Neap:

Trial Examination 2018

VCE Specialist Mathematics Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 8 pages

Formula sheet

Working space is provided throughout the booklet

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (3 marks)

Sketch the graph of $y = \dfrac{1}{x^2 + 4} + 1$ on the axes below. Label any asymptotes with their equations and label any intercepts with the axes and stationary points, expressing them as coordinates.
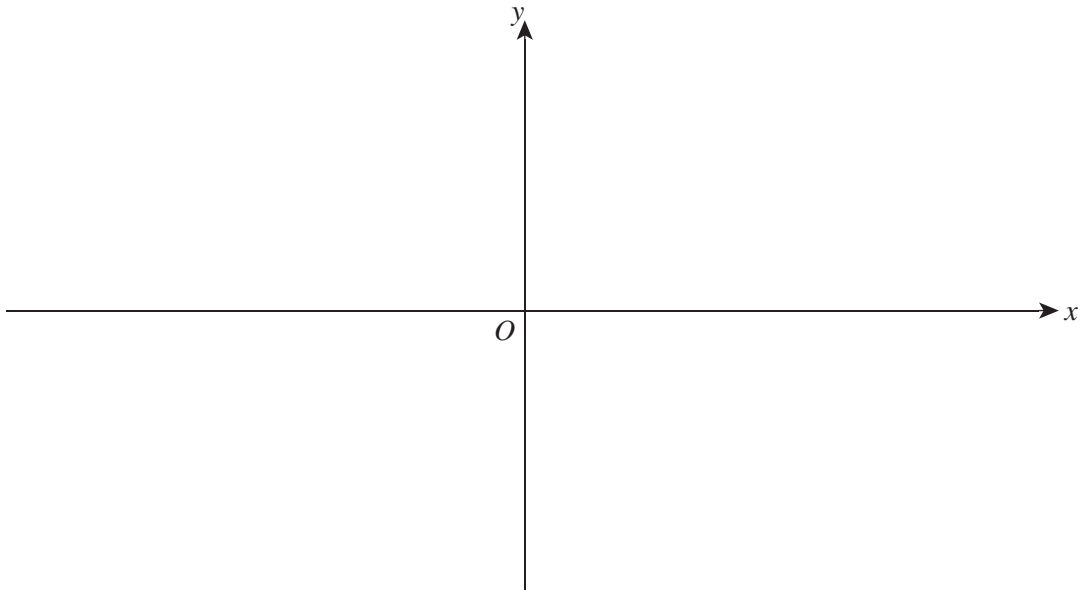
Question 2 (3 marks)

A curve C is defined by the parametric equations $x = 2 - t^2$ and $y = 4t$.

Find the equation of the normal to C at the point where $t = 1$. Give your answer in the form $y = mx + c$.

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Question 3 (3 marks)

The weights of lemons grown by a farmer follow a normal distribution with mean 58 g and standard deviation 9 g. The farmer sells his lemons in bags of 36.

Find the approximate probability that the mean weight of lemons in a randomly selected bag of 36 lemons is greater than 61 g. Give your answer correct to three decimal places.

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Question 4 (3 marks)

Two particles, *A* and *B*, are connected by a light, inextensible string which passes over a smooth, fixed pulley as shown in the following diagram.
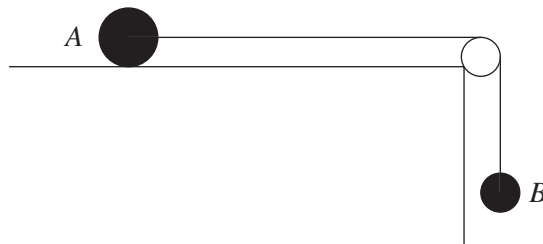
Particle *A* is of mass 3*m* kg and particle *B* is of mass 2*m* kg. Particle *A* is initially at rest on a smooth horizontal surface. The system is released from rest with the string taut and particle *A* moves towards the pulley.

Find the tension, *T*, in the string. Give your answer in terms of *m* and *g*.

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Question 5 (5 marks)

A curve C has the equation $x^3 + xy^2 - y^3 = 2$.

Show that there is no point on C at which $\dfrac{dy}{dx} = 0$.

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Question 6 (5 marks)

Solve the equation $z^3 = -2 + 2i$, $z \in C$. Give your answers in the form $r\mathrm{cis}(\theta)$ where $r > 0$ and $-\pi \le \theta < \pi$.

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Question 7 (3 marks)

The points O, P and Q lie on a circle with diameter OQ. The position vectors of P and Q, relative to O, are $\underset{\sim}{p}$ and $\underset{\sim}{q}$ respectively.
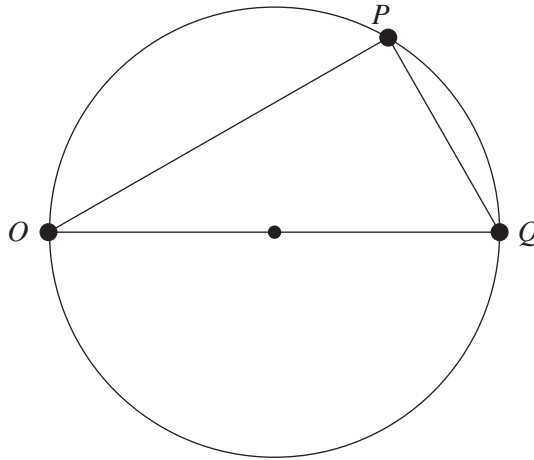
Prove that $\underset{\sim}{p} \cdot \underset{\sim}{q} = \left| \underset{\sim}{p} \right|^2$.

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Question 8 (5 marks)

A particle's acceleration, a m/s^2, in terms of its displacement, x m, from a fixed origin O is given by $a = 2x - \dfrac{1}{6}x^2$, $0 \le x \le 18$. The particle's velocity is v m/s.

a. Given that the particle starts from rest at O, find v^2 in terms of x.   3 marks

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b. Find the particle's displacement from O when it comes to rest for the first time.   2 marks

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Question 9 (5 marks)

a. Show that $\sin^4(\theta) = \frac{1}{8}(3 - 4\cos(2\theta) + \cos(4\theta))$.   3 marks

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b. Hence find $\displaystyle\int_{0}^{\frac{\pi}{4}} \sin^4(\theta)\,d\theta$. Give your answer in the form $\dfrac{a\pi + b}{c}$ where a, b and c are integers.   2 marks

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Question 10 (5 marks)

Solve the equation $\arccos(x) + \arccos(2x) = \dfrac{\pi}{2}$, $x \geq 0$.

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END OF QUESTION AND ANSWER BOOKLET