YEAR 12 *Trial Exam Paper*

2019

SPECIALIST MATHEMATICS

Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials provided
- Question and answer book of 27 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions
- Write your **name** in the box provided above and on the multiple-choice answer sheet.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination.

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SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

The implied domain and range of $f(x) = -a\cos^{-1}(bx+\pi)+c$, where $a, b, c > 0$, is

A. $\left[-b-\pi,\ b-\pi\right]$ and $[c,\ -a\pi+c]$

B. $\left[\dfrac{-1}{b}-\pi,\ \dfrac{1}{b}-\pi\right]$ and $[-a\pi+c,\ c]$

C. $\left[\dfrac{-1-\pi}{b},\ \dfrac{1-\pi}{b}\right]$ and $[-a\pi+c,\ ac]$

D. $\left[-b+\pi,\ b+\pi\right]$ and $[c,\ a\pi+c]$

E. $\left[\dfrac{-1-\pi}{b},\ \dfrac{1-\pi}{b}\right]$ and $[-a\pi+c,\ c]$

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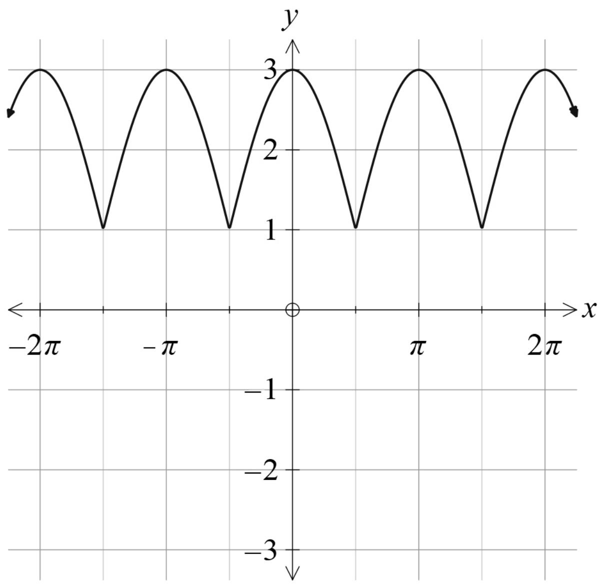
Question 2



A rule for the function shown in the graph above could be

A. $y = 2\left|\sin\left(x - \dfrac{\pi}{2}\right)\right| + 1$

B. $y = 2\left|\cos\left(x - \dfrac{\pi}{2}\right)\right| + 1$

C. $y = \left|\cos\left(x - \dfrac{\pi}{2}\right)\right| + 2$

D. $y = 2\left|\cos(x)\right|$

E. $y = 2\left|\sin\left(x + \dfrac{\pi}{2}\right)\right|$

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Question 3

If $\cos(\theta)=0.8$ and $\theta\in\left[\frac{3\pi}{2},2\pi\right]$, then $\sin\left(\frac{\theta}{2}\right)$ is equal to

A. $\frac{3}{10}$

B. $\frac{\sqrt{5}}{5}$

C. $-\frac{\sqrt{10}}{10}$

D. $\frac{\sqrt{10}}{10}$

E. $-\frac{3}{10}$

Question 4

Consider the graph of $y=\frac{x^2+1}{x^2-1}$.

Which one of the following statements is **true**?

A. The graph has a point of inflection.

B. The graph has an asymptote of $y=x^2-1$.

C. The graph is concave down at $x=0$.

D. The graph has only two asymptotes.

E. The graph is concave up at $x=0$.

Question 5

Which one of the following, where *A*, *B*, *C* and *D* are non-zero real numbers, is the partial fraction form for the expression $\frac{3x^2+8x+4}{(3x+2)^2(x^2-4)}$?

A. $\frac{A}{x-2}+\frac{B}{3x+2}$

B. $\frac{A}{x-2}+\frac{B}{3x+2}+\frac{C}{(3x+2)^2}$

C. $\frac{A}{3x+2}+\frac{B}{3x+2}+\frac{Cx+D}{x^2-4}$

D. $\frac{A}{x-2}+\frac{B}{x+2}$

E. $\frac{A}{x+2}+\frac{B}{3x+2}$

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Question 6

A quartic polynomial with all real coefficients has $z = 3ai$ and $z = 2a - 2ai$ as two of its solutions, where a is a non-zero real number.

The polynomial could be

A. $z^4 - 4az^3 + 17a^2z^2 - 36a^3z + 72a^4$

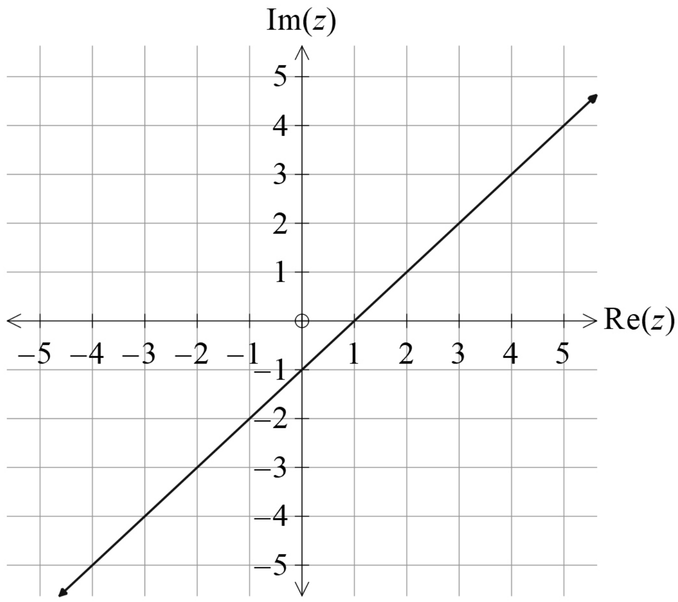
B. $z^2 - (2a + ai)z + 6a^2 + 6a^2i$

C. $z^4 + (13a^2)z^2 - 36a^2$

D. $z^4 + (17a^2 - 4a)z^2 - 36a^3z + 72a^4$

E. $z^2 + (17a^2 - 4a)z^2 - 36a^2 + 72a^4$

Question 7



Given that $z \in C$, the equation that best represents the graph shown above is

A. $\text{Arg}(z + i) = -\dfrac{3\pi}{4}$

B. $\{z : |z - 2 + 2i| = |z + 1 - i|\}$

C. $\{z : |z - 2 - i| = |z + 1 + 2i|\}$

D. $\{z : (z + 2 + 3i)(\overline{z} + 2 - 3i) = 4\}$

E. $\text{Arg}(z - 1) = \dfrac{\pi}{4}$

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Question 8

The region bounded by the curve $y=\sqrt{16x}$ and the lines $y=x$ and $x=4$ is rotated about the x-axis to form a volume of the solid of revolution.

The volume of the solid is

A. 120π cubic units

B. $\frac{40\pi}{3}$ cubic units

C. 4800π cubic units

D. $\frac{320\pi}{3}$ cubic units

E. $\frac{704\pi}{15}$ cubic units

Question 9

Using a suitable substitution, $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} -\sin^2(2x)\cos^2(x)+\sin^2(2x)\sin^2(x)\,dx$ can be written as

A. $\int_{\frac{\sqrt{3}}{2}}^{1} u^2\,du$

B. $\frac{1}{2}\int_{1}^{\frac{\sqrt{3}}{2}} u^2\,du$

C. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} u^2\,du$

D. $\frac{1}{2}\int_{\frac{\sqrt{3}}{2}}^{1} u^2\,du$

E. $-2\int_{1}^{\frac{\sqrt{3}}{2}} u^2\,du$

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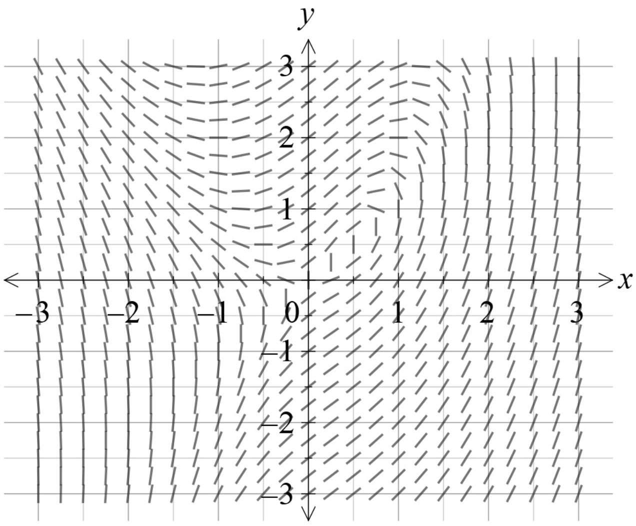
Question 10



The differential equation that best represents the direction field above is

A. $\dfrac{dy}{dx} = \dfrac{x + y^2}{x + y}$

B. $\dfrac{dy}{dx} = \dfrac{2x^2 - y}{x - y}$

C. $\dfrac{dy}{dx} = \dfrac{2x - y}{x + y^2}$

D. $\dfrac{dy}{dx} = \dfrac{2x^2 - y}{x + y}$

E. $\dfrac{dy}{dx} = \dfrac{x + y^2}{2x - y}$

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Question 11

The values of m for which $y = e^{2mx}$ satisfies the differential equation $\dfrac{d^2y}{dx^2} - 8\dfrac{dy}{dx} + 12y = 0$ are

A. $m = 2,\ m = 6$

B. $m = 1,\ m = 3$

C. $m = -1,\ m = -3$

D. $m = -2,\ m = -6$

E. $m = -1,\ m = 3$

Question 12

Let $\underset{\sim}{a} = -4\underset{\sim}{i} + 3\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + c\underset{\sim}{k}$, where c is a real constant.
If the scalar resolute of $\underset{\sim}{b}$ in the direction of $\underset{\sim}{a}$ is 8, then the value of c is

A. 16

B. $\dfrac{32}{3}$

C. 5

D. 8

E. 22

Question 13

The vectors $\underset{\sim}{a} = 4\underset{\sim}{i} - 6\underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{b} = -\dfrac{3}{2}\underset{\sim}{i} + 6\underset{\sim}{j} - 12\underset{\sim}{k}$ and $\underset{\sim}{c} = \underset{\sim}{i} + \underset{\sim}{j} + \gamma\underset{\sim}{k}$, where γ is a real constant, are linearly dependent if

A. $\gamma \in R$

B. $\gamma = 7$

C. $\gamma \in R \setminus \{-7\}$

D. $\gamma = -7$

E. $\gamma \in R \setminus \{7\}$

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Question 14

The angle that the vector $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ makes with the y-axis, correct to the nearest degree, is

A. 109°

B. 42°

C. 48°

D. 61°

E. 1°

Question 15

A block of mass 80 kg is being pulled along a rough surface by a light, inextensible rope at a constant velocity. The rope makes an angle of $15°$ to the horizontal. The resistance force in the opposite direction of motion due to the rough surface is $20g$.

The magnitude of the normal reaction force, in newtons, exerted by the ground on the block is closest to

A. 196

B. 784

C. 952

D. 731

E. 203

Question 16

A block of mass 4 kg, initially at rest, begins sliding down a ramp inclined at $30°$ to the horizontal. The magnitude of the resistive force due to the surface is $0.4x^2$ newtons, where x is the distance, in metres, that the block has travelled down the ramp.

The speed of the block after it has slid 5 m down the ramp is closest to

A. 6.38 ms^{-1}

B. 8.45 ms^{-1}

C. 6.88 ms^{-1}

D. 11.11 ms^{-1}

E. 9.46 ms^{-1}

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Question 17

A tennis ball of mass 250 grams begins moving with an initial velocity of $2\underset{\sim}{i} - 3\underset{\sim}{j}$ ms^{-1}.

If the tennis ball is accelerating at $-t\underset{\sim}{i} + 2\underset{\sim}{j}$ ms^{-2}, where $t \geq 0$ seconds, then the momentum, in kg ms^{-1}, of the tennis ball after two seconds is

A. $0.25\underset{\sim}{j}$

B. 1.52

C. 291.55

D. $-150\underset{\sim}{i} + 250\underset{\sim}{j}$

E. $-1.5\underset{\sim}{i} + 0.25\underset{\sim}{j}$

Question 18

A Mathematics teacher wishes to estimate the average exam score for the state. The teacher uses a sample size of 60 to create a 95% confidence interval for the population mean μ.

If a confidence interval is (65.18, 71.32), then the sample mean $\bar{x}$ and sample standard deviation s respectively, are closest to

A. 68.52 and 12.1

B. 68.25 and 93.98

C. 68.25 and 12.15

D. 68.25 and 14.5

E. 68.25 and 9.22

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Use the following information to answer Questions 19 and 20.

The times taken to recharge two different brands of electric car, brand X and brand Y, expressed in minutes, are both normally distributed. The mean recharge time and standard deviation of brand X are 125 and 8, respectively, and the mean recharge time and standard deviation of brand Y are 138 and 15 respectively. Assume that the recharge times for each brand are independent of each other.

Question 19

Given that $\frac{m+n}{2} = 125$, where $m < n$ and $\Pr(m < X < n) = 0.95$ for a randomly selected brand X electric car, the values of *m* and *n*, correct to one decimal place, are

A. 111.8 and 138.2

B. 114.7 and 135.3

C. 109.3 and 140.7

D. 104.4 and 145.6

E. 109.3 and 140.6

Question 20

The probability, correct to four decimal places, that a randomly chosen brand Y electric car has a recharge time within 10 minutes of a randomly chosen brand X car is closest to

A. 0.0014

B. 0.6581

C. 0.9120

D. 0.3419

E. 0.0880

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END OF SECTION A

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SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (10 marks)

Consider the function $f : D \to R$, where $f(x) = \dfrac{2x^3 - 3x^2 - 9x + 10}{x^2 - x - 2}$.

a. State the maximal domain D of f.

1 mark

b. Find the equations of any asymptotes of the graph of f.

2 marks

c. Find the coordinates of the point for which $f''(x) = 0$ and verify that this is a point of inflection. Give your answer correct to two decimal places.

2 marks

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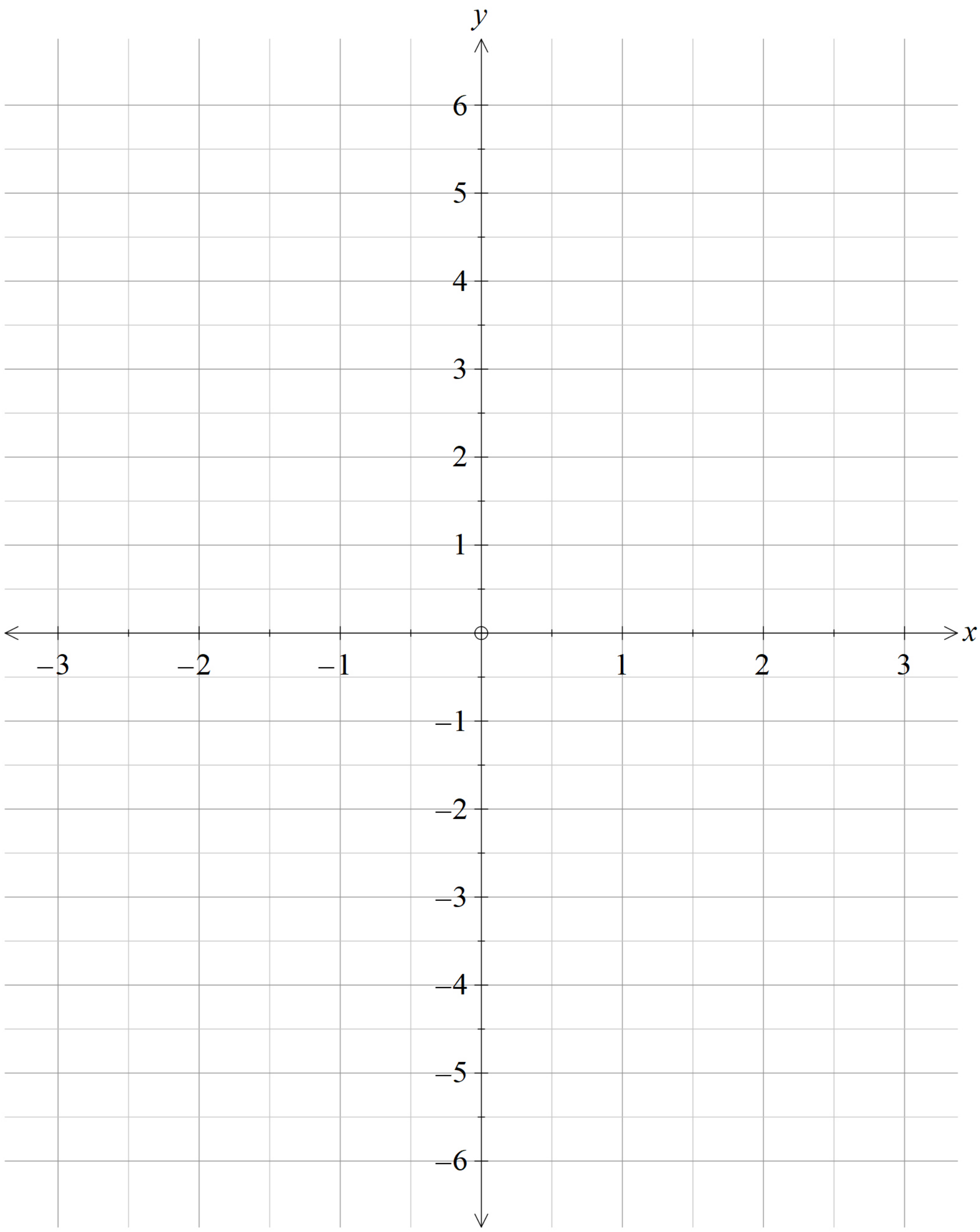
d. Sketch the graph of $y = f(x)$ on the axes below, labelling any asymptotes with their equations, and any intercepts and points of inflection with their coordinates.

3 marks



e. Write down a definite integral that gives the length L of the curve $f(x)$ over the interval $x \in [0.5, 1.5]$ and then state the value of the length of the curve, correct to two decimal places.

2 marks

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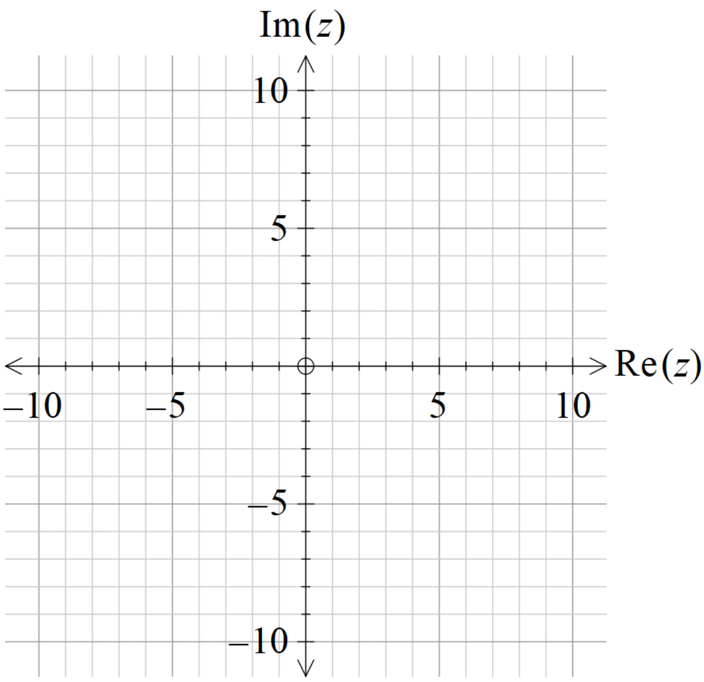
Question 2 (11 marks)

a. Find the cartesian equation of the relation given by $|z+1+i| = \sqrt{2}\,|z-1-i|$, where $z \in C$.

3 marks

b. Sketch the graph of $|z+1+i| = \sqrt{2}\,|z-1-i|$ on the axes below.

2 marks



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c. The solutions to the equation $z^2 - (10+2i)z + 24 + 2i = 0$, where $z \in C$, lie on the relation sketched in **part b**.

Solve the equation and plot the solutions on the axes provided in **part b.**, labelling the solutions as A and B, where A is the solution located in the first quadrant.

2 marks

d. Let z_1 be the complex number located at the point A.

The complex number $z_2 = \overline{z}_1 i$ also lies on the relation $|z+1+i| = \sqrt{2}\,|z-1-i|$.

Find and plot z_2 on the axes provided in **part b.** and label it as C.

2 marks

e. Consider the position vectors of the points A, B and C relative to the origin. Let $\underset{\sim}{\mathrm{i}}$ be the unit vector in the positive Re(z) direction and $\underset{\sim}{\mathrm{j}}$ the unit vector in the positive Im(z) direction.

Show that the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ intersect at a right angle.

2 marks

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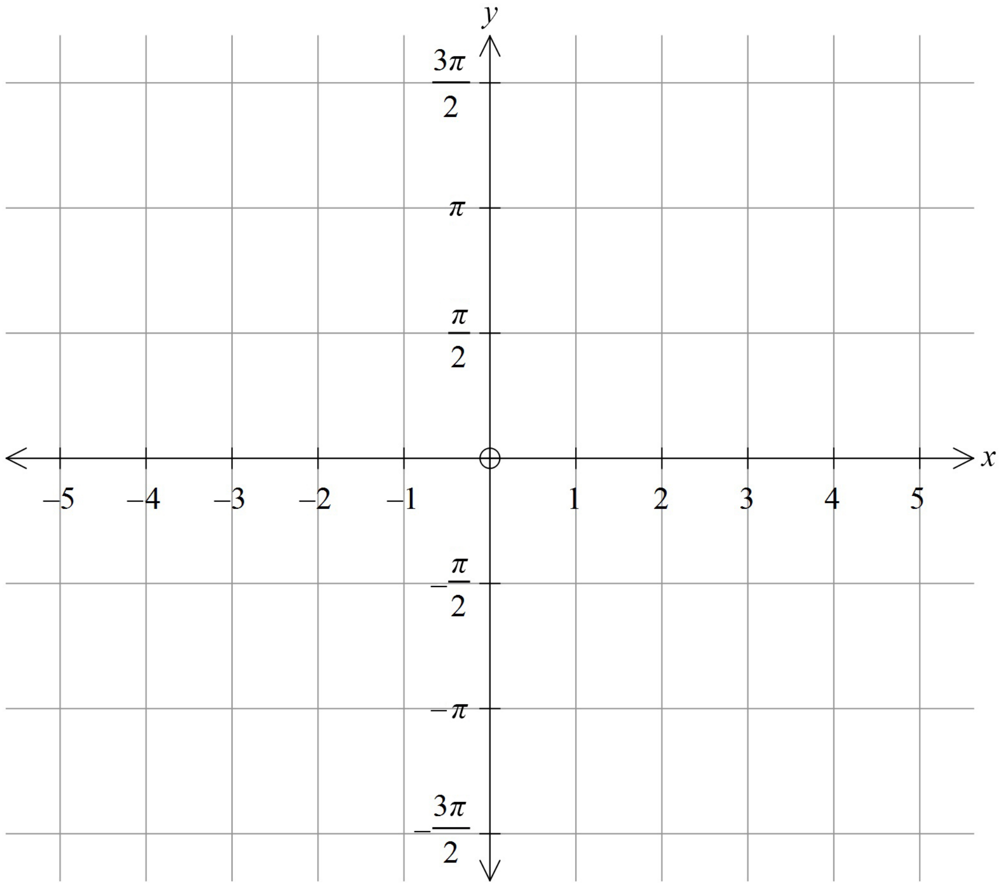
Question 3 (10 marks)

Consider the function $f:[0,\, a] \to R, f(x) = 2\arcsin\left(\frac{x}{4}\right)$, where a is the largest value for which f is defined.

a. Sketch the graph of $y = f(x)$ on the axes below, labelling any endpoints with their coordinates.

2 marks



The curve is rotated about the y-axis to form a volume of revolution that is to model the shape of a birdbath, where the length units are in centimetres.

b. Show that the volume, V cubic centimetres, of water in the birdbath when it is filled to a depth of h centimetres is given by $V = 8\pi[h - \sin(h)]$.

2 marks

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c. The birdbath is initially empty. Water is poured into the birdbath at a rate of $16\pi^2\sqrt{h}$ cm^3 s^{-1}, where h is the depth in centimetres at time t seconds.

i. Show that the expression for the rate at which the height of the water is changing is given by $\dfrac{dh}{dt}=\dfrac{2\pi\sqrt{h}}{1-\cos(h)}$.

2 marks

ii. Find the rate, in centimetres per second correct to two decimal places, at which the height of the water is changing at one quarter of the maximum depth.

1 mark

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d. Due to a hot summer, the water in the birdbath evaporates so that the depth of water in the birdbath is $\frac{\pi}{4}$ cm before being refilled.

i. Using Euler's method, with a step size of $\frac{\pi}{4}$ cm, find an estimate of the volume of water in the birdbath when $h = \frac{\pi}{2}$ cm. Give your answer in cubic centimetres, correct to two decimal places.

2 marks

ii. Determine if the estimate in **part d.i.** is an overestimate or an underestimate for the volume of water in the birdbath for a depth of $\frac{\pi}{2}$ cm.

1 mark

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TURN OVER

Question 4 (12 marks)

A surveillance drone S, and a delivery drone D, are observed flying at the same altitude in the airspace of a suburb. The position vectors of each drone t minutes after they are initially observed moving relative to some origin O are given by

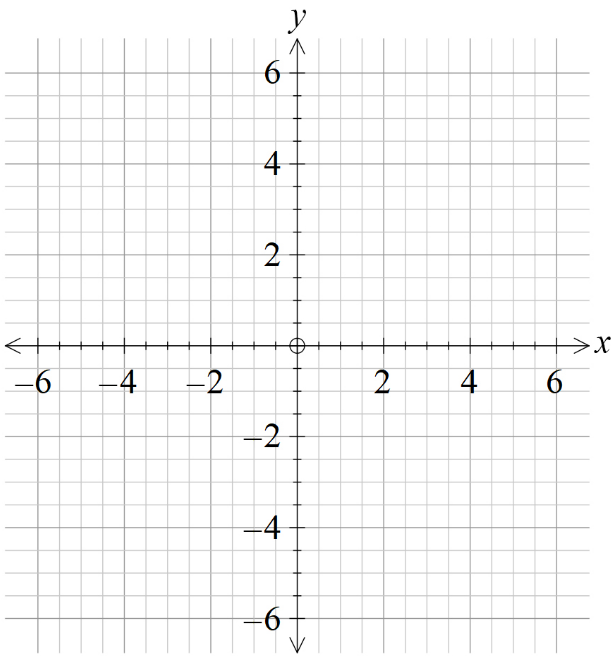
$$\underset{\sim}{r}_S(t) = \left(4 - \cos\left(\frac{t}{2}\right)\right)\underset{\sim}{i} + \left(2 - 2\sin\left(\frac{t}{2}\right)\right)\underset{\sim}{j} \text{ and } \underset{\sim}{r}_D(t) = \frac{t^2}{2}\underset{\sim}{i} + \left(4 - \frac{t^2}{3}\right)\underset{\sim}{j},\ t \geq 0.$$

All distances are measured in kilometres.

Both drones are initially observed at the same time.

a. Sketch the path of each drone on the axes below, labelling each curve, indicating the direction of movement and labelling any intercepts with their coordinates.

3 marks



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b. When $t = \sqrt{6}$ minutes, find the angle between the direction of motion of drone S and drone D. Give your answer in degrees correct to one decimal place.

2 marks

c. Show that the paths of drone S and drone D intersect but that the drones do not collide.

3 marks

d. Determine the maximum speed of drone S, in kilometres per minute.

2 marks

e. Find the minimum distance between the two drones and state at what time, in minutes, this occurs. Give your answers correct to one decimal place.

2 marks

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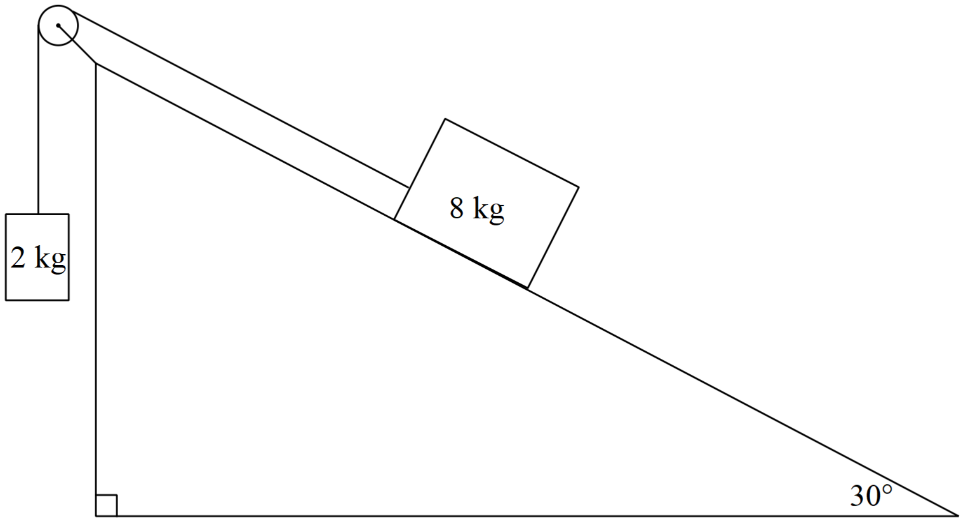
Question 5 (9 marks)

A mass of 8 kg slides down a rough ramp that is inclined at 30° to the horizontal plane. The mass is connected by a light, inextensible string that passes over a smooth pulley to a 2 kg mass, which is suspended vertically.

Initially, the 8 kg mass is at the top of the ramp and is sliding down the ramp with a velocity of $\sqrt{96}$ ms^{-1} parallel to the ramp. The size of the resistance force between the ramp and the 8 kg mass is kv^2 newtons, where v is the velocity of the 8 kg mass, in ms^{-1}, and k is a positive constant.

a. On the diagram below, show all forces acting on the masses.

1 mark



b. **i.** Write down the equations of motion for each mass parallel to their direction of motion.

2 marks

ii. Hence, show that the magnitude of the acceleration of the 8 kg mass is given by $\dfrac{2g-kv^2}{10}$ ms^{-2}.

2 marks

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c. The 8 kg mass reaches a constant velocity of 10 ms^{-1}.

Find the value of k.

1 mark

d. Using $k = 0.2$, find the velocity of the 8 kg mass in terms of the distance x metres travelled down the ramp. Express your answer in the form $v = \sqrt{ag + be^{-\frac{x}{c}}}$ where a, b and c are integers.

3 marks

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Question 6 (8 marks)

The height, X, of mature sunflowers in Eastern Victoria are normally distributed with a mean of 270 cm and a standard deviation of 15 cm.

A fertiliser company claims it has the best fertiliser on the market and advertises their fertiliser as being able to increase the height of mature sunflowers.

Farmer Geoff uses this fertiliser and decides to test this claim. He measures the heights of 50 mature sunflowers and finds the sample mean to be 274 cm.

Farmer Geoff then performs a statistical test.

Let $\bar{X}$ denote the mean height of a random sample of 50 mature sunflowers.

a. State suitable hypotheses H_0 and H_1 for the statistical test.

1 mark

b. Assume that the sample size is large enough that reasonable approximations for the standard deviation of the population and shape of the distribution of the sample means can be used.

Calculate the p value of the statistical test, correct to four decimal places, and state with a reason whether H_0 should be rejected at the 5% level of significance.

2 marks

c. Determine the largest sample size for which H_0 would **not** be rejected at the 5% level of significance.

2 marks

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d. For the statistical test conducted by Farmer Geoff, describe the conclusion that would be made about the fertiliser company's claim if a type I error occurred.

1 mark

e. Given that $\Pr(\bar{X} > 274.935 \mid \mu = 270) = 0.01$, calculate the probability of committing a type II error at the 1% level of significance, if the true mean height of all mature sunflowers in Eastern Victoria is 274 cm. Give your answer correct to four decimal places.

2 marks

END OF QUESTION AND ANSWER BOOK