TSSM
Creating VCE Success

2018 Trial Examination

THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER									Letter

SPECIALIST MATHEMATICS

Written Examination 2

Reading time: 15 minutes
Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	5	5	60
		Total	80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 28 pages.

Instructions

- Print your student number in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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SECTION A – Multiple Choice Questions

Instructions for Section A
Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1
The range of the function $y = \tan^{-1}(x+1) - \frac{\pi}{4}$ is:

A. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$
B. $\left(\frac{-3\pi}{4}, \frac{3\pi}{4}\right)$
C. $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$
D. $\left[\frac{-3\pi}{4}, \frac{3\pi}{4}\right]$
E. R

Question 2
Which pair of parametric equations is appropriate for a particle whose path is an ellipse, moving in a **clockwise** direction around the origin?

A. $x = 2\cos(t)$, $y = 2\sin(t)$
B. $x = 5\cos(t)$, $y = 2\sin(t)$
C. $x = 5\sin(t)$, $y = -2\cos(t)$
D. $x = 5\sin(t)$, $y = 2\cos(t)$
E. $x = \sin(5t)$, $y = \cos(2t)$

SECTION A- continued
TURN OVER

Question 3

Given that $w = 1 - i$, the **principal argument** of the complex number $\frac{w^2}{\bar{w}}$ is:

A. $\frac{-3\pi}{4}$
B. $\frac{-\pi}{4}$
C. $\frac{\pi}{4}$
D. $\frac{3\pi}{4}$
E. $\frac{5\pi}{4}$

Question 4

The circle defined by $|z - 3i| = 2|z + 3|$ has a centre at the point:

A. $(-3,3)$
B. $(3,-3)$
C. $(-6,-3)$
D. $(4,1)$
E. $(-4,-1)$

Question 5

Let $f(z) = z^5 - az^3 - 2z^2 + 2a$. The maximum number of **non-real** roots for $f(z)$ is:

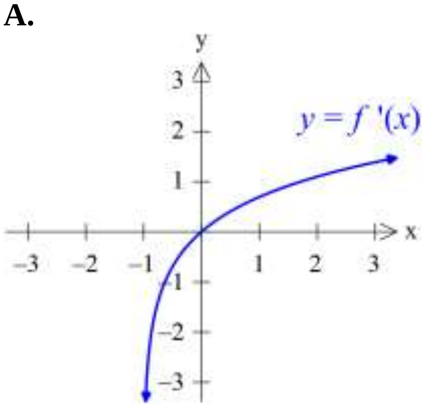
A. 1
B. 2
C. 3
D. 4
E. 5

SECTION A- continued

Question 6
The function g has a gradient function with rule $g'(x) = \sin^{-1} x$.
Which of the following statements about g must be true?

A. There is a point of inflection at the origin.
B. There is a maximum gradient at the origin.
C. The function is never concave down.
D. The function is never concave up.
E. The function is strictly increasing for the maximal domain.

Question 7
Each graph below shows a **gradient function** $y = f'(x)$. If Euler's method is used to estimate a y value on the **original function** $y = f(1.1)$ from $y = f(1)$, in which case will the estimate be above the true value?

A.


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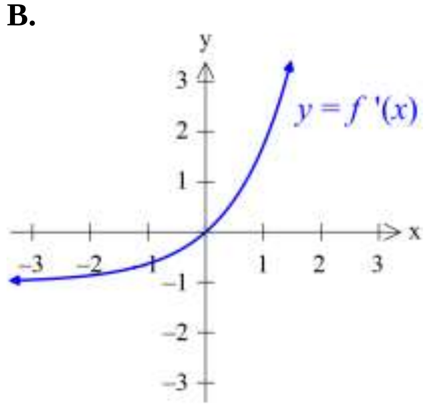

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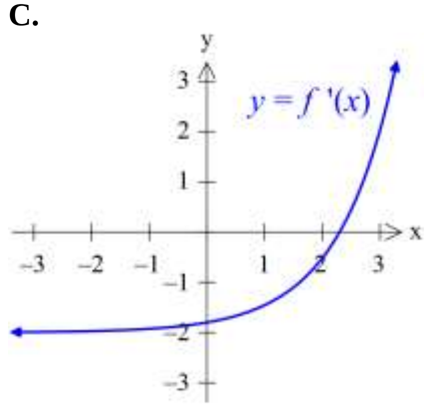

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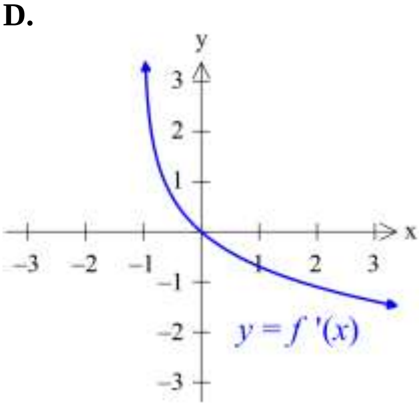

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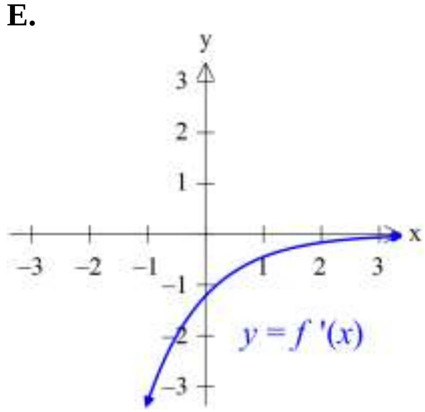


SECTION A- continued
TURN OVER

Question 8

Using an appropriate substitution, the integral $\int_0^1 e^{2x}\sqrt{e^x+1}\ dx$ can be expressed as:

A. $\int_1^e u\sqrt{u+1}\ du$

B. $\int_1^e u^2\sqrt{u+1}\ du$

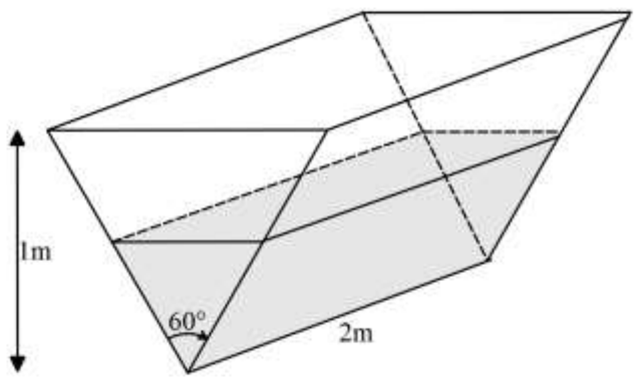
C. $\int_2^{e+1} u\sqrt{u}\ du$

D. $\int_2^{e+1} (u-1)\sqrt{u}\ du$

E. $\int_2^{e+1} (u-1)^2\sqrt{u}\ du$

Question 9

A water container has the shape of the triangular prism shown below, where the cross-section is an **equilateral** triangle.



The container is filled at a rate of $\frac{1}{2}$ m^3/min. Find the rate at which the height of the water is increasing, when the height of the water is 50cm (half a metre).

A. $\frac{2\sqrt{3}}{3}$ m/min

B. $\frac{4\sqrt{3}}{3}$ m/min

C. $\frac{\sqrt{3}}{2}$ m/min

D. $\frac{\sqrt{3}}{4}$ m/min

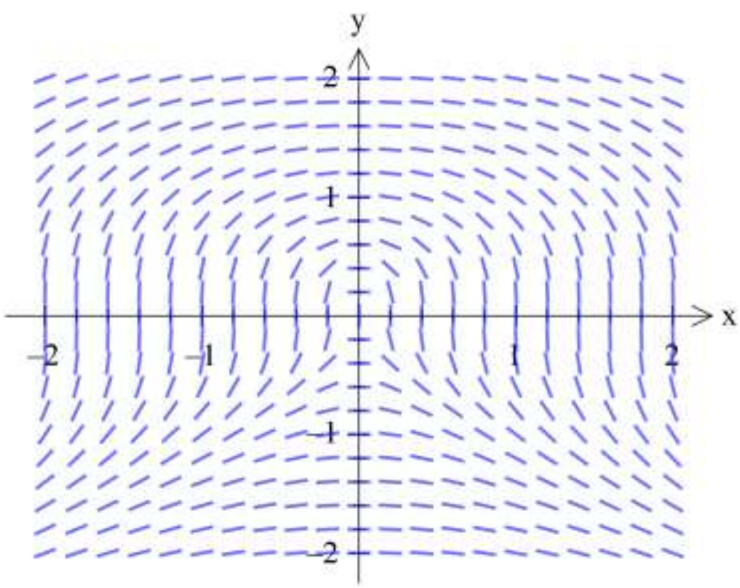
E. $\frac{\sqrt{3}}{8}$ m/min

SECTION A- continued

Question 10
The graph of which of these functions will have at least one intersection point with the graph of $y = \sec(x)$, for all values of $k \in R$?

A. $y = \operatorname{cosec}(x + k)$
B. $y = \cot(x + k)$
C. $y = \cos(x + k)$
D. $y = \cos^{-1}(x + k)$
E. $y = \tan^{-1}(x + k)$

Question 11
The slope field below could represent which first order differential equation?



A. $\frac{dy}{dx} = \frac{x^2}{y}$
B. $\frac{dy}{dx} = \frac{-x^2}{y}$
C. $\frac{dy}{dx} = \frac{x^2}{y^2}$
D. $\frac{dy}{dx} = \frac{x}{y^2}$
E. $\frac{dy}{dx} = \frac{-x}{y^2}$

SECTION A- continued
TURN OVER

Question 12
The vector resolute of $3\underset{\sim}{i} - 2\underset{\sim}{j}$ parallel to $\underset{\sim}{i} + \underset{\sim}{k}$ is:

A. $\frac{1}{2}\underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}$

B. $\frac{\sqrt{2}}{2}\underset{\sim}{i} + \frac{\sqrt{2}}{2}\underset{\sim}{j}$

C. $\frac{3}{2}\underset{\sim}{i} + \frac{3}{2}\underset{\sim}{k}$

D. $\frac{3\sqrt{2}}{2}\underset{\sim}{i} + \frac{3\sqrt{2}}{2}\underset{\sim}{k}$

E. $\frac{3\sqrt{2}}{2}$

Question 13
A quadrilateral $ABCD$ has $\left(\overrightarrow{AB} + \overrightarrow{BC}\right) \cdot \left(\overrightarrow{BC} + \overrightarrow{CD}\right) = 0$ but $\overrightarrow{AB} \cdot \overrightarrow{AD} \neq 0$.
The quadrilateral could be:

A. A rhombus, but not a square.
B. A rectangle, but not a square.
C. A parallelogram, but not a rhombus.
D. A trapezium, but not a parallelogram.
E. A trapezium, but not a rectangle.

Question 14
Any three-dimensional vector can be expressed as a linear combination of the three vectors $\underset{\sim}{a} = \underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$, $\underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c}$. The vector $\underset{\sim}{c}$ could be:

A. $4\underset{\sim}{i} + 2\underset{\sim}{k}$

B. $2\underset{\sim}{i} + 4\underset{\sim}{j}$

C. $\underset{\sim}{i} + 6\underset{\sim}{j} - \underset{\sim}{k}$

D. $-2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$

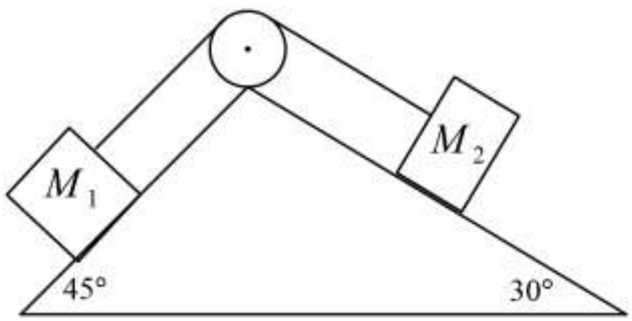
E. $5\underset{\sim}{i} + 6\underset{\sim}{j} + \underset{\sim}{k}$

SECTION A- continued

Question 15
An object is initially 7m from the origin with a velocity of 1m/s. Its motion is then given by the differential equation $a = \sqrt{x-3}$. The velocity when the object is 12m from the origin is:

A. $\frac{79}{3}$ m/s
B. $\frac{\sqrt{237}}{3}$ m/s
C. $\frac{82}{3}$ m/s
D. $\frac{\sqrt{246}}{3}$ m/s
E. $\frac{41}{3}$ m/s

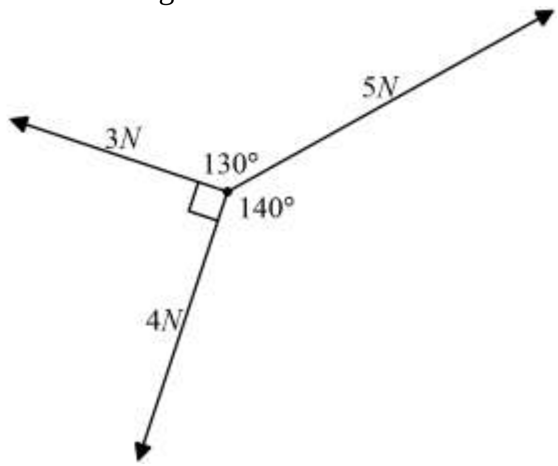
Question 16
The system below shows object with mass M_1 and M_2 kg resting on smooth inclined planes, connected by a smooth pulley. In order for the system to be in equilibrium, the ratio of masses $M_1 : M_2$ must be:



A. $1 : 1$
B. $\sqrt{2} : 2$
C. $2 : \sqrt{2}$
D. $\sqrt{2} : \sqrt{3}$
E. $\sqrt{3} : \sqrt{2}$

SECTION A- continued
TURN OVER

Question 17
An object is under the influence of three forces, with magnitude $3, 4$ and 5 newtons, and whose directions are shown in the diagram below.



The particle will:

A. Accelerate in the direction of the $5N$ force
B. Accelerate in a direction between the $3N$ and $5N$ force
C. Accelerate in a direction between the $3N$ and $4N$ force
D. Accelerate in a direction between the $4N$ and $5N$ force
E. Remain in equilibrium

Question 18
A biologist wants to estimate the average lifespan of a certain beetle. She uses a sample of size 50 to create a 99% confidence interval for the population mean μ days, which is $(35.32 , 41.58)$. The sample mean $\bar{x}$ and sample standard deviation s were closest to:

A. $\bar{x} = 38.45$, $s = 8.59$
B. $\bar{x} = 38.45$, $s = 1.22$
C. $\bar{x} = 38.45$, $s = 11.29$
D. $\bar{x} = 38.45$, $s = 1.62$
E. $\bar{x} = 38.45$, $s = 60.76$

SECTION A- continued

Question 19

A two-tailed z-test is used to to test a null hypothesis H_0 that $\mu = 120$. Given that the sample size is 25 and assuming that the population standard deviation is 15, which of these sample means would be sufficient to reject H_0 at a 5% significance level, but not at a 1% significance level?

A. 107
B. 109
C. 111
D. 113
E. 115

Question 20

A 100g golf ball is struck so that its initial speed is U m/s. Let U be a normal random variable with $U \sim N(5, 0.5^2)$. After it is struck, the ball moves along a flat golf green, subject only to a constant resistance force of 0.1 newtons.
After 2 seconds, what is the probability the ball has travelled at least 10 metres?

A. 0.421
B. 0.245
C. 0.056
D. 0.023
E. 0.0003

END OF SECTION A
TURN OVER

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SECTION B – Extended Response Questions

Instructions for Section B

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (10 marks)

The questions below refer to the function: $f(x) = \frac{2x-1}{x^2-2x+1}$

a. Find $f'(x)$. Also find the co-ordinates of any stationary points.

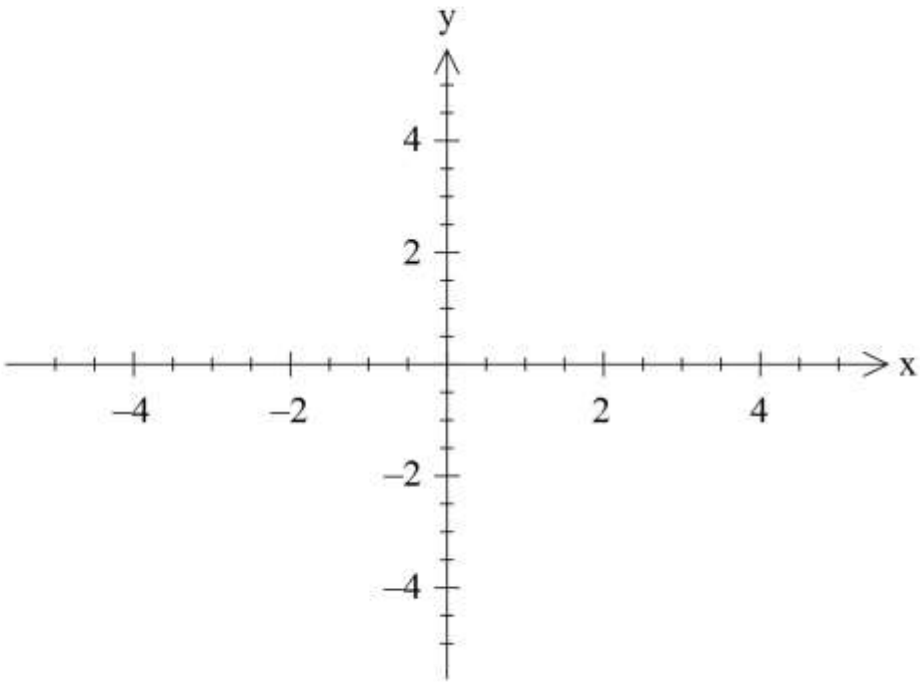
2 marks

b. Find $f''(x)$. Also find the co-ordinates of any points of inflection.

2 marks

SECTION B- Question 1- continued
TURN OVER

c. Sketch $f(x)$ on the axis below, labelling any axis intercepts, asymptotes, stationary points and points of inflection.



3 marks

d. Express $f(x)$ in partial fraction form.

1 mark

e. Find the exact area bound by $f(x)$, the x axis and the line with equation $x = -2$.

1 mark

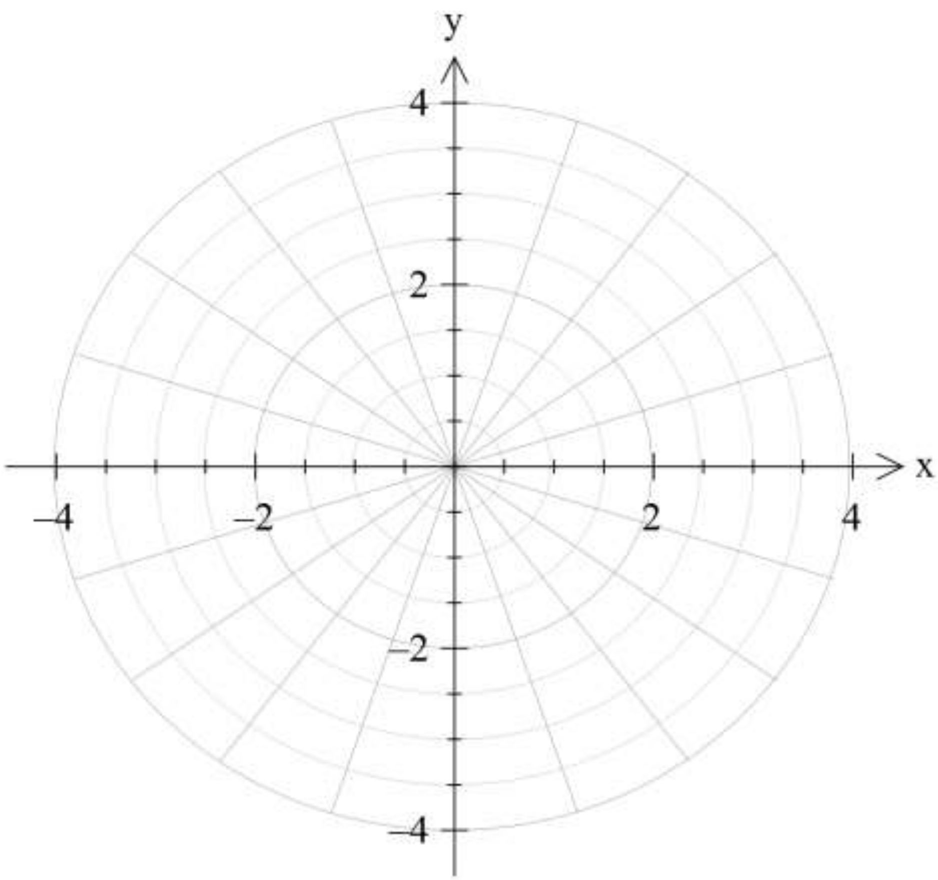
SECTION B- continued

Question 2 (14 marks)

a. Write the solutions to $z^5 = 32$ in polar form.

2 marks

b. Plot the solutions to $z^5 = 32$ on the Argand plane below. Label the solutions z_1, z_2, z_3, z_4 and z_5 in an **anti-clockwise** direction, starting from z_1 which is on the real axis.



1 mark

SECTION B- Question 2- continued
TURN OVER

c. Find the equation of the ray, with an endpoint at the origin, which passes through z_2.

1 mark

d. Find the Cartesian equation of the line defined by $|z - z_2| = |z - z_3|$.

2 marks

e. How many different straight lines satisfy $|z - z_a| = |z - z_b|$, where z_a, z_b can be any two distinct solutions to $z^5 = 32$?

1 mark

SECTION B- Question 2- continued

f. Find the equation for the circle that passes through the origin, z_2 and z_5.
Give your answer in the form $|z - w| = r$.

2 marks

g. A function $f(z) = pz + q$ can be applied to each of the points z_1, z_2, z_3, z_4 and z_5, such that $f(z)$ for each point will lie on the circle described in part **e.** State the values of p and q.

2 marks

SECTION B- Question 2- continued
TURN OVER

h. There is a cubic polynomial with real coefficients whose roots are z_1, z_2, and z_5.
Write this cubic polynomial in the form $ax^3 + bx^2 + cx + d$.

2 marks

i. Is there is a cubic polynomial with real coefficients whose roots are z_2, z_3, and z_4?
Justify your answer.

1 mark

SECTION B- continued

Question 3 (11 marks)
Lightbulbs sold by Bright Idea Pty Ltd are advertised to have an average lifetime of 3000 hours.

A keen student decides to undertake a statistical test to see whether the lifetime is actually shorter than the advertised claim.

a. Assuming the lightbulbs are normally distributed with $\mu = 3000$ and $\sigma = 250$, find the probability a single lightbulb would have a lifetime of 2920 hours or less. Round to four decimal places.

1 mark

b. Write appropriate hypotheses H_0 and H_1 for the statistical test undertaken by the student.

1 mark

c. In one test, the student takes a sample of 50 lightbulbs, and finds that this sample of 50 has a mean of 2920 hours. Using $\sigma = 250$, calculate the p value for this test, correct to four decimal places. Hence state, with a reason, whether the student should accept or reject the advertised claim, at a 1% significance level.

2 marks

SECTION B- Question 3- continued
TURN OVER

d. What is the largest value of the sample size n, for which a sample mean of 2920 would result in the student accepting the advertised claim, at the 1% significance level?

2 marks

e. Given that the student is testing the claim about the lightbulbs at the 1% significance level, state the value(s) of p and the population mean μ, which would result in a Type I error.
Also state the probability of making a Type I error in this statistical test.

2 marks

f. Imagine now that the true population mean is actually $\mu_2 = 2900$. Given that the student is using a sample of size 50, and testing at the 1% significance level, find the probability she makes a statistical error in this test. Round to four decimal places.

2 marks

g. If the lifetimes for the Bright Idea Pty Ltd. lightbulbs were **not** normally distributed, would the student still be able to carry out the statistical test described? Explain.

1 mark

SECTION B- continued
TURN OVER

Question 4 (12 marks)
Eric has a YouTube channel on which he posts interesting and entertaining videos about mathematics. Let P be the percentage of his subscribers who have seen a video, t days after it is posted.

For one of his best videos, Eric noticed that P can be modelled by the differential equation

$$\frac{dP}{dt} = \frac{100 - P}{10}\ , \text{ for } \ t \geq 0$$

a. Use **integration to** show that the general solution to the differential equation above is

$$P = Ae^{\frac{-t}{10}} + 100$$

3 marks

b. Given that Eric has shown the video to 4% of his subscribers before he posts it on YouTube, find the value of A.

1 marks

SECTION B- Question 4- continued

Eric posts a new video on his YouTube channel, and notices that this time the percentage of his subscribers who have seen the video is more closely modelled by

$$\frac{dP}{dt} = \frac{P}{10}\left(\frac{100 - P}{100}\right), \text{ for } t \geq 0$$

c. Use **differentiation** and substitution to verify that the function P below satisfies the differential equation above, and also the condition that 4% of his subscribers have seen the video before it is posted.

$$P = \frac{100}{1 + 24e^{\frac{-t}{10}}}, \text{ for } t \geq 0$$

3 marks

SECTION B- Question 4- continued
TURN OVER

d. What percentage of Eric's subscribers have seen his new video after 30 days?

Round to the nearest whole percentage.

1 mark

e. Use a **definite integral** to find the number of whole days it takes for at least 75% of Eric's subscribers to watch the new video.

2 marks

f. Find the number of days t, and the percentage P, when the percentage of subscribers is increasing at a maximum rate. Round to two decimal places where necessary.

2 marks

SECTION B- continued

Question 5 (13 marks)

Two friends are playing an online game called "Mini Tank Wars".
Player A programs Tank A to move according to the vector function

$$\underset{\sim}{A}(t) = \left(25\cos\left(\frac{\pi t}{30}\right)\right)\underset{\sim}{i} + \left(10\sin\left(\frac{\pi t}{15}\right)\right)\underset{\sim}{j} \quad \text{for } t \in [0,60],$$

where $\underset{\sim}{i}$ is a unit vector due East, $\underset{\sim}{j}$ is a unit vector due North,
t represents time in seconds, and distances are measured in metres.

a. State the initial position of Tank A, in co-ordinate form.

1 mark

b. Find the time(s) when Tank A passes through the origin (0,0).

2 marks

SECTION B- Question 5- continued
TURN OVER

c. Find the exact time(s) when Tank A is moving at a maximum speed, and also find the maximum speed (in metres per second, correct to two decimal places).

3 marks

d. Find the total distance travelled by Tank A in the 60 seconds (correct to two decimal places).

2 marks

SECTION B- Question 5- continued

Player B controls Tank B, which is stationary at the point (0,-10).

35 seconds after Tank A starts moving, Tank B shoots a missile with path:

$$\underset{\sim}{M}(t) = (60 - 2t)\underset{\sim}{i} + (4t - 140)\underset{\sim}{j} + \left(\frac{49}{20}(35 - t)(2t - 75)\right)\underset{\sim}{k} \quad \text{for} \quad t \in [35, a],$$

where $\underset{\sim}{k}$ is a unit vector in the positive vertical direction,
a is the time when the missile hits the ground.

e. When the missile is fired, ie. when $t = 35$, find its initial speed (correct to 2 decimal places), and its initial angle of elevation (to the nearest degree).

2 marks

SECTION B- Question 5- continued
TURN OVER

f. Find the time a when the missile hits the ground, hence find the distance that the missile lands from Tank A, correct to 2 decimal places.

3 marks

END OF QUESTION AND ANSWER BOOK