

## 2018 Trial Examination

STUDENT  
NUMBER

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Letter

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# SPECIALIST MATHEMATICS

## Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

### QUESTION & ANSWER BOOK

#### Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

#### Materials supplied

- Question and answer book of 12 pages.

#### Instructions

- Print your student number in the space provided on the top of this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.**

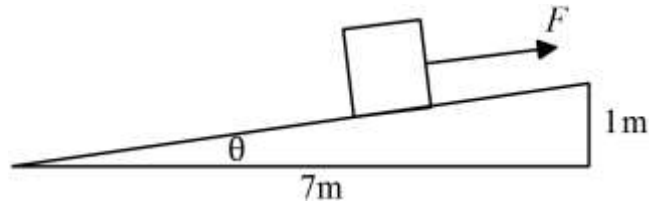
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**Instructions**

Answer **all** questions in the spaces provided.  
 Unless otherwise specified, an **exact** answer is required to a question.  
 In questions where more than one mark is available, appropriate working **must** be shown.  
 Unless otherwise indicated, the diagram in this book are not drawn to scale.  
 Take the acceleration due to gravity to have magnitude  $g \text{ ms}^{-2}$ , where  $g = 9.8$ .

**Question 1 (3 marks)**

A constant force of  $F$  newtons is applied to a body of mass  $10\text{kg}$ , parallel to a smooth inclined plane with angle of elevation  $\theta$  as shown on the diagram below.



a. Show that  $\sin \theta = \frac{\sqrt{2}}{10}$ .

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1 mark

b. Find the force  $F$  required to give the body an acceleration of  $\sqrt{2} \text{ m/s}$  up the plane.  
 Write your answer in the form  $a(b + g)$ , where  $a, b \in R$  and  $g$  is acceleration due to gravity.

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2 marks

**TURN OVER**

**Question 2 (3 marks)**

Find all possible values for the gradient of the curve  $\sin(x) \sin(y) = \frac{1}{2}$  at the points where  $x = \frac{\pi}{4}$ ,  $0 < y < 2\pi$ .

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**Question 3 (3 marks)**

Solve the equation  $z^3 = (1 - i)^6$  for  $z$ , writing the solutions in Cartesian form.

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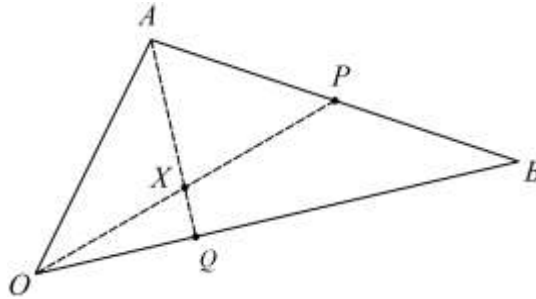
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**Question 4 (5 marks)**

In triangle  $OAB$ , let  $P$  be the midpoint of  $AB$  and  $Q$  be the point on  $OB$  such that  $\overrightarrow{QB} = 2\overrightarrow{OQ}$ . Let  $X$  be the point of intersection of  $AQ$  and  $OP$ .



a. Given that  $\underline{a} = \overrightarrow{OA}$  and  $\underline{b} = \overrightarrow{OB}$ , express  $\overrightarrow{AQ}$  in terms of  $\underline{a}$  and  $\underline{b}$ .

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1 mark

b. Express  $\overrightarrow{OP}$  in terms of  $\underline{a}$  and  $\underline{b}$ .

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1 mark

c. Let  $k$  be the rational number such that  $\overrightarrow{AX} = k\overrightarrow{AQ}$ . Find  $k$ .

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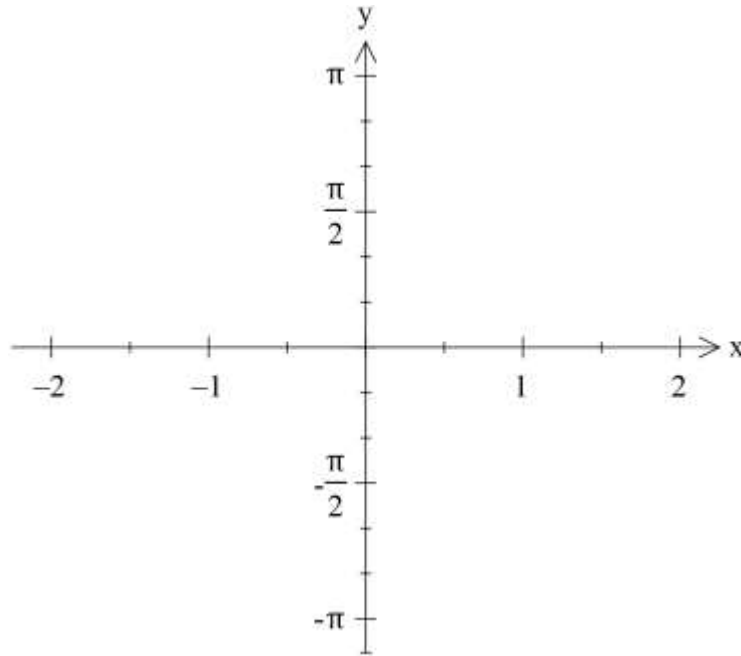
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3 marks

**TURN OVER**

**Question 5 (5 marks)**

- a. Sketch the graph of  $y = \arcsin\left(\frac{1}{2} - x\right)$  on the axis below.  
Label endpoints in coordinate form.



2 marks

- b. Find the exact volume of the solid obtained when the section of the graph of  $y = \arcsin\left(\frac{1}{2} - x\right)$  where  $x \geq 0$  is rotated around the  $y$  axis.

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3 marks

**Question 6 (4 marks)**

Two teams of soccer players are timed running 100m sprints. Team A's times are normally distributed with mean 14.7 seconds and standard deviation 0.8 seconds. Team B's times are normally distributed with mean 13.7 seconds and standard deviation 0.6 seconds.

- a. Using the normal random variable  $D = A - B$  to represent the difference in times, find the probability that a randomly selected player from Team A would be faster than a randomly selected player from Team B. Correct your answer to two decimal places.

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2 marks

- b. If a random sample of four players is taken from each team, find the probability that the average time for the four Team A players would be faster than the average time for the four Team B players. Correct your answer to three decimal places.  
You may assume that each selection is independent and normally distributed.

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2 marks

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**Question 9 (6 marks)**

- a. Find an anti-derivative for the function  $g$ , where  $g(x) = \frac{\log_e|x-1|}{x-1}$ .

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1 mark

- b. Find  $x$  intercepts for the function  $g$ .

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1 mark

- c. Find the area bound by the graph of  $g$ , the  $x$  axis and the line  $x = -2$ .

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1 mark

**TURN OVER**

