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# YEAR 12 *Trial Exam Paper*

# 2019

# SPECIALIST MATHEMATICS

## Written examination 1

**Reading time: 15 minutes**
**Writing time: 1 hour**

**STUDENT NAME:**

## QUESTION AND ANSWER BOOK

### Structure of book

| *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|
| 10 | 10 | 40 |

- Students are permitted to bring the following items into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

**Materials supplied**

- Question and answer book of 15 pages
- Formula sheet
- Working space is provided throughout the book.

**Instructions**

- Write your **name** in the box provided above.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

**At the end of the examination**

- You may keep the formula sheet.

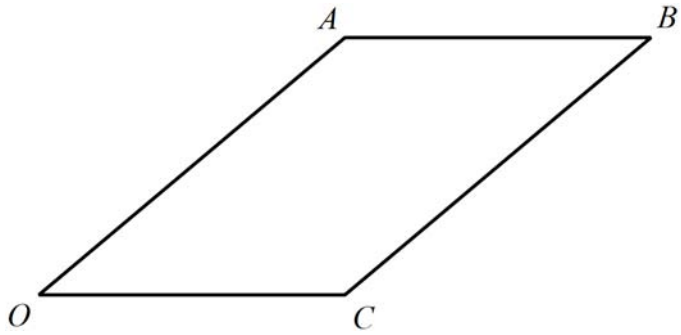
**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

## Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude $g$ ms$^{-2}$, where $g = 9.8$.

**Question 1** (4 marks)

Consider the parallelogram $OABC$ shown below where $\overrightarrow{OA} = m\underset{\sim}{\mathrm{i}} + \sqrt{39}\underset{\sim}{\mathrm{k}}$ and $\overrightarrow{OC} = -6\underset{\sim}{\mathrm{i}} + 8\underset{\sim}{\mathrm{j}}$.




**a.** Find the possible values of $m$ if $\left|\overrightarrow{OA}\right| = 2\left|\overrightarrow{OC}\right|$.

2 marks

**b.** Find the possible values of $m$ if the angle between $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is $\arccos\left(\frac{1}{5}\right)$. Give your answer in the form $\frac{\sqrt{a}}{b}$, where $a$ and $b$ are integers.

2 marks

**Question 2** (3 marks)

Find the gradient of the curve with equation $2xy^2 + \arccos\left(\frac{x}{4}\right) - \frac{\pi xy}{3} = 1$ at the point $\left(2, \frac{1}{2}\right)$.

Give your answer in the form $\frac{\pi - a + \sqrt{a}}{b - c\pi}$, where $a$, $b$ and $c$ are integers.

**Question 3** (4 marks)

Let $P(z) = z^3 - (a-1)z^2 + (b^2+4)z - 8,\ z \in C$, where *a* and *b* are real constants.

Given that $P(1-\sqrt{3}i) = 0$, determine all the solutions to $P(z) = 0$ and find the values of *a* and *b*.

**Question 4** (4 marks)

Two types of chocolate blocks are selected and their masses are measured. Block *X*'s masses are normally distributed with a mean of 175 grams and standard deviation of 3 grams. Block *Y*'s masses are normally distributed with a mean of 165 grams and standard deviation of 4 grams. The distributions are independent of each other.

**a.** Find the approximate probability that the mass of a randomly selected block *X* chocolate is greater than the mass of a randomly selected block *Y* chocolate.

2 marks

**b.** A quality control officer suspects that the mean difference in mass of the two chocolate blocks is not 10 grams. The officer collects 100 samples of block *X* chocolate and 100 samples of block *Y* chocolate.

Calculate a 95% confidence interval for the mean difference in the mass of block *X* chocolate and block *Y* chocolate. Use an integer multiple of the standard deviation in your calculations.

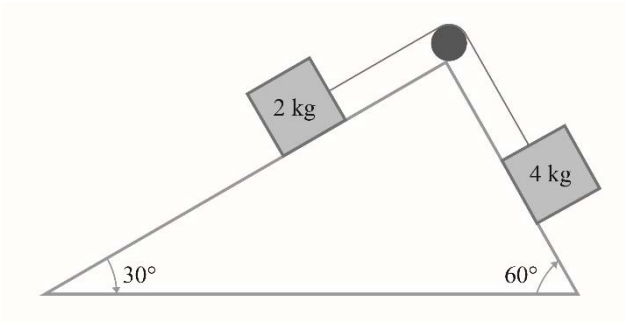
2 marks

**Question 5** (3 marks)

Find $\tan(t)$ given that $t = \arcsin\left(\frac{4}{5}\right) - \arccos\left(\frac{5}{13}\right)$.

**Question 6** (3 marks)

Two objects of masses 2 kg and 4 kg are attached by a light inextensible string that passes over a smooth pulley. The 2 kg mass is on a smooth plane inclined at $30°$ to the horizontal and the 4 kg mass is on a smooth plane inclined at $60°$ to the horizontal, as shown in the diagram below. Initially the pulley is locked, preventing the objects from moving. The pulley is unlocked and the 4 kg block accelerates down the plane.




**a.** Find the magnitude of the acceleration, $a$ ms$^{-2}$, of the 4 kg block.

2 marks

**b.** Determine the tension in the string connecting the two objects in newtons.

1 mark

**Question 7** (4 marks)

A fish tank initially holds 200 L of water in which 10 kg of salt has been dissolved. Pure water then flows into the tank at a rate of 5 L per minute. The mixture is stirred continuously and flows out at a rate of 10 L per minute.

**a.** Show that the differential equation for $x$, the number of kilograms of salt in the tank after $t$ minutes, is given by

$$\frac{dx}{dt} = \frac{-2x}{40-t}$$

1 mark

**b.** Solve the differential equation given in **part a.** to find $x$ as function of $t$.

3 marks

**Question 8** (4 marks)

The acceleration, $a$ ms$^{-2}$, of a motorcycle moving in a straight line relative to the origin with a velocity of $v$ ms$^{-1}$ is given by $a = v^3 + 16v$.

**a.** Given that $v = 0$ when $x = \frac{\pi}{16}$, where $x$ is the displacement in metres, find an equation for the velocity of the motorcycle in terms of $x$.

3 marks

**b.** Hence, calculate the velocity, in ms$^{-1}$, of the motorcycle when $x = \frac{\pi}{8}$.

1 mark

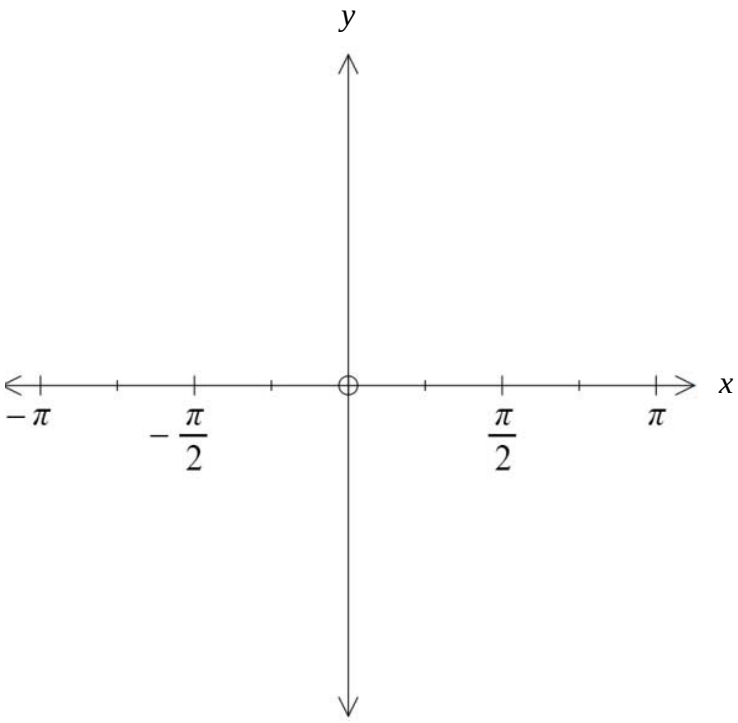
**CONTINUES OVER PAGE**

**Question 9** (6 marks)

Consider the function $f(x) = \arctan(2x - \pi)$.

**a.** Sketch the graph of the function on the axes provided below, labelling any asymptotes with their equations and any intercepts with their coordinates.

2 marks



**b.** The function $f(x)$ is translated $\frac{\pi}{2}$ units to the left to form the function $y = g(x)$. The function $y = g(x)$ is then rotated about the *y*-axis.

Determine the volume of the solid generated when the region bounded by the graph of $y = g(x)$, the *y*-axis and the line $y = \frac{\pi}{3}$ is rotated about the *y*-axis. Give your answer in the form $\frac{\pi}{a}\left(b\sqrt{c} - \pi\right)$, where *a*, *b* and *c* are integers.

4 marks

**Question 10** (5 marks)

The position vector of a particle moving along a curve at time $t$ seconds is given by

$\underset{\sim}{r}(t) = (2t^6 + 3)\underset{\sim}{i} + \left(\frac{6}{7}t^7 + 1\right)\underset{\sim}{j},\ t \geq 0$, where distances are measured in metres.

**a.** Show that the distance $d$ that the particle travels along the curve in the first 2 seconds is given by

$$d = \int_0^2 6t^5\sqrt{t^2 + 4}\ dt$$

1 mark

**b.** Using a suitable substitution, show that the distance travelled by the particle along the curve in the first 2 seconds can be expressed as

$$d = 3\left[ au^{\frac{7}{2}} - bu^{\frac{5}{2}} + cu^{\frac{3}{2}} \right]_4^8$$

where $u$ is the substituted variable and $a$, $b$ and $c$ are real numbers. State the values of $a$, $b$ and $c$.

4 marks

**END OF QUESTION AND ANSWER BOOK**