Year 2018

VCE

Specialist Mathematics

Trial Examination 2

Solutions

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- While every care has been taken, no guarantee is given that these answers are free from error. Please contact us if you believe you have found an error.

SECTION 1

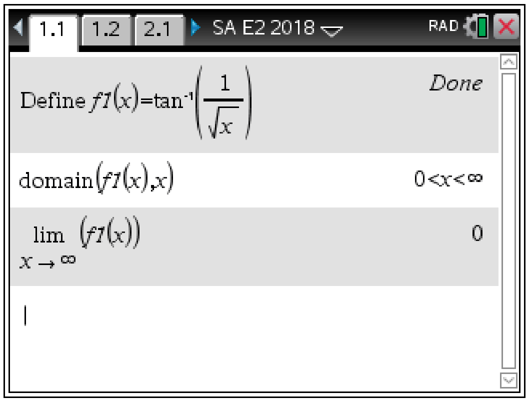
ANSWERS

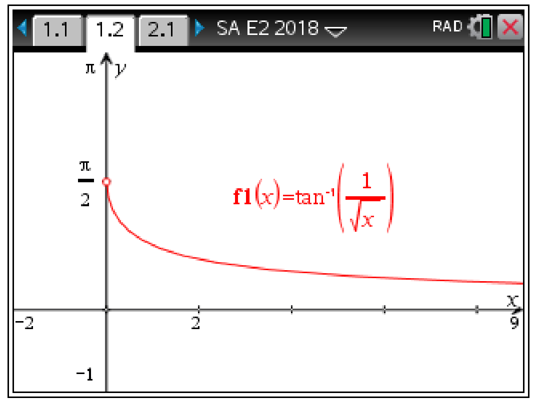
1	A	B	**C**	D	E
2	A	B	C	D	**E**
3	A	B	C	**D**	E
4	**A**	B	C	D	E
5	A	B	C	D	**E**
6	A	B	C	**D**	E
7	A	B	**C**	D	E
8	A	**B**	C	D	E
9	A	**B**	C	D	E
10	**A**	B	C	D	E
11	A	**B**	C	D	E
12	A	B	C	**D**	E
13	A	B	**C**	D	E
14	A	B	C	D	**E**
15	A	B	C	D	**E**
16	A	B	**C**	D	E
17	**A**	B	C	D	E
18	**A**	B	C	D	E
19	A	**B**	C	D	E
20	A	B	C	**D**	E

SECTION A

Question 1 **Answer C**

$f(x)=\tan^{-1}\left(\frac{1}{\sqrt{x}}\right)$, domain $(0,\infty)$, range $\left(0\,,\,\frac{\pi}{2}\right)$



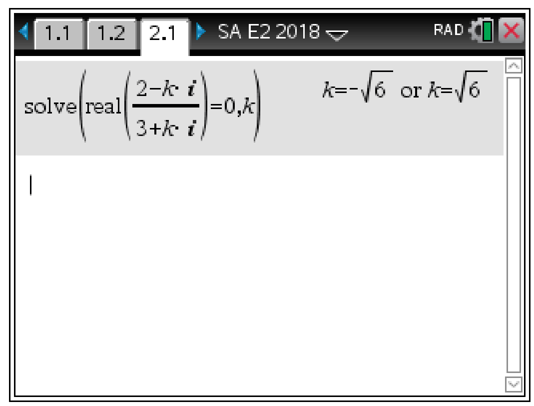


Question 2 **Answer E**

$$\frac{2-ki}{3+ki}=\frac{2-ki}{3+ki}\times\frac{3-ki}{3-ki}=\frac{6-k^2-5ki}{9+k^2}$$

$$\text{Re}\left(\frac{6-k^2-5ki}{9+k^2}\right)=0 \quad\Rightarrow\quad 6-k^2=0$$

$k=\pm\sqrt{6}$ only.



Question 3 **Answer D**

vertical asymptotes at $x=a$ and $x=b$ and a horizontal asymptote at $y=0$

and a maximum turning point at $x=\frac{a+b}{2}$

Question 4 **Answer A**

$z=3a-ai$ is a root, since the polynomial has real co-efficients, so is the conjugate $\overline{z}=3a+ai$

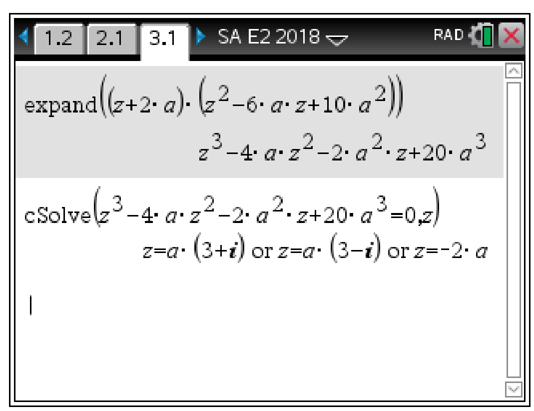
$z+\overline{z}=6a \qquad z.\overline{z}=a^2\left(9-i^2\right)=10a^2$

The polynomial has factors

$\left(z+2a\right)\left(z^2-6az+10a^2\right)$,

expanding gives

$z^3-4az^2-2a^2z+20a^3$



Question 5 **Answer E**

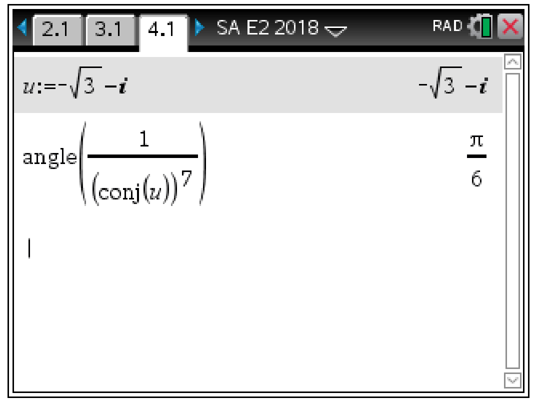
$$u=-\sqrt{3}-i=2\text{cis}\left(-\frac{5\pi}{6}\right)$$

$$\bar{u}=-\sqrt{3}+i=2\text{cis}\left(\frac{5\pi}{6}\right)$$

$$\frac{1}{\bar{u}}=\frac{1}{2}\text{cis}\left(-\frac{5\pi}{6}\right)$$

$$\arg\left(\frac{1}{\bar{u}^7}\right)=7\times-\frac{5\pi}{6}=-\frac{35\pi}{6}$$

$$\text{Arg}\left(\frac{1}{\bar{u}^7}\right)=-\frac{35\pi}{6}+6\pi=\frac{\pi}{6}$$



Question 6 **Answer D**

There are 5 roots, one of the roots is $z=i$, $z^5=i^5=i$

The polynomial must be $z^5-i=0$

Question 7 **Answer C**

If $m=1$, $\underset{\sim}{a}=\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}$ and $\underset{\sim}{b}=-\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{k}$, $\underset{\sim}{a}=-\underset{\sim}{b}$

then the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel, Matilda is correct.

If $m=4$, $\underset{\sim}{a}=4\underset{\sim}{i}+2\underset{\sim}{j}-2\underset{\sim}{k}$ and $\underset{\sim}{b}=-2\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{k}$, $\underset{\sim}{a}=-2\underset{\sim}{b}$

then the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel, Nick is correct.

$\underset{\sim}{a}.\underset{\sim}{b}=-m\sqrt{m}-\sqrt{m}-\sqrt{m}=-\sqrt{m}\,(m+2)$ but $m>0$

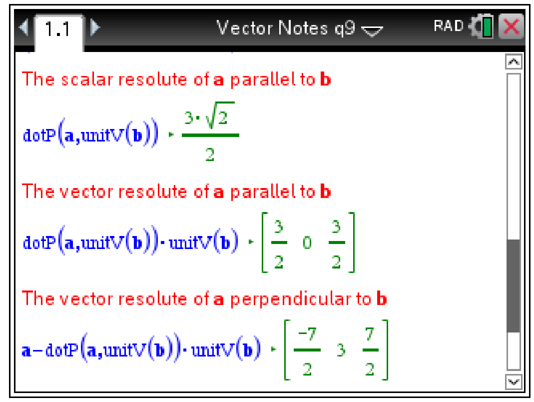
If $m=1$ or $m=-2$ then $\underset{\sim}{a}.\underset{\sim}{b}\neq 0$ the vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are not perpendicular,

Yvonne and Zach are both incorrect

Question 8 **Answer B**

The part of the curve under the *x*-axis becomes positive, and the graph is steeper then $g(x)=\left[f(x)\right]^2$

Question 9 **Answer B**



$\underset{\sim}{a} = -2\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}$, $|\underset{\sim}{a}| = \sqrt{4+9+25} = \sqrt{38}$

the length of the vector $\underset{\sim}{a}$ is $\sqrt{38}$, **A.** is true

$\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{k}$, $\underset{\sim}{a} + \underset{\sim}{b} = -\underset{\sim}{i} + 3\underset{\sim}{j} + 6k$

$|\underset{\sim}{a} + \underset{\sim}{b}| = \sqrt{1+9+36} = \sqrt{46}$ **B.** is false

The scalar resolute of $\underset{\sim}{a}$ in the direction $\underset{\sim}{b}$ is

$\underset{\sim}{a}.\hat{\underset{\sim}{b}} = \frac{\underset{\sim}{a}.\underset{\sim}{b}}{|\underset{\sim}{b}|} = \frac{-2+5}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$ **C.** is true

The vector resolute of $\underset{\sim}{a}$ in the direction $\underset{\sim}{b}$ is $\left(\underset{\sim}{a}.\hat{\underset{\sim}{b}}\right)\hat{\underset{\sim}{b}} = \frac{3\sqrt{2}}{2} \times \frac{1}{\sqrt{2}}(\underset{\sim}{i} + \underset{\sim}{k}) = \frac{3}{2}(\underset{\sim}{i} + \underset{\sim}{k})$

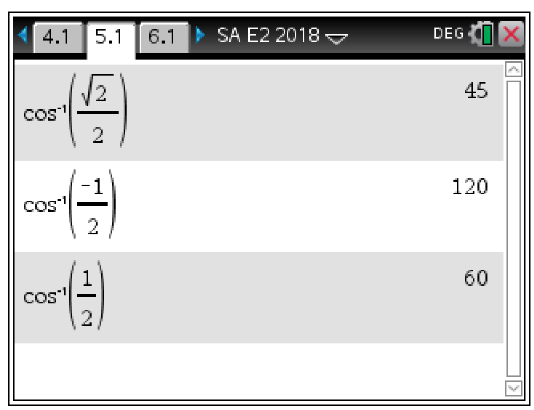
D. is true. The vector resolute of $\underset{\sim}{a}$ perpendicular $\underset{\sim}{b}$ is

$\underset{\sim}{a} - \left(\underset{\sim}{a}.\hat{\underset{\sim}{b}}\right)\hat{\underset{\sim}{b}} = \left(-2\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}\right) - \frac{3}{2}(\underset{\sim}{i} + \underset{\sim}{k}) = \frac{1}{2}\left(-7\underset{\sim}{i} + 6\underset{\sim}{j} + 7\underset{\sim}{k}\right)$ **E.** is true.

Question 10 **Answer A**

$f(2) < 2$, the slope of the tangent at $x = 2$ is positive approximately a slope of 45^0,
so $f'(2) \approx 1$, the second derivative is negative as the function is increasing, so
$f''(2) < f'(2) < f(2) < 2$

Question 11 **Answer B**



$\underset{\sim}{a} = \frac{1}{2}\left(\sqrt{2}\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}\right)$, and

$\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 1$

so $\underset{\sim}{a}$ is a unit vector, $|\underset{\sim}{a}| = 1$. Using direction cosines, the vector makes an angle of

$\alpha = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45^0$ with the *x*-axis,

$\beta = \cos^{-1}\left(-\frac{1}{2}\right) = 120^0$ with the *y*-axis, and

$\chi = \cos^{-1}\left(\frac{1}{2}\right) = 60^0$ with the *z*-axis.

Question 12 **Answer D**

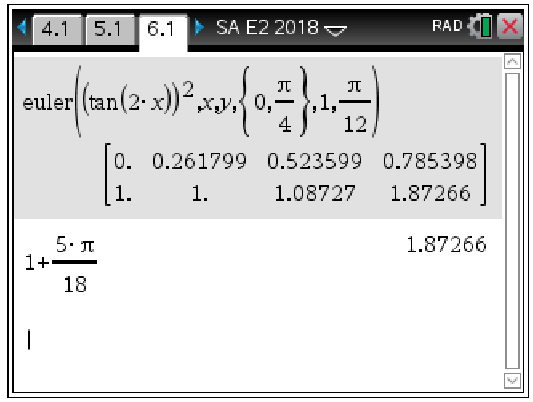
when $x=1$, $y=0$, $m=1$, when $y=\pm 1$, $m=\infty$, when $x=2$, $m=0$

is only satisfied by $m=\dfrac{dy}{dx}=\dfrac{x-2}{y^2-1}$

Question 13 **Answer C**

$\dfrac{dy}{dx}=f(x)=\tan^2(2x) \quad y_0=a \quad x_0=0 \quad h=\dfrac{\pi}{12}$ using Euler's Method

$y_1=y_0+hf(x_0)=a+\dfrac{\pi}{12}\tan^2(0)=a$



$y_2=y_1+hf(x_1)$ and $x_1=\dfrac{\pi}{12}$

$=a+\dfrac{\pi}{12}\tan^2\left(\dfrac{\pi}{6}\right)=a+\dfrac{\pi}{12}\times\dfrac{1}{3}=a+\dfrac{\pi}{36}$

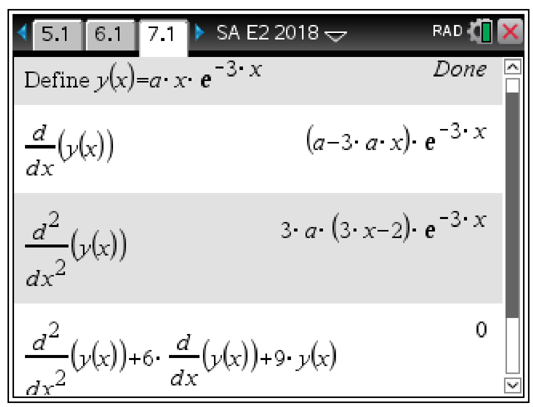
$y_3=y_2+hf(x_2)$ and $x_2=\dfrac{\pi}{6}$

$=a+\dfrac{\pi}{36}+\dfrac{\pi}{12}\tan^2\left(\dfrac{\pi}{3}\right)=a+\dfrac{\pi}{36}+\dfrac{\pi}{12}\times 3$

$=a+\pi\left(\dfrac{1}{36}+\dfrac{1}{4}\right)$

$=a+\dfrac{5\pi}{18}$

Question 14 **Answer E**



$y=axe^{-3x}$

$\dfrac{dy}{dx}=a(1-3x)e^{-3x}$

$\dfrac{d^2y}{dx^2}=a(9x-6)e^{-3x}$

$\dfrac{d^2y}{dx^2}+b\dfrac{dy}{dx}+cy=0$

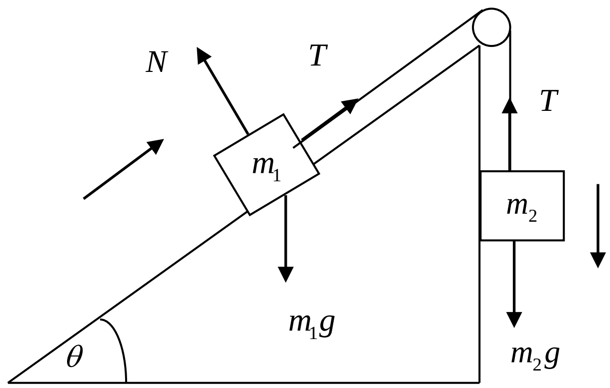
$ae^{-3x}\left[9x-6+b(1-3x)+cx\right]$

$ae^{-3x}\left[x(9+c-3b)+b-6\right]=0$

$\Rightarrow b-6=0 \qquad \Rightarrow b=6$

$9+c-3b=0 \quad \Rightarrow c=9$

Question 15 **Answer E**



resolving up parallel to plane around the m_1 kg mass $(1)\ \ T - m_1 g\sin(\theta) = m_1 a$

resolving downwards around the m_2 kg mass $(2)\ \ m_2 g - T = m_2 a$

adding to eliminate the tension in the string, to find the acceleration a, of the system

$(1)+(2)\ \ m_2 g - m_1 g\sin(\theta) = m_1 a + m_2 a \quad \Rightarrow \quad a = \dfrac{g\left(m_2 - m_1\sin(\theta)\right)}{m_1 + m_2}$

Checking the alternatives

$a > 0$ when $m_2 > m_1\sin(\theta) \Rightarrow \dfrac{m_2}{m_1} > \sin(\theta)$

and $a = 0$ when $\dfrac{m_2}{m_1} = \sin(\theta)$

If $\theta = 30^0$ and $\dfrac{m_2}{m_1} = \sin\left(30^0\right) = \dfrac{1}{2}$ then the system is in equilibrium, **A.** is correct

If $\theta = 30^0 \quad a = \dfrac{g\left(m_2 - \dfrac{m_1}{2}\right)}{m_1 + m_2}$ if $\dfrac{m_2}{m_1} < \dfrac{1}{2}$ then $a < 0$

therefore the mass m_2 moves upwards, **B.** is correct

If $\theta = 45^0$ and $\dfrac{m_2}{m_1} = \sin\left(45^0\right) = \dfrac{\sqrt{2}}{2}$ then the system is in equilibrium, **C.** is correct

If $\theta = 60^0$ and $\dfrac{m_2}{m_1} = \sin\left(60^0\right) = \dfrac{\sqrt{3}}{2}$ then the system is in equilibrium, **D.** is correct

If $\theta = 60^0 \quad a = \dfrac{g\left(m_2 - \dfrac{\sqrt{3}\,m_1}{2}\right)}{m_1 + m_2}$ if $\dfrac{m_2}{m_1} < \dfrac{\sqrt{3}}{2}$ then $a < 0$

therefore the mass m_2 moves upwards, **E.** is incorrect

Question 16 **Answer C**

$$t = e^{kx} \quad \Rightarrow \quad \frac{dt}{dx} = ke^{kx}$$

$$v = \frac{dx}{dt} = \frac{1}{k}e^{-kx} \quad \Rightarrow \quad \frac{dv}{dx} = -e^{-kx}$$

$$a = v\frac{dv}{dx} = -\frac{1}{k}e^{-2kx}$$

Question 17 **Answer A**

$v = \dfrac{dx}{dt} = t\sqrt{x}$ separating the variables

$$\int \frac{1}{\sqrt{x}}\,dx = \int t\,dt$$

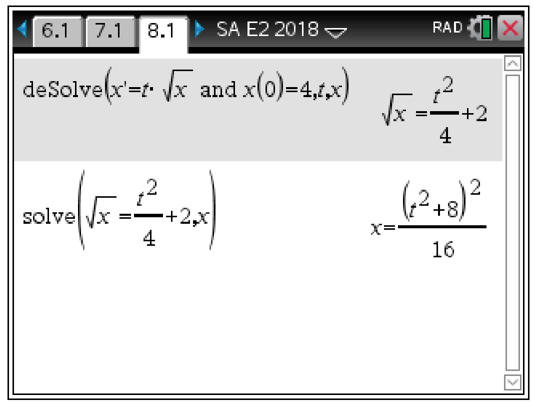
$$2\sqrt{x} = \frac{1}{2}t^2 + c$$

when $t = 0$, $x = 4 \Rightarrow 2\sqrt{4} = 0 + c \Rightarrow c = 4$

$$2\sqrt{x} = \frac{1}{2}t^2 + 4$$

$$\sqrt{x} = \frac{1}{4}t^2 + 2 = \frac{t^2 + 8}{4}$$

$$x = \frac{1}{16}\left(t^2 + 8\right)^2$$



Question 18 **Answer A**

$X \sim (30, 9)$, $Y \sim (20, 4)$

$P = 2X + 2Y$

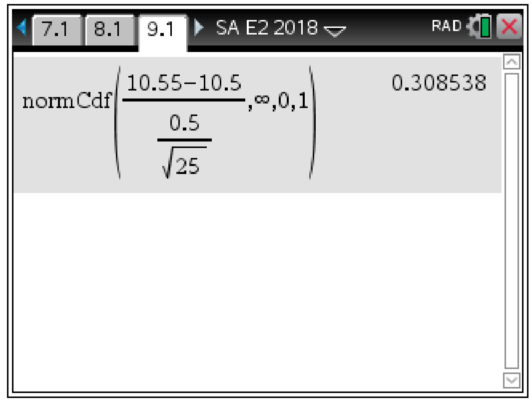
$E(P) = 2E(X) + 2E(Y) = 2 \times 30 + 2 \times 20$

$E(P) = 100$

$\text{Var}(P) = 4\text{Var}(X) + 4\text{Var}(Y) = 4 \times 9 + 4 \times 4$

$\text{Var}(P) = 52$

Question 19 **Answer B**



$$n = 25 \quad , \quad \bar{X} \sim N\left(10.5, \frac{0.5^2}{25}\right) \quad \sigma_{\bar{X}} = \frac{0.5}{5} = 0.1$$

$$\Pr\left(\bar{X} > 10.55\right) = \Pr\left(Z > \frac{10.55 - 10.5}{0.1}\right)$$

$$= \Pr\left(Z > 0.5\right)$$

$$= 0.3085$$

Question 20 **Answer D**

$$\bar{x} = 150 \quad , \quad z = 1.96 \quad , \quad s = 5 \quad , \quad n = 25$$

$$\bar{x} \pm z \times \frac{s}{\sqrt{n}}$$

$$150 \pm 1.96 \times \frac{5}{\sqrt{25}}$$

$$148.04 - 151.96$$

END OF SECTION A SUGGESTED ANSWERS

SECTION B

Question 1

a.i. $f(x)=\dfrac{2x^3+10x^2+4x-16}{x^3-2x^2-x+2}$

$f(x)=\dfrac{2(x-1)(x+2)(x+4)}{(x-2)(x-1)(x+1)}$

$f(x)=2+\dfrac{2(7x+10)}{x^2-x-2}=2+\dfrac{16}{x-2}-\dfrac{2}{x+1}\quad , \quad x\neq 1$

the domain $D=R\setminus\{-1,1,2\}$ A1

ii. the vertical asymptotes are $x=-1$ and $x=2$

the horizontal asymptote is $y=2$ A1

iii. $f'(x)=\dfrac{-2(7x^2+20x+4)}{(x^2-x-2)^2}$ A1

for stationary points $f'(x)=0$

$7x^2+20x+4=0 \quad \Rightarrow x=-2.64,-0.22$

$f(-2.64)=-0.23 \quad , \quad f(-0.22)=-7.77$

the stationary points are $(-2.64,-0.23)$ and $(-0.22,-7.77)$ A1

iv. $f''(x)=\dfrac{4(7x^3+30x^2+12x+16)}{(x^2-x-2)^3}$ A1

for inflexion points $f''(x)=0$

$7x^3+30x^2+12x+16=0 \quad \Rightarrow x=-4$

$f(-4)=0$ the inflexion point is $(-4\ ,\ 0)$ A1

b. Note that when $x=1$, $\lim_{x\to1} f(x)=-15$

the point $(1,-15)$ is a point of discontinuity, open circle at $(1,-15)$

Also the graph crosses the horizontal asymptote $y=2$

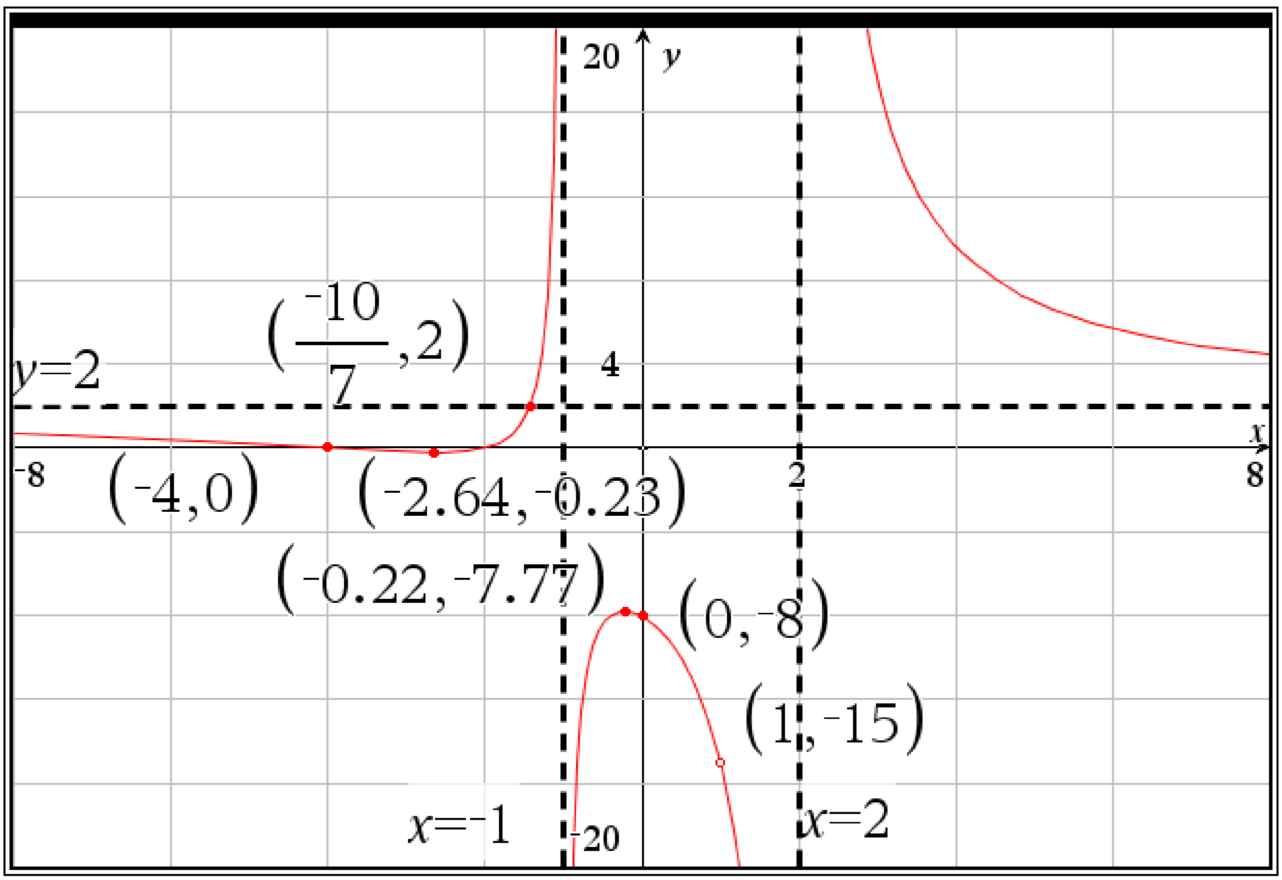
when $7x+10=0$ at $x=-\frac{10}{7}$ $\left(-\frac{10}{7},2\right)$

the graph crosses the x-axis when $y=0$ at $x=-2$ and $x=-4$

axial intercepts $(-4,0)$ $(-2,0)$

the graph crosses the y-axis when $x=0$ at $y=-8$ $(0,-8)$

correct graph, shape, asymptotes, axial intercepts, point of discontinuity G3



Define $f1(x)=\dfrac{2\cdot x^3+10\cdot x^2+4\cdot x-16}{x^3-2\cdot x^2-x+2}$	*Done*
$f1(0)$	$^-8$
domain$(f1(x),x)$	$x\neq {}^-1$ and $x\neq 1$ and $x\neq 2$
factor$(2\cdot x^3+10\cdot x^2+4\cdot x-16)$	$2\cdot(x-1)\cdot(x+2)\cdot(x+4)$
factor$(x^3-2\cdot x^2-x+2)$	$(x-2)\cdot(x-1)\cdot(x+1)$
$f1(x)$	$\dfrac{2\cdot(x^2+6\cdot x+8)}{x^2-x-2}$
expand$(f1(x))$	$\dfrac{^-2}{x+1}+\dfrac{16}{x-2}+2$
solve$(f1(x)=2,x)$	$x=\dfrac{^-10}{7}$
$\dfrac{d}{dx}(f1(x))$	$\dfrac{^-2\cdot(7\cdot x^2+20\cdot x+4)}{(x^2-x-2)^2}$
zeros$\left(\dfrac{d}{dx}(f1(x)),x\right)$	$\{^-2.64,{}^-0.22\}$
$f1(\{^-2.64075,{}^-0.2163\})$	$\{^-0.23,{}^-7.77\}$
$\dfrac{d^2}{dx^2}(f1(x))$	$\dfrac{4\cdot(7\cdot x^3+30\cdot x^2+12\cdot x+16)}{(x^2-x-2)^3}$
solve$\left(\dfrac{d^2}{dx^2}(f1(x))=0,x\right)$	$x={}^-4$

Question 2

a. $S = \{z : |z+3+i| = 5\}$ let $z = x + yi$

$|(x+3)+(y+1)i| = 5$

$\sqrt{(x+3)^2+(y+1)^2} = 5$

$(x+3)^2+(y+1)^2 = 25$ circle centre $(-3,-1)$ radius 5 A1

b. $T = \{z : \text{Arg}(z+3) = -\frac{3\pi}{4}\}$

$\tan^{-1}\left(\frac{x+3}{y}\right) = -\frac{3\pi}{4}$

$\tan\left(-\frac{3\pi}{4}\right) = 1 = \frac{x+3}{y}$ M1

$y = x+3$ for $x < -3$ ray not including the point $(-3,0)$ making

an angle of -135^0 with the positive end of the real axis. A1

c. $R = \{z : |z| = |z+3+3i|\}$

$|x+yi| = |(x+3)+(y+3)i|$

$\sqrt{x^2+y^2} = \sqrt{(x+3)^2+(y+3)^2}$ M1

$x^2+y^2 = x^2+6x+9+y^2+6y+9$

$6x+6y+18 = 0$

$y = -(x+3)$ line A1

d. $u \in S \cap T$

solving $(x+3)^2+(y+1)^2 = 25$ and $y = x+3$ for $x < -3$

$(x+3)^2+(x+4)^2 = 25$

$x^2+6x+9+x^2+8x+16 = 25$

$2x^2+14x = 0$ M1

$2x(x+7) = 0$ $x < -3$

$x = -7$ $y = -4$

$u = -7-4i$ A1

e. $v \in S \cap R$

solving $(x+3)^2+(y+1)^2=25$ and $y=-(x+3)$

$(x+3)^2+(-x-2)^2=25$

$x^2+6x+9+x^2+4x+4=25$

$2x^2+10x-12=0$

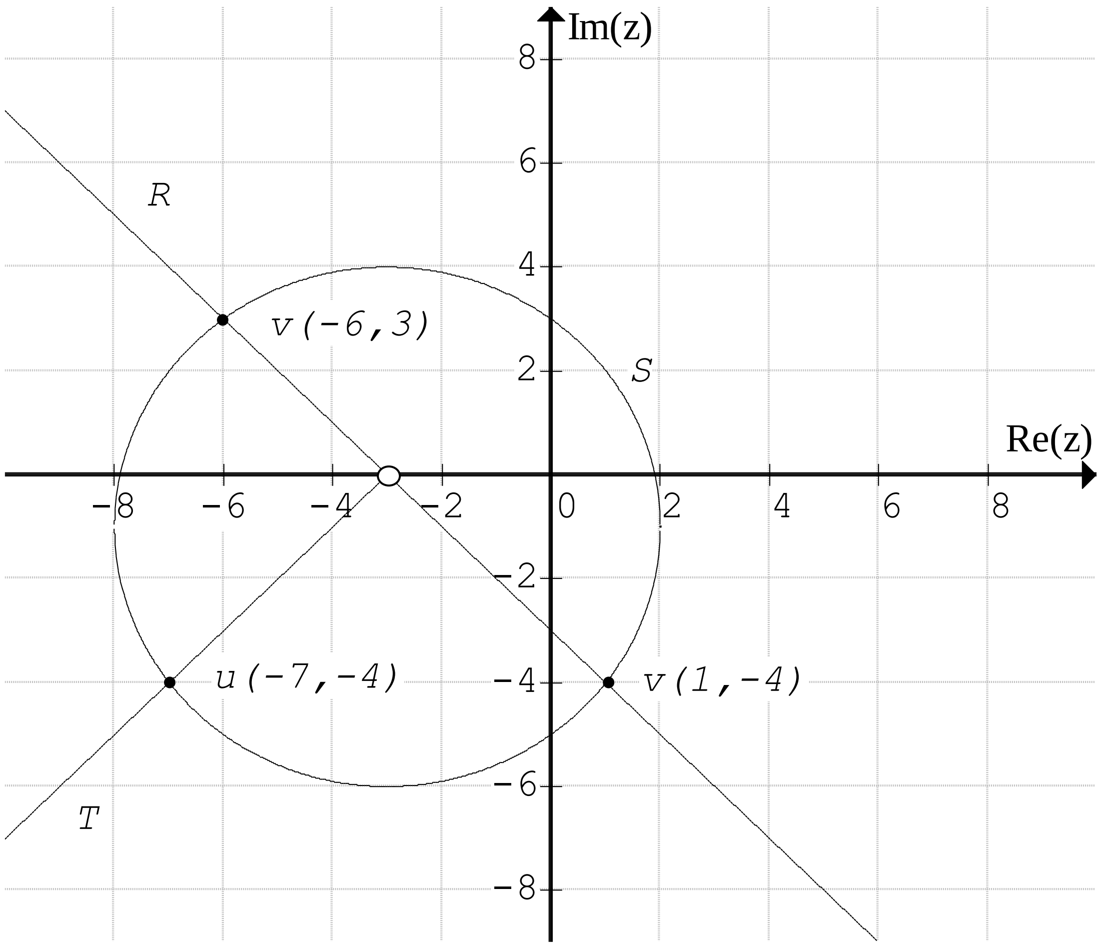
$2(x^2+5x-6)$ M1

$2(x-1)(x+6)=0$

$x=-6\ ,\ 1 \ \Rightarrow\ y=3\ ,\ -4$

$v=-6+3i\ ,\ 1-4i$ A1

f. G2



g. It is the area of a sector, $\theta=\dfrac{\pi}{4}$, $r=5$

$A=\dfrac{1}{2}r^2\theta=\dfrac{1}{2}\times 25\times\dfrac{\pi}{4}$

$A=\dfrac{25\pi}{8}$ A1

$z:=x+y\cdot i$	$x+y\cdot i$
$\lvert z+3+i\rvert=5$	$\sqrt{x^2+6\cdot x+y^2+2\cdot y+10}=5$
$\left(\sqrt{x^2+6\cdot x+y^2+2\cdot y+10}=5\right)^2$	$x^2+6\cdot x+y^2+2\cdot y+10=25$
$\text{completeSquare}\left(x^2+6\cdot x+y^2+2\cdot y+10=25,\{x,y\}\right)$	$(x+3)^2+(y+1)^2=25$
$\text{angle}(z+3)=\frac{-3\cdot\pi}{4}$	$\frac{\pi\cdot\text{sign}(y)}{2}-\tan^{-1}\left(\frac{x+3}{y}\right)=\frac{-3\cdot\pi}{4}$
$\text{solve}\left(\text{angle}(z+3)=\frac{-3\cdot\pi}{4},y\right)$	$y=x+3 \text{ and } x<-3$
$\lvert z\rvert=\lvert z+3+3\cdot i\rvert$	$\sqrt{x^2+y^2}=\sqrt{x^2+6\cdot x+y^2+6\cdot y+18}$
$\left(\sqrt{x^2+y^2}=\sqrt{x^2+6\cdot x+y^2+6\cdot y+18}\right)^2$	$x^2+y^2=x^2+6\cdot x+y^2+6\cdot y+18$
$\text{solve}\left(x^2+y^2=x^2+6\cdot x+y^2+6\cdot y+18,y\right)$	$y=-(x+3)$
$\text{solve}\left((x+3)^2+(y+1)^2=25 \text{ and } y=x+3,\{x,y\}\right)\vert x<-3$	$x=-7 \text{ and } y=-4$
$\text{zeros}\left(\begin{cases}\lvert z+3+i\rvert-5\\ \text{angle}(z+3)+\frac{3\cdot\pi}{4}\end{cases},\{x,y\}\right)$	$\begin{bmatrix}-7 & -4\end{bmatrix}$
$\text{solve}\left(y=-(x+3) \text{ and } (x+3)^2+(y+1)^2=25,\{x,y\}\right)$	$x=-6 \text{ and } y=3 \text{ or } x=1 \text{ and } y=-4$
$\text{zeros}\left(\begin{cases}\lvert z+3+i\rvert-5\\ \lvert z\rvert-\lvert z+3+3\cdot i\rvert\end{cases},\{x,y\}\right)$	$\begin{bmatrix}1 & -4\\ -6 & 3\end{bmatrix}$
$\text{zeros}\left(\begin{cases}\lvert z+3+i\rvert-5\\ \text{real}(z)+\text{imag}(z)+3\end{cases},\{x,y\}\right)$	$\begin{bmatrix}1 & -4\\ -6 & 3\end{bmatrix}$

Question 3

a. when $y = 3t^3 - 14t^2 + 11t = t(t-1)(3t-11) = 0 \;\Rightarrow\; t = 0,1$

$x(0) = 0$, $x(1) = 9$, the width of the cave is 9 metres. A1

b.i. $x(t) = t^3 - 8t^2 + 16t$ $\qquad$ $y(t) = 3t^3 - 14t^2 + 11t$

$\dot{x} = \dfrac{dx}{dt} = 3t^2 - 16t + 16$ $\qquad$ $\dot{y} = \dfrac{dy}{dt} = 9t^2 - 28t + 11$ A1

$\dfrac{dy}{dx} = \dfrac{dy}{dt}\dfrac{dt}{dx} = \dfrac{\dot{y}}{\dot{x}} = \dfrac{9t^2 - 28t + 11}{3t^2 - 16t + 16}$ A1

ii. for turning points $\dfrac{dy}{dx} = 0 \;\Rightarrow\; \dfrac{dy}{dt} = 0$ solving $9t^2 - 28t + 11 = 0$ with $0 < t < 1$

gives $t = 0.4612$ and $x(0.4612) = 5.776$, $y(0.4612) = 2.38962$,

the coordinates of the highest point on the cave is $(5.78, 2.39)$ A1

Define $x1(t)=t^3-8\cdot t^2+16\cdot t$	*Done*
Define $y1(t)=3\cdot t^3-14\cdot t^2+11\cdot t$	*Done*
$\text{factor}(y1(t))$	$t\cdot(t-1)\cdot(3\cdot t-11)$
$x1(0)$	0
$x1(1)$	9
$\frac{d}{dt}(x1(t))$	$3\cdot t^2-16\cdot t+16$
$\frac{d}{dt}(y1(t))$	$9\cdot t^2-28\cdot t+11$
$\dfrac{\frac{d}{dt}(y1(t))}{\frac{d}{dt}(x1(t))}$	$\dfrac{9\cdot t^2-28\cdot t+11}{3\cdot t^2-16\cdot t+16}$
$\text{solve}\left(\frac{d}{dt}(y1(t))=0,t\right)\mid 0<t<1$	$t=0.46123$▸
$x1(0.461238$▸	5.7760▸
$y1(0.461238$▸	2.3896▸

c.i. $y(t)\times\dfrac{dx}{dt}=\left(3t^3-14t^2+11t\right)\left(3t^2-16t+16\right)=9t^5-90t^4+305t^3-400t^2+176t$ M1

$$A=\int_0^1 y\frac{dx}{dt}\,dt=\int_0^1\left(9t^5-90t^4+305t^3-400t^2+176t\right)dt$$

$b_5=9$, $b_4=-90$, $b_3=305$, $b_2=-400$, $b_1=176$ and $b_0=0$ A1

ii. $A=\int_0^1\left(9t^5-90t^4+305t^3-400t^2+176t\right)dt$

$A=\dfrac{173}{12}=14\dfrac{5}{12}$ A1

expand$\left(y1(t)\cdot\dfrac{d}{dt}\left(x1(t)\right)\right)$ $\quad 9\cdot t^5-90\cdot t^4+305\cdot t^3-400\cdot t^2+176\cdot t$

$\displaystyle\int_0^1\left(9\cdot t^5-90\cdot t^4+305\cdot t^3-400\cdot t^2+176\cdot t\right)dt$ $\quad \dfrac{173}{12}$

d.i. Solving $x(t_2)-x(t_1)=2$, $y(t_1)=h$, $y(t_2)=h$ with $0<t_1<1$, $0<t_2<1$

gives $h=2.2746$, $t_1=0.3549$, $t_2=0.5712$ A1

Now $y(0.354872)=y(0.57117)=2.274586$

Now the triangle *ABS* is an equilateral triangle with all sides 2,

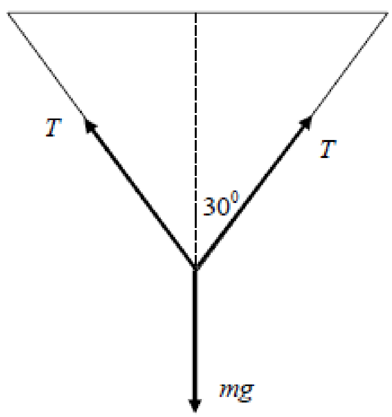
so S is $\sqrt{3}$ below *AB*.

The distance of S above the ground is $2.274586-\sqrt{3}$

0.5425 metres. A1

$\text{solve}\left(\begin{cases} x1(t2)-x1(t1)=2. \\ y1(t1)=h \\ y1(t2)=h \end{cases},\{t1,t2,h\}\right)\mid 0<t1<1 \text{ and } 0<t2<1$	$h=2.27459$ and $t1=0.354872$ and $t2=0.57117$
$x1(0.354872)$	4.71517
$x1(0.57117)$	6.71517
$x1(0.57117)-x1(0.354872)$	2.
$y1(0.354872)$	2.27459
$y1(0.57117)$	2.27458
$2.2745855871646-\sqrt{3}$	0.542535
$\dfrac{2\cdot 400\cdot \cos(30°)}{9.8}$	70.696

ii.



Resolving vertically,

$2T\cos\left(30^0\right)-mg=0$ A1

$$m=\frac{2T\cos\left(30^0\right)}{g}=\frac{2\times 400\times\frac{\sqrt{3}}{2}}{9.8}=70.696$$

so the largest mass is 69.7 kg. A1

Question 4

a.i. $\sqrt{x}+\sqrt{y}=\sqrt{a}$ using implicit differentiation

$$\frac{d}{dx}\left(\sqrt{x}\right)+\frac{d}{dx}\left(\sqrt{y}\right)=\frac{d}{dx}\left(\sqrt{a}\right)$$

$$\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}} \quad \text{at } P(c,d) \quad m_T=-\frac{\sqrt{d}}{\sqrt{c}}$$ A1

$$T: \quad y-d=-\frac{\sqrt{d}}{\sqrt{c}}(x-c)$$ A1

$$\sqrt{c}(y-d)=-\sqrt{d}(x-c)$$

$$y\sqrt{c}+x\sqrt{d}=c\sqrt{d}+d\sqrt{c}$$

ii. at A, $y=0 \Rightarrow x=\dfrac{c\sqrt{d}+d\sqrt{c}}{\sqrt{d}}=c+\sqrt{dc}$ since $c>0$ and $d>0$

at B, $x=0 \Rightarrow y=\dfrac{c\sqrt{d}+d\sqrt{c}}{\sqrt{c}}=d+\sqrt{dc}$ since $c>0$ and $d>0$

$$A\left(c+\sqrt{dc},0\right) \quad B\left(0,\ d+\sqrt{dc}\right)$$ A1

b.i. $x=a\cos^4(t)$ and $y=a\sin^4(t)$ where $0\le t\le\dfrac{\pi}{2}$

$$\sqrt{x}=\sqrt{a}\cos^2(t) \text{ and } \sqrt{y}=\sqrt{a}\sin^2(t)$$

$$\sin^2(t)+\cos^2(t)=1$$

$$\frac{\sqrt{x}}{\sqrt{a}}+\frac{\sqrt{y}}{\sqrt{a}}=1$$

$$\sqrt{x}+\sqrt{y}=\sqrt{a}$$ A1

ii. $\dot{x}=\dfrac{dx}{dt}=-4a\cos^3(t)\sin(t)$ and $\dot{y}=\dfrac{dy}{dt}=4a\sin^3(t)\cos(t)$

$\dfrac{dy}{dx}=\dfrac{\dot{y}}{\dot{x}}=\dfrac{4a\sin^3(t)\cos(t)}{-4a\cos^3\sin(t)}=-\tan^2(t)$

the line $y=-3x$ has a gradient of -3,

$-\tan^2(t)=-3$

$\tan(t)=\sqrt{3}\ ,\quad 0<t<\dfrac{\pi}{2}$ M1

$t=\tan^{-1}\left(\sqrt{3}\right)=\dfrac{\pi}{3}$

$x\left(\dfrac{\pi}{3}\right)=a\cos^4\left(\dfrac{\pi}{3}\right)=a\left(\dfrac{1}{2}\right)^4=\dfrac{a}{16}$

$y\left(\dfrac{\pi}{3}\right)=a\sin^4\left(\dfrac{\pi}{3}\right)=a\left(\dfrac{\sqrt{3}}{2}\right)^4=\dfrac{9a}{16}$

the point is $\left(\dfrac{a}{16},\dfrac{9a}{16}\right)$ A1

the tangent $y\sqrt{c}+x\sqrt{d}=c\sqrt{d}+d\sqrt{c}$ where $c=\dfrac{a}{16}$ and $d=\dfrac{9a}{16}$

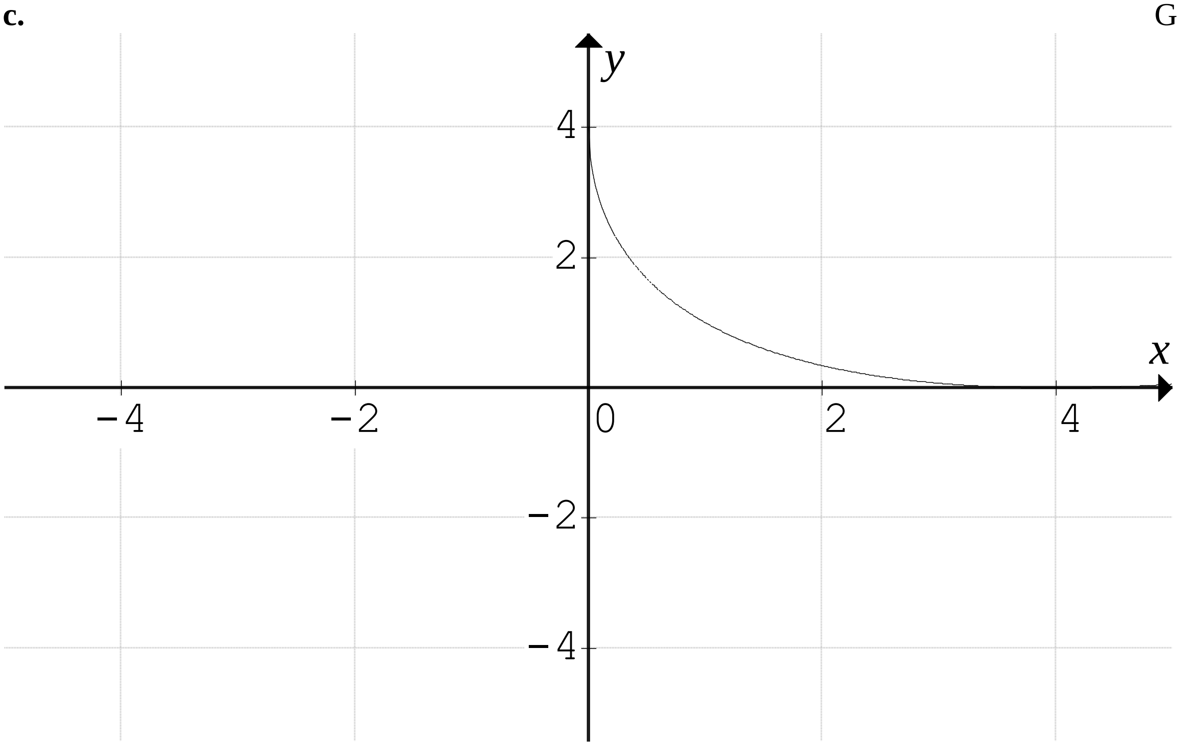
$t(x)=y\dfrac{\sqrt{a}}{4}+x\dfrac{3\sqrt{a}}{4}=\dfrac{a}{16}\times\dfrac{3\sqrt{a}}{4}+\dfrac{9a}{16}\times\dfrac{\sqrt{a}}{4}$

$y=t(x)=-3x+\dfrac{3a}{4}$ A1

Define $x1(t)=a\cdot(\cos(t))^4$	*Done*
Define $y1(t)=a\cdot(\sin(t))^4$	*Done*
$\dfrac{\frac{d}{dt}(y1(t))}{\frac{d}{dt}(x1(t))}$	$-(\tan(t))^2$

$\text{solve}\left(-(\tan(t))^2=-3,t\right)\mid 0<t<\frac{\pi}{2}$	$t=\frac{\pi}{3}$
$x1\left(\frac{\pi}{3}\right)$	$\frac{a}{16}$
$y1\left(\frac{\pi}{3}\right)$	$\frac{9\cdot a}{16}$
Define $t(x)=-3\cdot x+\frac{3\cdot a}{4}$	*Done*
Define $y(x)=\left(\sqrt{a}-\sqrt{x}\right)^2$	*Done*

c. G1



d.i. $\sqrt{y}=\sqrt{a}-\sqrt{x}$, $y(x)=\left(\sqrt{a}-\sqrt{x}\right)^2$, $t(x)=-3x+\frac{3a}{4}$

This tangent crosses the x-axis at $c+\sqrt{dc},\ x=\frac{a}{4}$ and the y-axis $d+\sqrt{dc},\ y=\frac{3a}{4}$

$$A=\int_0^{\frac{a}{4}}\left(y(x)-t(x)\right)dx+\int_{\frac{a}{4}}^{a}y(x)\,dx=\int_0^{\frac{a}{4}}\left(\left(\sqrt{a}-\sqrt{x}\right)^2-\left(-3x+\frac{3a}{4}\right)\right)dx+\int_{\frac{a}{4}}^{a}\left(\sqrt{a}-\sqrt{x}\right)^2dx$$

$$A=\int_0^{\frac{a}{4}}\left(\left(\sqrt{a}-\sqrt{x}\right)^2+3x-\frac{3a}{4}\right)dx+\int_{\frac{a}{4}}^{a}\left(\sqrt{a}-\sqrt{x}\right)^2dx$$ A1

ii. $A=\frac{7a^2}{96}$ A1

alternatively, the area is the area bounded by C the coordinate axes, minus the area of the triangle formed by the tangent and the coordinate axes.

$$A=\int_0^{a}\left(\sqrt{a}-\sqrt{x}\right)^2dx-\frac{1}{2}\times\frac{3a}{4}\times\frac{a}{4}=\frac{a^2}{6}-\frac{3a^2}{32}=\frac{7a^2}{96}$$

e. $s=\int_0^{\frac{\pi}{2}}\sqrt{\dot{x}^2+\dot{y}^2}\,dt$

$$s=\int_0^{\frac{\pi}{2}}\sqrt{\left(-4a\cos^3(t)\sin(t)\right)^2+\left(4a\sin^3(t)\cos(t)\right)^2}\,dt$$

M1

$$s=\int_0^{\frac{\pi}{2}}\sqrt{16a^2\sin^2(t)\cos^2(t)\left(\cos^4(t)+\sin^4(t)\right)}\,dt$$

$$s=2a\int_0^{\frac{\pi}{2}}\sin(2t)\sqrt{\cos^4(t)+\sin^4(t)}\,dt=1.623a \qquad L=1.623$$ A1

$$\int_0^{\frac{a}{4}}\left(y(x)-t(x)\right)dx+\int_{\frac{a}{4}}^{a}y(x)\,dx \qquad \frac{7\cdot a^2}{96}$$

$$\int_0^{a}y(x)\,dx \qquad \frac{a^2}{6}$$

$$\int_0^{\frac{\pi}{2}}\sqrt{\left(\frac{d}{dt}(x1(t))\right)^2+\left(\frac{d}{dt}(y1(t))\right)^2}\,dt|a>0 \qquad 1.62323\cdot a$$

Question 5

a. $\ddot{x} = v\dfrac{dv}{dx} = -g - kv^2$

$\dfrac{dv}{dx} = \dfrac{-\left(g + kv^2\right)}{v}$ M1

$\dfrac{dx}{dv} = \dfrac{-v}{g + kv^2}$

$x = \displaystyle\int \frac{-v}{g + kv^2}\,dv$

$x = -\dfrac{1}{2k}\log_e\left(g + kv^2\right) + c$

when $x = 0$ $v = \dfrac{1}{3}\sqrt{\dfrac{g}{k}}$ $\Rightarrow kv^2 = \dfrac{g}{9}$ $\Rightarrow$ $c = \dfrac{1}{2k}\log_e\left(\dfrac{10g}{9}\right)$ A1

$x = -\dfrac{1}{2k}\log_e\left(g + kv^2\right) + \dfrac{1}{2k}\log_e\left(\dfrac{10g}{9}\right) = \dfrac{1}{2k}\log_e\left(\dfrac{10g}{9\left(g + kv^2\right)}\right)$

at the top $v = 0$ $x = H = \dfrac{1}{2k}\log_e\left(\dfrac{10}{9}\right)$ A1

b.i. $\ddot{x} = \dfrac{dv}{dt} = -\left(g + kv^2\right)$

$\dfrac{dt}{dv} = \dfrac{-1}{\left(g + kv^2\right)}$

$T = \displaystyle\int_{\frac{1}{3}\sqrt{\frac{g}{k}}}^{0} \frac{-1}{\left(g + kv^2\right)}\,dv = \int_{0}^{\frac{1}{3}\sqrt{\frac{g}{k}}} \frac{1}{\left(g + kv^2\right)}\,dv$ A1

ii. $k = 0.02 = \dfrac{1}{50}$, $g = 9.8$ $\sqrt{\dfrac{g}{k}} = \sqrt{490} = 7\sqrt{10}$

$T = \displaystyle\int_{0}^{\frac{7\sqrt{10}}{3}} \frac{1}{9.8 + \frac{v^2}{50}}\,dv = \int_{0}^{\frac{7\sqrt{10}}{3}} \frac{50}{490 + v^2}\,dv$ A1

$T = \dfrac{50}{7\sqrt{10}}\left[\tan^{-1}\left(\dfrac{v}{7\sqrt{10}}\right)\right]_0^{\frac{7\sqrt{10}}{3}} = \dfrac{50}{7\sqrt{10}}\left[\tan^{-1}\left(\dfrac{7\sqrt{10}}{3\times 7\sqrt{10}}\right) - \tan^{-1}(0)\right]$ A1

$T = \dfrac{5\sqrt{10}}{7}\tan^{-1}\left(\dfrac{1}{3}\right)$ A1

c.i. $H = \frac{1}{2k}\log_e\left(\frac{10}{9}\right) = 25\log_e\left(\frac{10}{9}\right) = 2.63401$

$v^2 = u^2 + 2as$

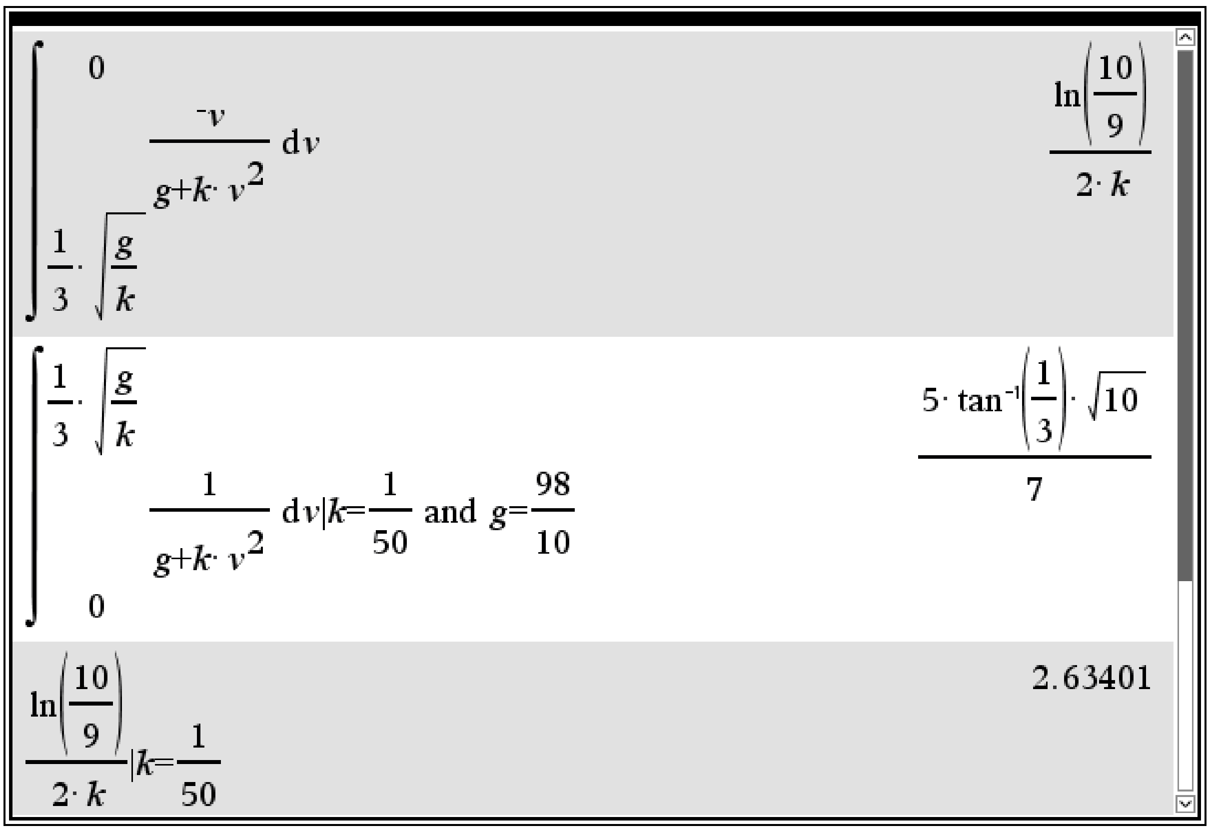
$v = 0 + \sqrt{2 \times 9.8 \times 2.63401}$

$v = 7.19 \text{ m/s}$ A1

ii. $s = ut + \frac{1}{2}at^2 \quad s = -2.63401 \ , \ a = -9.8 \ , \ u = 0$

$t = \sqrt{\frac{2 \times 2.63401}{9.8}}$

$t = 0.73 \text{ sec}$ A1



Question 6

a. Let T be toast $T \sim (50, 3^2)$, G eggs $G \sim (60, 5^2)$, B bacon $B \sim (65, 8^2)$

A be the total amount

$A = T1 + T2 + T3 + G1 + G2 + B1 + B2 + B3 + B4$

$E(A) = 3E(T) + 2E(G) + 4E(B)$

$= 3 \times 50 + 2 \times 60 + 4 \times 65$

$= 530$

$\text{Var}(A) = 3\text{Var}(T) + 2\text{Var}(G) + 4\text{Var}(B)$

$= 3 \times 9 + 2 \times 25 + 4 \times 64$ A1

$= 333$

$A \sim (530, 333)$, $\Pr(A > 500) = 0.9499$ A1

b. $\bar{T} \sim N\left(51, \dfrac{3}{\sqrt{n}}\right)$

$\Pr(\bar{T} > 50) = 0.83 \quad \Rightarrow \quad \dfrac{50 - 51}{\frac{3}{\sqrt{n}}} = -0.9542$ A1

$n = (3 \times 0.9542)^2$

$n = 8$ A1

c. $\bar{x} = 999$, $\sigma = 5$, $z_{0.9} = 1.64485$, $\bar{x} \pm \dfrac{z\sigma}{\sqrt{n}} = (997.16, 1000.84)$ A1

Now $1000.84 - 997.16 = 3.68$, so $2 \times \dfrac{z\sigma}{\sqrt{n}} = 3.68$ A1

$n = \left(\dfrac{2 \times 1.64485 \times 5}{3.68}\right)^2$

$n = 20$ A1

$\text{normCdf}(500, \infty, 530, \sqrt{333})$	0.949911
$\text{invNorm}(0.17, 0, 1)$	$^{-}0.954165$
$(3 \cdot 0.954165)^2$	8.19388
$\text{invNorm}(0.95, 0, 1)$	1.64485
$1000.84 - 997.16$	3.68
$\left(\dfrac{2 \cdot 1.64485 \cdot 5}{3.68}\right)^2$	19.9782

END OF SECTION B SUGGESTED ANSWERS