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Trial Examination 2019

# **VCE Specialist Mathematics Units 3&4**

Written Examination 1

**Suggested Solutions**

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**Question 1** (3 marks)

In the direction of motion we have  $2pt + q = ma$ .

A1

$$a = \frac{2pt + q}{m} \Rightarrow \frac{dv}{dt} = \frac{2pt + q}{m}$$

$$v = \frac{pt^2 + qt}{m} + c$$

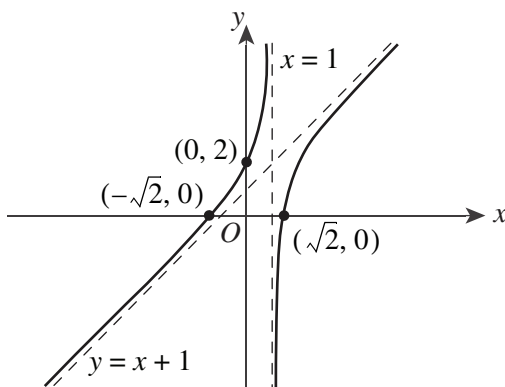
A1

When  $t = 0$ ,  $v = 0$  and so  $c = 0$ .

$$\text{So } v = \frac{pt^2 + qt}{m}.$$

A1

**Question 2** (4 marks)



*correct shape (two branches and asymptotic behaviour) A1*  
*correct intercepts with the axes A1*  
*vertical asymptote is  $x = 1$  A1*  
*non-vertical asymptote is  $y = x + 1$  A1*

**Question 3** (3 marks)

a. 
$$\begin{aligned} E(4X - 3Y) &= 4E(X) - 3E(Y) \\ &= 4 \times 30 - 3 \times 20 \\ &= 60 \end{aligned}$$

A1

b. 
$$\begin{aligned} \text{Var}(4X - 3Y) &= 16\text{Var}(X) + 9\text{Var}(Y) \\ &= 16 \times 9 + 9 \times 4 \\ &= 180 \end{aligned}$$

M1

A1

**Question 4** (3 marks)

$$\vec{AB} = 3\vec{i} - 2\vec{j} + (m + 3)\vec{k}$$

$$|\vec{OC}| = 7$$

A1

Attempting to solve  $3^2 + (-2)^2 + (m + 3)^2 = 49$  for  $m$ .

M1

$m = -9$  or  $3$

A1

**Question 5** (3 marks)

$$\text{Use of } \tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)} \text{ and } \cot(x) = \frac{1}{\tan(x)} \text{ to obtain } \frac{10 \tan(x)}{1 - \tan^2(x)} = \frac{4}{\tan(x)}. \quad \text{M1}$$

$$\text{Attempting to simplify gives } \tan^2(x) = \frac{2}{7}. \quad \text{M1}$$

$$\tan(x) = \pm \sqrt{\frac{2}{7}} \quad \text{A1}$$

**Question 6** (4 marks)

$$\int_0^{\frac{\pi}{6}} \frac{1 + \cos^4(2x)}{\cos^2(2x)} dx = \int_0^{\frac{\pi}{6}} \sec^2(2x) + \cos^2(2x) dx \quad \text{A1}$$

$$\cos(4x) = 2\cos^2(2x) - 1 \Rightarrow \cos^2(2x) = \frac{1}{2} + \frac{1}{2} \cos(4x) \quad \text{M1}$$

$$= \left[ \frac{1}{2} \tan(2x) + \frac{x}{2} + \frac{1}{8} \sin(4x) \right]_0^{\frac{\pi}{6}} \quad \text{A1}$$

$$= \frac{\sqrt{3}}{2} + \frac{\pi}{12} + \frac{\sqrt{3}}{16}$$

$$= \frac{27\sqrt{3} + 4\pi}{48} \quad \text{A1}$$

**Question 7** (5 marks)

$$\text{a. } \frac{d}{dx} \left( \frac{1}{5} e^{2x} (2 \sin(x) - \cos(x)) \right) = \frac{2}{5} e^{2x} (2 \sin(x) - \cos(x)) + \frac{1}{5} e^{2x} (2 \cos(x) + \sin(x)) \quad \text{M1}$$

$$= \frac{4}{5} e^{2x} \sin(x) + \frac{1}{5} e^{2x} \sin(x) - \frac{2}{5} e^{2x} \cos(x) + \frac{2}{5} e^{2x} \cos(x) \quad \text{A1}$$

$$\text{So } \frac{d}{dx} \left( \frac{1}{5} e^{2x} (2 \sin(x) - \cos(x)) \right) = e^{2x} \sin(x).$$

$$\text{b. } \int \frac{dy}{\sqrt{1-y^2}} = \int e^{2x} \sin(x) dx \quad \text{A1}$$

$$\sin^{-1}(y) = \frac{1}{5} e^{2x} (2 \sin(x) - \cos(x)) + c \quad \text{M1}$$

$$\text{When } x = 0, y = 0 \text{ and so } c = \frac{1}{5}.$$

$$y = \sin \left( \frac{1}{5} e^{2x} (2 \sin(x) - \cos(x)) + \frac{1}{5} \right) \quad \text{A1}$$

**Question 8** (5 marks)

$$(a + bi)^2 = a^2 - b^2 + 2abi \text{ and so } a^2 - b^2 = 7 \text{ and } 2ab = -6\sqrt{2}. \quad \text{M1 A1}$$

$$\text{Attempting to eliminate a variable, for example } b: b = -\frac{3\sqrt{2}}{a} \text{ to form } a^4 - 7a^2 - 18 = 0. \quad \text{M1}$$

(Alternatively, eliminating variable  $a$  forms  $b^4 + 7b^2 - 18 = 0$ .)

$$\text{Solving gives } a = 3, b = -\sqrt{2}; \text{ that is, } z = 3 - \sqrt{2}i. \quad \text{A1}$$

$$\text{And } a = -3, b = \sqrt{2}; \text{ that is, } z = -3 + \sqrt{2}i. \quad \text{A1}$$

**Question 9** (5 marks)

$$\text{Attempting to differentiate implicitly to obtain } e^{-y} - xe^{-y} \frac{dy}{dx} + e^y \frac{dy}{dx} - 1 = 0. \quad \text{M1}$$

$$\text{At the point } (k, \log_e(k)), \frac{1}{k} - \frac{dy}{dx} + k \frac{dy}{dx} - 1 = 0. \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{1}{k} \Rightarrow m_N = -k, \text{ where } m_N \text{ is the gradient of the normal.} \quad \text{A1}$$

$$\text{The equation of the normal is } y - \log_e(k) = -k(x - k).$$

$$\text{When } x = 0, y = \frac{2k^2 + 1}{2} \text{ and so } \frac{2k^2 + 1}{2} - \log_e(k) = k^2. \quad \text{M1}$$

$$\text{So } k = \sqrt{e}. \quad \text{A1}$$

**Question 10** (5 marks)

a.  $1 - x^2 \geq 0$   
 $-1 \leq x \leq 1$  but  $x \neq 0$

The maximal domain of  $f$  is  $-1 \leq x \leq 1, x \neq 0$ .

A1

b.  $f'(x) = \left( \frac{1}{1 + \frac{1-x^2}{x^2}} \right) \left( \frac{-x^2(1-x^2)^{-\frac{1}{2}} - (1-x^2)^{\frac{1}{2}}}{x^2} \right)$

M1 A1

$$= \left( \frac{x^2}{x^2 + 1 - x^2} \right) \left( \frac{-x^2(1-x^2)^{-\frac{1}{2}} - (1-x^2)^{\frac{1}{2}}}{x^2} \right)$$

$$= x^2 \left( \frac{\frac{-x^2}{\sqrt{1-x^2}} - \sqrt{1-x^2}}{x^2} \right)$$

$$= \frac{-x^2}{\sqrt{1-x^2}} - \sqrt{1-x^2}$$

M1

$$= \frac{-x^2 - (1-x^2)}{\sqrt{1-x^2}} \text{ and so } f'(x) = -\frac{1}{\sqrt{1-x^2}}$$

A1