have exactly 1 real solution.
**D.** P(z) **must** have a complex conjugate pair of solutions.
**E.** P(z) **could** have two real and one non-real solution.

**Question 8**
Let $m = 4 + i$ and $n = a + bi$ where $m, n \in C$ and $a, b \in R$.
Given $m \times n = -11 + 10i$ then $m + \bar{n} =$

**A.** $2 + i$
**B.** $2 - 2i$
**C.** $1 - 2i$
**D.** $2 - i$
**E.** $1 + 2i$

**SECTION A -** continued

**Question 9**

Consider the two points A and B where point A is on the curve defined by $|z - 2i| = 2$ and point B is on the line $Re(z) - Im(z + 4i) = 2$, where $z \in C$.
The smallest possible distance between points A and B is:

**A.** $4\sqrt{2} - 2$
**B.** $4\sqrt{2} + 2$
**C.** $4\sqrt{2}$
**D.** $2\sqrt{2} + 2$
**E.** $2\sqrt{2} - 2$

**Question 10**

$z^3 + az^2 - iz^2 + bz + iz + c = 0$ $where\ a, b, c \in R$ has solutions of
$z = 1, z = -i$ $and$ $z = 2i$. The values of a, b and c respectively are:

**A.** $a = 1, b = 2, c = -2$
**B.** $a = 1, b = -2, c = -2$
**C.** $a = -1, b = 2, c = -2$
**D.** $a = -1, b = -2, c = 2$
**E.** $a = -1, b = 2, c = 2$

**Question 11**

Given $f(x) = x\tan^{-1} 2x$; $f'\left(\frac{1}{2a}\right)$ for appropriate values of $a$ equals:

**A.** $\frac{2a}{a^2+1} + \tan^{-1}(\frac{1}{a})$
**B.** $\frac{a}{a^2+1} + \tan^{-1}(\frac{1}{a})$
**C.** $\frac{a}{a^2+1} + \tan^{-1}(\frac{1}{2a})$
**D.** $\frac{a}{4a^2+1} + \tan^{-1}(\frac{2}{a})$
**E.** $\frac{2a}{4a^2+1} + \tan^{-1}(\frac{2}{a})$

**SECTION A -** continued
**TURN OVER**

**Question 12**

$\int_0^1 \frac{2x}{\sqrt{2x+1}}dx$ can be rewritten as:

**A.** $2\int_0^1 u^{½} - u^{-½}du \ \ where\, u = 2x+1$

**B.** $2\int_0^1 u^{½} - u^{-½}du \ \ where\, u = \sqrt{2x+1}$

**C.** $½\int_1^3 u^{½} - u^{-½}du \ \ where\, u = \sqrt{2x+1}$

**D.** $½\int_1^3 u^{½} - u^{-½}du \ \ where\, u = 2x+1$

**E.** $½\int_1^3 u^{½} + u^{-½}du \ \ where\, u = 2x+1$

**Question 13**

The rule for the volume of a cone of radius *r* units and height *h* units can be generated by rotating:

**A.** $y = \frac{h}{r}x \ \ around\ the\ x - axis\ from\ x = r\ \ to\ \ x = h$

**B.** $y = \frac{h}{r}x \ \ around\ the\ x - axis\ from\ x = 0\ \ to\ \ x = r$

**C.** $y = \frac{h}{r}x \ \ around\ the\ y - axis\ from\ y = 0\ \ to\ \ y = r$

**D.** $y = \frac{h}{r}x \ \ around\ the\ y - axis\ from\ y = 0\ \ to\ \ y = h$

**E.** $y = \frac{r}{h}x \ \ around\ the\ y - axis\ from\ y = 0\ \ to\ \ y = r$

**Question 14**

The area bound by the curves $y = x^3 - 2x^2$ and $y = x^2 - 1$ is closest to:

**A.** 1 square unit

**B.** 2 square units

**C.** 3 square units

**D.** 4 square units

**E.** 5 square units

**SECTION A -** continued

**Question 15**

Which statement is **NOT** true of the curve $x^2 - 6x + y^2 + 8y = 0$ ?

**A.** It is a circle centred at $(3, -4)$

**B.** It passes through the origin.

**C.** A point on the curve has parametric equations $(5\cos\theta + 3,\ \ 5\sin\theta - 4)$

**D.** The gradient at any point on the curve is given by $\frac{-x+3}{y+4}$

**E.** $y = \frac{3}{4}x$ is a tangent to the curve.

**Question 16**

The maximum negative gradient on $f(x) = x^2e^x$ occurs when x =

**A.** $\sqrt{2} + 2$

**B.** $\sqrt{2} - 2$

**C.** $2 - \sqrt{2}$

**D.** $1 - \sqrt{2}$

**E.** $\sqrt{2} + 1$

**Question 17**

Consider the function $a(x) = f(x) \times (g(x))^2$

Given $f(2) = 2,\ g(2) = 0,\ f'(2) = 5,\ g'(2) = 3,$ then $a''(2) =$

**A.** 36

**B.** 24

**C.** 18

**D.** 12

**E.** 0

**SECTION A -** continued

**TURN OVER**

**Question 18**

The lengths of timber pickets are normally distributed with a mean of $\mu\ cm$ and a standard deviation of $1.5\ cm$.

What is the smallest sample size required to be at least 99% certain that the sample mean differs by less than 1 cm from the population mean?

**A.** 3
**B.** 4
**C.** 12
**D.** 15
**E.** 25

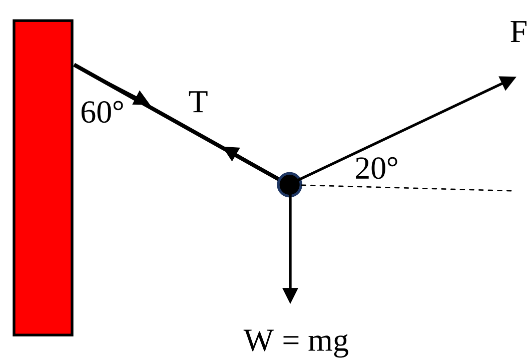
**Question 19**

A 10 $kg$ mass is just on the point of slipping when placed on a rough plane inclined at $20°$ to the horizontal. What force, to the nearest newton, needs to be applied directly up the plane for the mass to accelerate at $2\ ms^{-2}$ ?

**A.** 93
**B.** 87
**C.** 81
**D.** 54
**E.** 20

**Question 20**

An 8 kg mass attached to a string is pulled away from a vertical wall by force (F) inclined at $20°$ to the horizontal. Find, correct to the nearest newton, the magnitude of F so that the string maintains an angle of $60°$




**A.** 89
**B.** 96
**C.** 100
**D.** 104
**E.** 107

**END OF SECTION A**

## SECTION B – Extended response questions

**Instructions for Section B**

Answer **all** questions in the spaces provided.

In **all** questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

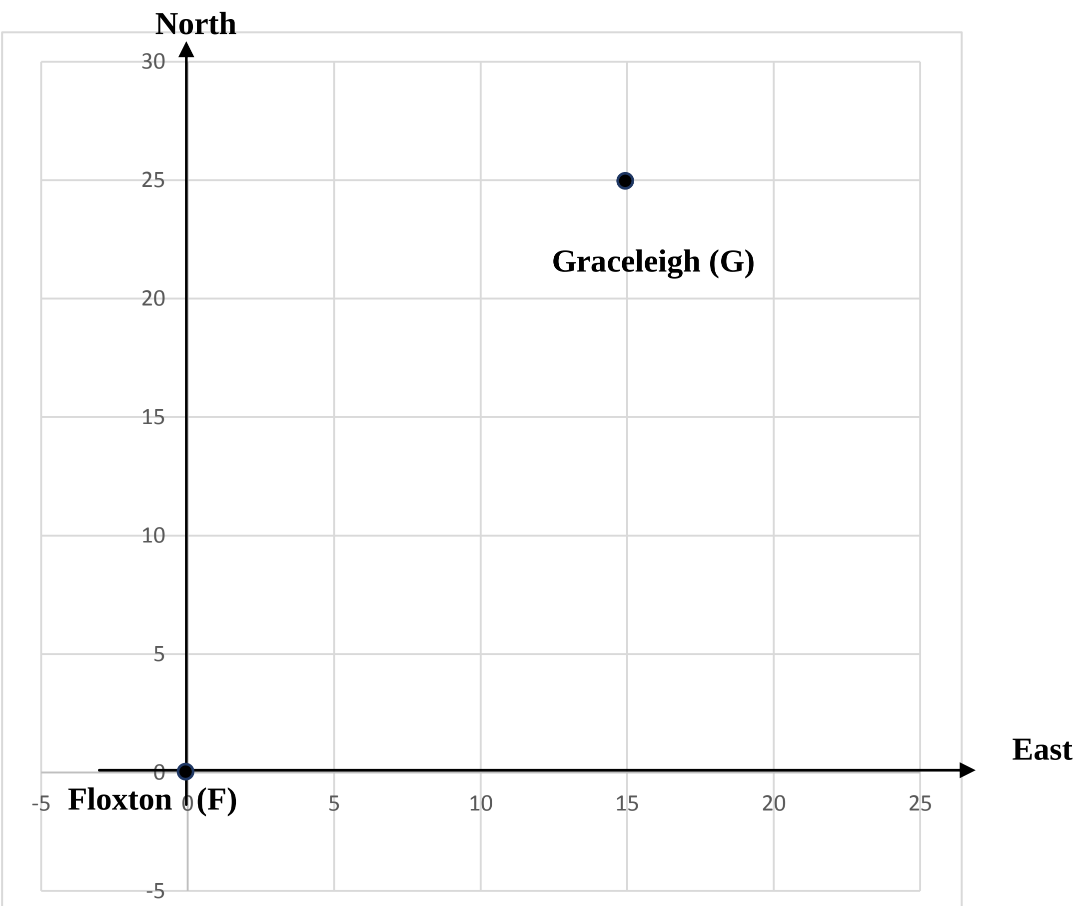
Unless otherwise indicated, the diagrams in this book are **not** to scale.

**Question 1 (17 marks)**

The following map shows the position of two islands, Floxton (F) and Graceleigh (G).

Floxton is located at $(0, 0)$ and Graceleigh is located at $(15, 25)$

Let 1 $km$ East be $\underset{\sim}{i}$ and 1 $km$ North be $\underset{\sim}{j}$




**SECTION B – Question 1 -** continued
**TURN OVER**

**a.** Write the position of Graceleigh $\overrightarrow{OG}$ in vector notation.

1 mark

A sight-seeing cruise boat leaves Floxton and sails at $20\ kmh^{-1}$on a bearing of $North\ 30^{\circ} East$.

**b.** Mark the boat's path on the graph provided at the beginning of the question. 1 mark

**c.** Prove that the boat passes just to the north of Graceleigh.

2 marks

**SECTION B – Question 1 -** continued

**d.** The position of the boat P, as a function of $t$ where $t$ is the number of hours since the boat has left Floxton is given by $\overrightarrow{OP} = \underset{\sim}{p} = kt(\underset{\sim}{i} + \sqrt{3}\underset{\sim}{j})$
Find the value of k.

2 marks

**e.** When is the boat due north of Graceleigh?

1 mark

**SECTION B – Question 1 -** continued
**TURN OVER**

**f.** Find $\overrightarrow{PG}$ in terms of $t$ .

2 marks



**g.** Hence use a vector method to find when, to the nearest minute, the boat is closest to Graceleigh and how close, to the nearest metre, the boat passes Graceleigh at that time.

3 marks

A patrol boat has position (B) given by $\overrightarrow{OB} = (15\cos 2t + 15)\,\underset{\sim}{i} + (10\sin 2t)\,\underset{\sim}{j}$ where $t$ is the number of hours after the sight-seeing cruise boat leaves Floxton.

**h.** How far apart are the two boats when the sight-seeing cruise boat first leaves Floxton?

2 marks

**i.** Find the Cartesian equation of the patrol boat.

2 marks

**SECTION B – Question 1 -** continued

**j.** Sketch and label the graph of the distance (D) between the two boats for $t \in [0,2]$
Interpret the local minimum.

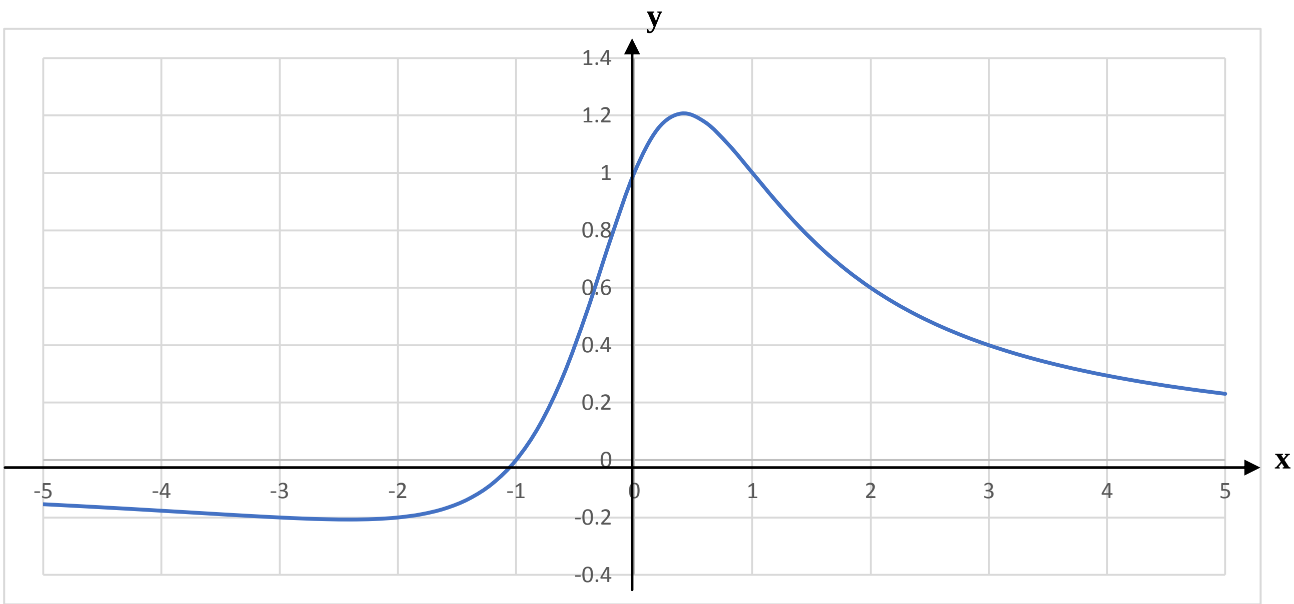

3 marks

1 + 1 + 2 + 2 + 1 + 2 + 3 + 1 + 1 + 3 = 17 marks

**SECTION B –** continued
**TURN OVER**

**Question 2 (14 marks)**

The following graph shows the function $f: [-5,5] \to R$ where $f(x) = \frac{x+1}{x^2+1}$




**a.** Find the derivative of $f: [-5,5] \to R$ where $f(x) = \frac{x+1}{x^2+1}$

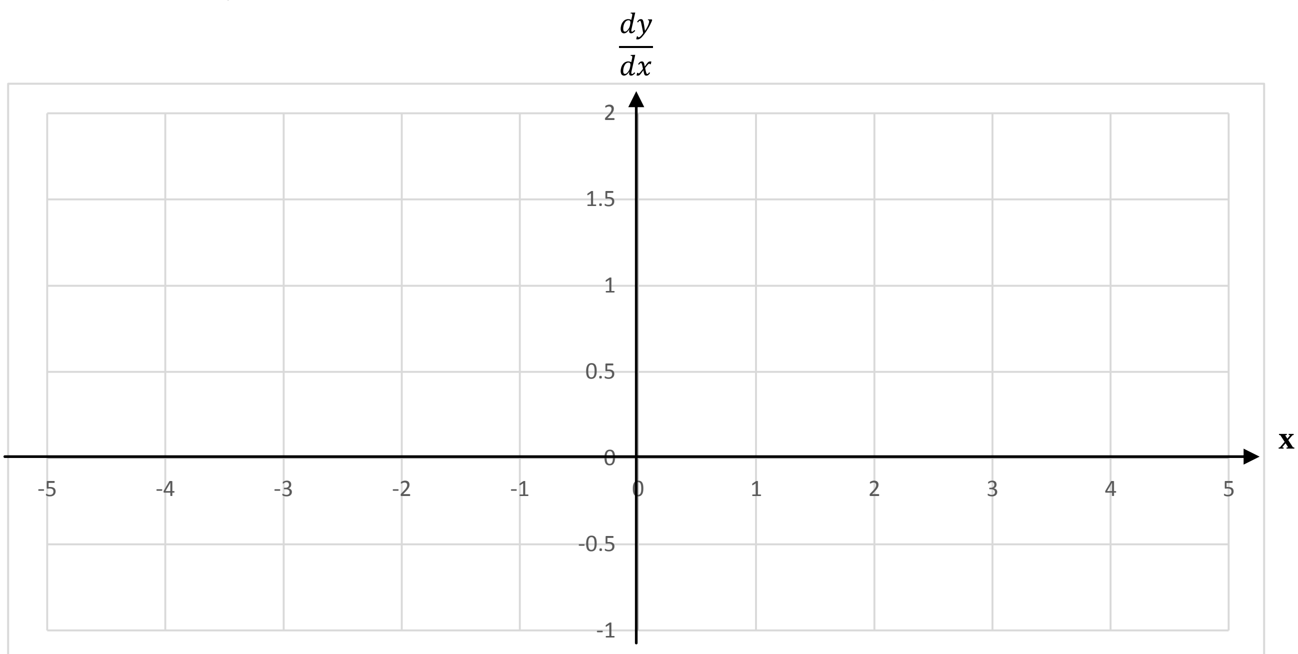
1 mark

**SECTION B – Question 2 -** continued

**b.** Find the exact co-ordinates of the stationary points to $f:[-5,5] \rightarrow R$ where $f(x) = \frac{x+1}{x^2+1}$
Mark these on the graph above.

3 marks

**c.** Sketch $y = f'(x)$ on the axes below. Label intercepts and stationary points in exact form.




3 marks

**SECTION B – Question 2 -** continued
**TURN OVER**

**d.** Let $g(x) = f'(x)$
Write down the exact range of $y = g(x)$.

1 mark

**e.** Explain in words what this range of values represents, in terms of the original function $y = f(x)$.

1 mark

**SECTION B – Question 2 -** continued

**f.** $y = f(x)$ and $y = g(x)$ intersect at $x = n$.
**Use calculus** to find the exact area bound by $y = f(x)$ and $y = g(x)$ in the domain $x \epsilon [n, 1]$

3 marks

**g.** Find the volume generated when the region enclosed by the $x$-axis, the $y$-axis and $y = f(x)$ is rotated around the $x$-axis.

2 marks

1 + 3 + 3 + 1 + 1 + 3 + 2 = 14 marks

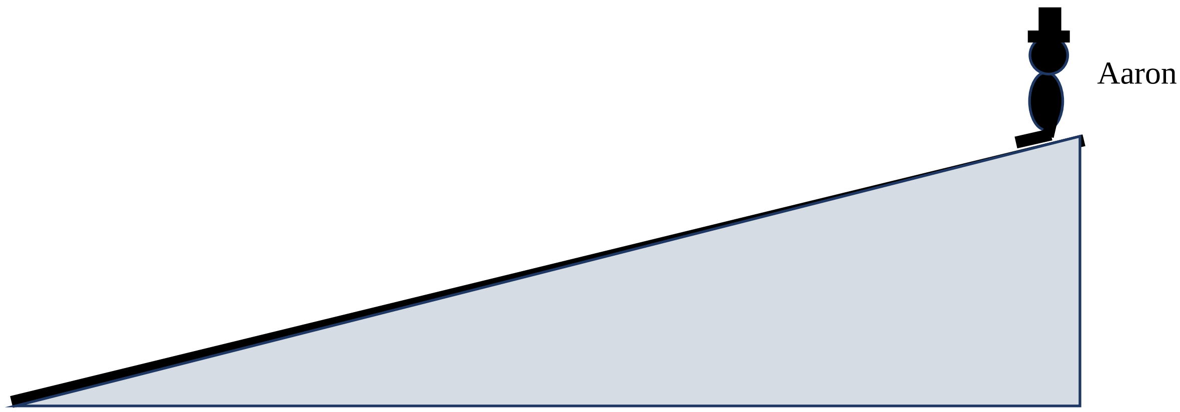
**SECTION B** – continued

**Question 3 (12 marks)**

A 5-metre playground slide is inclined at $30°$ to the horizontal. A 40 *kg* boy (Aaron) sits at the very top of the slide.

Initially assume that the contact between the boy and the surface of the slide is smooth.

**a.** Show all the forces acting on Aaron on the diagram below.




1 mark

**b.** How long will it take Aaron to reach the bottom of the slide?

Give your answer correct to one decimal place.

1 mark

**SECTION B – Question 3 -** continued
**TURN OVER**

**c.** Find Aaron's speed at the bottom of the slide

1 mark

In fact, the contact between Aaron and the surface of the slide is not smooth. A resistive force of $F_R$ newtons (due to friction) acts directly up the plane.

**d.** If Aaron's speed at the bottom of the slide is 5 $ms^{-1}$ find $F_R$

2 marks

**SECTION B – Question 3 -** continued

Aaron wants to have a faster ride. He asks his sister to give him a push from the top of the slide. She provides a pushing force (P newtons) directly down the slide where:

$P = 960t, \qquad 0 \leq t \leq 0.5$

$P = 0$ otherwise

**e.** Show that the Aaron's speed 0.5 $seconds$ after the start of being pushed is 4.25 $ms^{-1}$

2 marks

**f.** Find the distance that Aaron has travelled down the slide 0.5 $seconds$ after the start of being pushed.

1 mark

**SECTION B – Question 3 -** continued

**TURN OVER**

**g.** Find Aaron's speed at the bottom of the slide now.

2 marks

**h.** A horizontal rubber mat at the bottom of the slide slows Aaron down by providing a resistive force of $100x\ newtons$ where $x$ is the horizontal distance from the end of the slide in metres.

Find, to the nearest cm, how far Aaron comes to rest from the base of the slide.

2 marks

1 + 1 + 1 + 2 + 2 + 1 + 2 + 2 = 12 marks

**SECTION B** – continued

**Question 4 (17 marks)**

Mr Braggs, a Specialist Mathematics teacher has found an old exam question relating to the solutions to $z^3 = -64i$. He wants to compare how long his class takes to answer the question compared to the published data.

**a.** Show by substitution that $z = 4i$ is a solution to $z^3 = -64i$

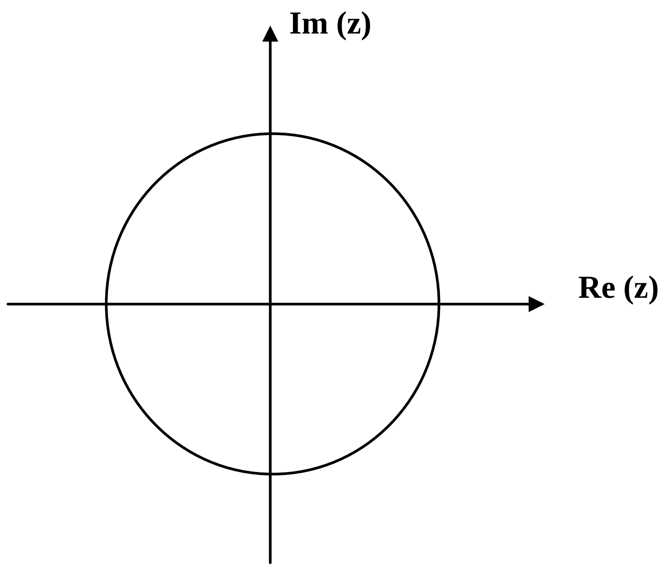
1 mark

**b.** Write $4i$ in $r\ cis\ \theta$ form. (Call this $z_1$).

1 mark

**SECTION B – Question 4 -** continued
**TURN OVER**

**c.** Plot $z_1$ on the axes below then plot $z_2$ and $z_3$ the other two solutions to $z^3 = -64i$




2 marks

**d.** Write $z_2$ and $z_3$ in both polar and rectangular form.

2 marks

**SECTION B – Question 4 -** continued

**e.** Solve $z^2 = -64i$, writing your answers in polar form.

2 marks

**f.** Using one of your roots from each of $z^3 = -64i$ and $z^2 = -64i$, find one solution to $z^6 = -64i$ in rectangular form.

2 marks

**SECTION B – Question 4 -** continued
**TURN OVER**

Parts **a.** to **f.** of Question 4 were originally from an old Applied Mathematics exam paper.

Time (t minutes) to complete the question from the state-wide population of students is well modelled by a continuous random variable with a pdf given by:

$$m(t) = at(t-20);\ \ 0 \le t \le 20$$
$$= 0, \text{ otherwise}$$

**g.** Find the mean and standard deviation of m.

3 marks

**SECTION B – Question 4 -** continued

**h.** Mr Braggs is worried that his students take too long to answer application questions. To test this hypothesis, what mean time (correct to 2 decimal places) would Mr Bragg's class of 23 students need to record so that this time is higher than the state-wide mean at the 0.01 level of significance?

Assume that Mr Bragg's class is an independent sample from the entire population of students, who have attempted the question.

2 marks

**i.** In fact, Mr Braggs has a total of 25 students in his class. The two students who were absent on the day of the test, attempt the question the following day. What mean time (correct to 1 decimal place) would these two students need to record so that the entire class of 25 would achieve the same level of significance as the original 23 students?

2 marks

$1 + 1 + 2 + 2 + 2 + 2 + 3 + 2 + 2 = 17$ marks

**END OF QUESTION AND ANSWER BOOK**