Year 2018

VCE

Specialist Mathematics

Trial Examination 1

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Victorian Certificate of Education
2018

STUDENT NUMBER

										Letter
Figures										
Words										

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 20 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (4 marks)

Given that $z = ai$, where $a \in R$, is a solution of the equation $z^4 + 6z^3 + 41z^2 + 96z + 400 = 0$, find the value of a and determine all the solutions of $z^4 + 6z^3 + 41z^2 + 96z + 400 = 0$, $z \in C$.

Question 2 (4 marks) Let $f(x)=\sqrt{\arcsin\left(\frac{3x}{4}\right)}$

a. State the maximal domain and the range of the function *f*.

2 marks

b. Find $f'(x)$ and hence evaluate $\displaystyle\int_0^{\frac{4}{3}} \frac{1}{\sqrt{\left(16-9x^2\right)\sin^{-1}\left(\frac{3x}{4}\right)}}\,dx$,

giving your answer in the form $\dfrac{\sqrt{b\pi}}{c}$ where b and c are positive integers.

2 marks

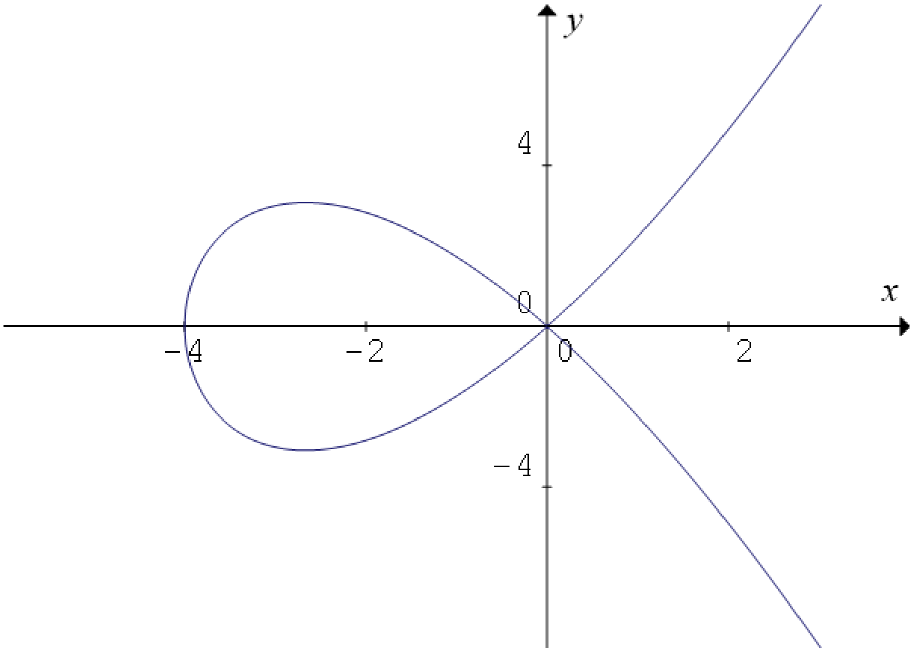
Question 3 (4 marks)

Solve the differential equation $\frac{dy}{dx}+\frac{4y^2}{16-9x^2}=0$, given that $y(0)=1$.

Express *y* as a function of *x*.

Question 4 (6 marks)

The diagram shows the graph of the relation $y^2 = x^3 + 4x^2$.



a. Write down a definite integral which gives the total area of the loop in the second and third quadrants.

1 mark

b. Find the total area of the loop in the second and third quadrants.

3 marks

c. A particle is moving along the curve $y^2 = x^3 + 4x^2$. When it is at the point where $x = 2$, the downward component of its speed is $7\ \text{ms}^{-1}$. Find the horizontal component of its speed at this point.

2 marks

Question 5 (4 marks)

The weights of medium sized dogs are normally distributed with a mean of 10 kg and a standard deviation of 2.5 kg. The weights of cats are normally distributed with a mean of 5 kg and a standard deviation of 1.5 kg. Assume that the weights of medium sized dogs and cats are independent random variables.

a. Find the mean and variance of a medium sized dog and two cats.

1 mark

b. A dog breeder is concerned that his medium sized dogs are over-weight. A sample of 25 dogs is found to have a mean weight of 11 kg. Assume that the population standard deviation for medium sized dogs is still 2.5 kg

i. State the appropriate null and alternative hypotheses for the weights of medium sized dogs in this situation.

1 mark

ii. The *p* value for this test is given by $\Pr(Z \geq a)$, where *Z* has the standard normal distribution. Find the value of a and hence determine whether the null hypothesis should be rejected at the 0.05 level of significance.

2 marks

Question 6 (3 marks)

Find the value of the gradient of the curve $x^2y^2+\frac{4}{\pi}\arctan(2x)=2$ at the point in the fourth quadrant where $x=\frac{1}{2}$.

Question 7 (4 marks)

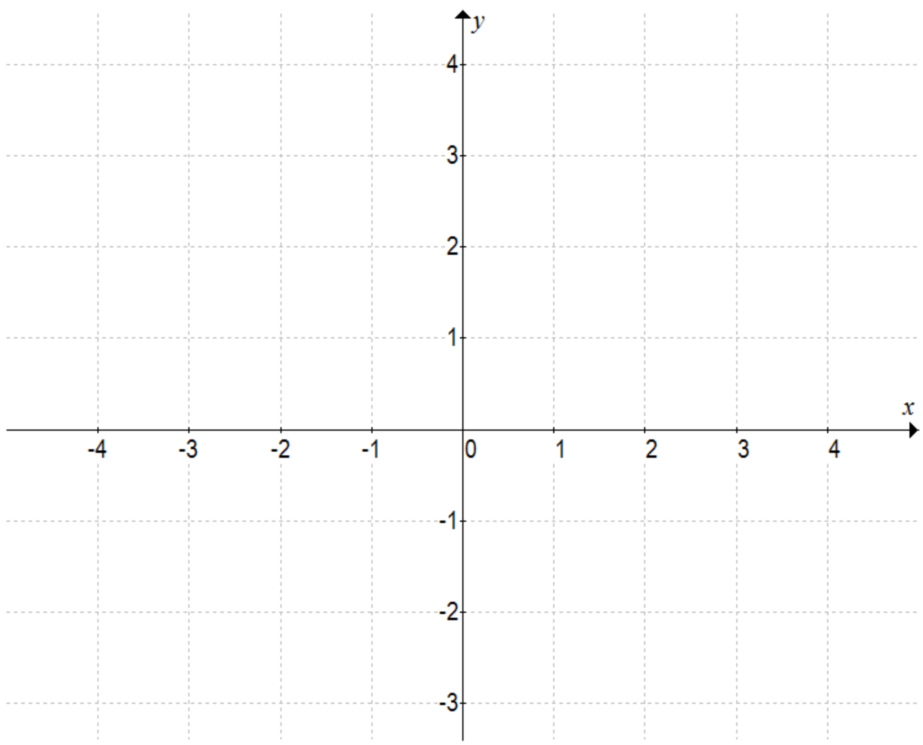
A particle moves so that its velocity vector is $\underset{\sim}{v}(t) = -2e^{-t}\underset{\sim}{i} + 2e^{2t}\underset{\sim}{j}$ ms^{-1} at time t seconds, where $t \geq 0$.

a. Given that $\underset{\sim}{r}(0) = 2\underset{\sim}{i}$, find the position vector $\underset{\sim}{r}(t)$ and hence show that the particle moves on the part of the curve $y = \dfrac{4}{x^2} - 1$, stating the domain and range.

3 marks

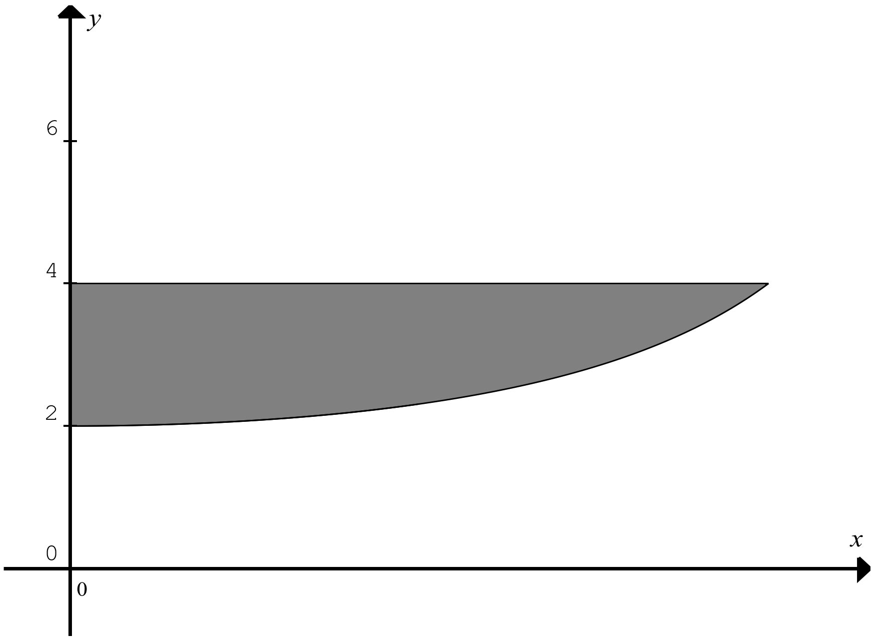
b. On the diagram below, sketch the graph of the path of the particle, indicating the direction of motion.

1 mark



Question 8 (3 marks)

The diagram shows part of the graph of $y = 2\sec\left(\frac{x}{3}\right)$.



The shaded region R is the area bounded by the graph of $y = 2\sec\left(\frac{x}{3}\right)$, the y-axis and the line $y = 4$. Find the volume generated when the region R is rotated about the x-axis.

Question 9 (4 marks)

A curve is defined parametrically by $x = 5t^4 + 1$, $y = 2t^5 + 3$, for $t \geq 0$.

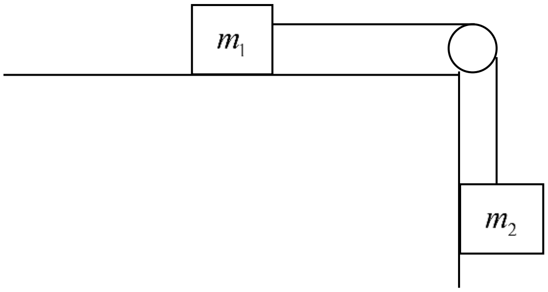
a. Show that the arc length between the points where $t = 0$ and $t = \sqrt{5}$ on the curve is given by $s = \int_0^{\sqrt{5}} 10t^3 \sqrt{t^2 + 4}\, dt$

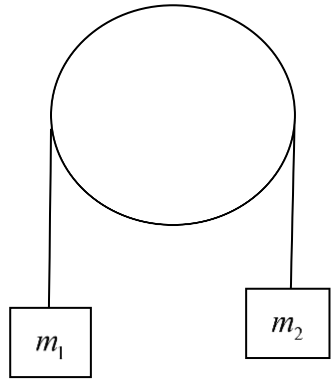
1 mark

b. Evaluate the definite integral in part a**,** to find the arc length *s*.

3 marks

Question 10 (4 marks)





A particle of mass m_1 kg is on a smooth table and is connected by a light string which passes around a smooth pulley to another mass of m_2 kg hanging vertically. The mass m_2 moves downwards with an acceleration of $3a\ \text{ms}^{-2}$. In another situation both masses are hanging vertically and connected by a light string which passes around another smooth pulley. In this situation the mass m_2 moves downwards with an acceleration of $a\ \text{ms}^{-2}$. Determine the ratio $\dfrac{m_2}{m_1}$.

END OF EXAMINATION

EXTRA WORKING SPACE

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a pyramid	$\frac{1}{3}Ah$
area of triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$
cosine rule	$c^2=a^2+b^2-2ab\cos(C)$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$	
$1+\tan^2(x)=\sec^2(x)$	$\cot^2(x)+1=\operatorname{cosec}^2(x)$
$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$	$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$
$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$	$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$
$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$	
$\sin(2x)=2\sin(x)\cos(x)$	$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

Circular (trigonometric) functions - continued

Function	$\sin^{-1}$ (arcsin)	$\cos^{-1}$ (arcos)	$\tan^{-1}$ (arctan)
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$
$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$E(aX + b) = aE(X) + b$ $E(aX + bY) = aE(X) + bE(Y)$ $\text{Var}(aX + b) = a^2\,\text{Var}(X)$
for independent random variables X and Y	$\text{Var}(aX + bY) = a^2\,\text{Var}(X) + b^2\,\text{Var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $E(\bar{X}) = \mu$ variance $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert\underset{\sim}{r}\rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r_1} \cdot \underset{\sim}{r_2} = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e\left(x\right)\right)=\frac{1}{x}$	$\int\frac{1}{x}dx=\log_e\lvert x\rvert+c$
$\frac{d}{dx}\left(\sin\left(ax\right)\right)=a\cos\left(ax\right)$	$\int\sin\left(ax\right)dx=-\frac{1}{a}\cos\left(ax\right)+c$
$\frac{d}{dx}\left(\cos\left(ax\right)\right)=-a\sin\left(ax\right)$	$\int\cos\left(ax\right)dx=\frac{1}{a}\sin\left(ax\right)+c$
$\frac{d}{dx}\left(\tan\left(ax\right)\right)=a\sec^2\left(ax\right)$	$\int\sec^2\left(ax\right)dx=\frac{1}{a}\tan\left(ax\right)+c$
$\frac{d}{dx}\left(\sin^{-1}\left(x\right)\right)=\frac{1}{\sqrt{1-x^2}}$	$\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\cos^{-1}\left(x\right)\right)=\frac{-1}{\sqrt{1-x^2}}$	$\int\frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$
$\frac{d}{dx}\left(\tan^{-1}\left(x\right)\right)=\frac{1}{1+x^2}$	$\int\frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$
	$\int\left(ax+b\right)^n dx=\frac{1}{a\left(n+1\right)}\left(ax+b\right)^{n+1}+c,\ n\neq -1$
	$\int\left(ax+b\right)^{-1}dx=\frac{1}{a}\log_e\lvert ax+b\rvert+c$
product rule	$\frac{d}{dx}\left(uv\right)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b,$ then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf\left(x_n\right)$
acceleration	$a=\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2}\sqrt{1+\left(f'\left(x\right)\right)^2}\,dx$ or $\int_{t_1}^{t_2}\sqrt{\left(x'\left(t\right)\right)^2+\left(y'\left(t\right)\right)^2}\,dt$

END OF FORMULA SHEET