

**Year 2018**

**VCE**

**Specialist Mathematics**

**Trial Examination 1**

**Solutions**



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**Question 1****Method 1**

$(z - ai)(z + ai) = z^2 + a^2$  is a factor

$$(z^2 + a^2)(z^2 + bz + c)$$

$$= z^4 + bz^3 + (c + a^2)z^2 + ba^2z + ca^2 = z^4 + 6z^3 + 41z^2 + 96z + 400 \quad \text{M1}$$

$$\Rightarrow b = 6 \quad 41 = c + a^2, \quad ba^2 = 96, \quad a^2 = 16 \Rightarrow a = \pm 4, \quad c = 25$$

$$(z^2 + 16)(z^2 + 6z + 25) = 0 \quad \text{A1}$$

$$(z^2 + 16)((z + 3)^2 - 16i^2) = 0 \quad \text{M1}$$

$$(z + 4i)(z - 4i)(z + 3 + 4i)(z + 3 - 4i) = 0$$

$$z = \pm 4i, -3 \pm 4i \quad \text{A1}$$

**Method II**

Let  $P(z) = z^4 + 6z^3 + 41z^2 + 96z + 400 = 0$

$$P(ai) = (ai)^4 + 6(ai)^3 + 41(ai)^2 + 96ai + 400 = 0$$

$$= a^4 - 6a^3i - 41a^2 + 96ai + 400 = 0 \quad \text{M1}$$

$$= a^4 - 41a^2 + 400 + (96a - 6a^3)i = 0$$

the real part must be zero

$$a^4 - 41a^2 + 400 = 0$$

$$(a^2 - 25)(a^2 - 16) = 0$$

$$a = \pm 5, \pm 4$$

and the imaginary part must also be zero

$$96a - 6a^3 = 0$$

$$6a(16 - a^2) = 0$$

$$a = 0, \pm 4$$

the only common solution which satisfy both are  $a = \pm 4$  A1

so  $(z - 4i)(z + 4i) = z^2 - 16i^2 = z^2 + 16 = 0$  is a factor

$$z^4 + 6z^3 + 41z^2 + 96z + 400 = 0$$

$$(z^2 + 16)(z^2 + 6z + 25) = 0$$

$$(z^2 + 16)(z^2 + 6z + 9 + 16) = 0 \quad \text{M1}$$

$$(z^2 + 16)((z + 3)^2 - 16i^2) = 0$$

$$(z + 4i)(z - 4i)(z + 3 + 4i)(z + 3 - 4i) = 0$$

$$z = \pm 4i, -3 \pm 4i \quad \text{A1}$$

**Question 2**

a.  $f(x) = \sqrt{\arcsin\left(\frac{3x}{4}\right)}$  domain  $0 \leq \frac{3x}{4} \leq 1$

$$0 \leq x \leq \frac{4}{3} \text{ or } \text{dom } f = \left[0, \frac{4}{3}\right] \quad \text{A1}$$

$$f(0) = \sqrt{\arcsin(0)} = 0, \quad f\left(\frac{4}{3}\right) = \sqrt{\arcsin(1)} = \sqrt{\frac{\pi}{2}} = \frac{\sqrt{2\pi}}{2}$$

since it is a one-one function, the range is

$$0 \leq y \leq \sqrt{\frac{\pi}{2}} \text{ or } \text{range } f = \left[0, \frac{\sqrt{2\pi}}{2}\right] \quad \text{A1}$$

b.  $f(x) = \sqrt{\arcsin\left(\frac{3x}{4}\right)} = \left(\arcsin\left(\frac{3x}{4}\right)\right)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2} \left(\arcsin\left(\frac{3x}{4}\right)\right)^{-\frac{1}{2}} \frac{d}{dx} \left(\arcsin\left(\frac{3x}{4}\right)\right) = \frac{1}{2} \left(\arcsin\left(\frac{3x}{4}\right)\right)^{-\frac{1}{2}} \frac{3}{\sqrt{16-9x^2}}$$

$$f'(x) = \frac{3}{2\sqrt{(16-9x^2)\sin^{-1}\left(\frac{3x}{4}\right)}} \quad \text{A1}$$

$$\text{hence } \int_0^{\frac{4}{3}} \frac{1}{\sqrt{(16-9x^2)\sin^{-1}\left(\frac{3x}{4}\right)}} dx = \frac{2}{3} \left[ \sqrt{\arcsin\left(\frac{3x}{4}\right)} \right]_0^{\frac{4}{3}}$$

$$= \frac{2}{3} \left[ f\left(\frac{4}{3}\right) - f(0) \right] = \frac{2}{3} \times \frac{\sqrt{2\pi}}{2}$$

$$= \frac{\sqrt{2\pi}}{3} \text{ so } b=2, \quad c=3 \quad \text{A1}$$

**Question 3**

$$\frac{dy}{dx} + \frac{4y^2}{16-9x^2} = 0, \text{ given that } y(0) = 1$$

$$\frac{dy}{dx} = \frac{-4y^2}{16-9x^2}, \text{ separating the variables}$$

$$-\int \frac{1}{y^2} dy = \int \frac{4}{16-9x^2} dx \quad \text{A1}$$

by partial fractions

$$\begin{aligned} \frac{4}{16-9x^2} &= \frac{A}{4-3x} + \frac{B}{4+3x} \\ &= \frac{A(4+3x) + B(4-3x)}{(4-3x)(4+3x)} = \frac{4(A+B) + 3x(A-B)}{16-9x^2} \end{aligned} \quad \text{M1}$$

$$(1) A+B=1 \quad (2) A-B=0 \quad \Rightarrow A=B=\frac{1}{2}$$

$$\frac{1}{y} = \int \frac{4}{16-9x^2} dx = \frac{1}{2} \int \left( \frac{1}{4+3x} + \frac{1}{4-3x} \right) dx$$

$$\frac{1}{y} = \frac{1}{2} \left[ \frac{1}{3} \log_e(|4+3x|) - \frac{1}{3} \log_e(|4-3x|) \right] + c$$

$$\frac{1}{y} = \frac{1}{6} \log_e \left( \frac{|4+3x|}{|4-3x|} \right) + c$$

$$\text{To find } c \text{ use } x=0, y=1 \Rightarrow 1 = \frac{1}{6} \log_e(1) + c \Rightarrow c=1 \quad \text{A1}$$

$$\frac{1}{y} = \frac{1}{6} \log_e \left( \frac{|4+3x|}{|4-3x|} \right) + 1 = \frac{\log_e \left( \frac{|4+3x|}{|4-3x|} \right) + 6}{6}$$

$$y = \frac{6}{\log_e \left( \frac{|4+3x|}{|4-3x|} \right) + 6} \quad \text{A1}$$

**Question 4**

a.  $y^2 = x^3 + 4x^2 = x^2(x+4) \Rightarrow y = -x\sqrt{x+4}$  and  $y = x\sqrt{x+4}$ ,

these two functions represent the upper and lower parts of the curve in the second and third quadrants respectively, by symmetry

$$A = 2 \int_{-4}^0 -x\sqrt{x+4} dx = 2 \int_0^{-4} x\sqrt{x+4} dx \quad \text{A1}$$

b. let  $u = x+4$ ,  $x = u-4$ ,  $\frac{du}{dx} = 1$

terminals when  $x = -4$ ,  $u = 0$  when  $x = 0$ ,  $u = 4$

$$A = 2 \int_0^{-4} x\sqrt{x+4} dx = 2 \int_4^0 (u-4)\sqrt{u} du$$

$$A = 2 \int_4^0 \left( u^{\frac{3}{2}} - 4u^{\frac{1}{2}} \right) du \quad \text{M1}$$

$$A = 2 \left[ \frac{2}{5} u^{\frac{5}{2}} - \frac{8}{3} u^{\frac{3}{2}} \right]_4^0 = 4 \left[ u^{\frac{3}{2}} \left( \frac{u}{5} - \frac{4}{3} \right) \right]_4^0 \quad \text{A1}$$

$$A = 4 \left[ u^{\frac{3}{2}} \left( \frac{3u-20}{15} \right) \right]_4^0 = 4 \left[ \left( 0 - 4^{\frac{3}{2}} \left( \frac{12-20}{15} \right) \right) \right]$$

$$A = \frac{256}{15} \text{ units}^2 \quad \text{A1}$$

c.  $y^2 = x^3 + 4x^2$  using implicit differentiation

$$2y \frac{dy}{dx} = 3x^2 + 8x, \quad \frac{dy}{dx} = \frac{3x^2 + 8x}{2y} \quad \text{A1}$$

now when  $x = 2$   $y = x\sqrt{x+4} = 2\sqrt{6}$  so  $\frac{dy}{dx} = \frac{12+16}{4\sqrt{6}} = \frac{7}{\sqrt{6}}$

also given  $\frac{dy}{dt} = -7$ , then  $\frac{dx}{dt} = \frac{dx}{dy} \cdot \frac{dy}{dt} = \frac{\sqrt{6}}{7} \times -7 = -\sqrt{6}$

the particle is moving inwards at  $\sqrt{6} \text{ ms}^{-1}$  A1

alternatively  $y = x\sqrt{x+4}$  using the product rule

$$\frac{dy}{dx} = \sqrt{x+4} + \frac{x}{2\sqrt{x+4}} = \frac{2x+8+x}{2\sqrt{x+4}} = \frac{3x+8}{2\sqrt{x+4}}$$

when  $x = 2$   $\frac{dy}{dx} = \frac{6+8}{2\sqrt{6}} = \frac{7}{\sqrt{6}}$

**Question 5**

a. Let  $D$  be the weights of medium sized dogs,  $D \stackrel{d}{=} N(10, 2.5^2)$

and let  $C$  be the weights of cats,  $C \stackrel{d}{=} N(5, 1.5^2)$ .

Let  $T$  be the total weight of one dog and two actually different and independent cats

$$T = D + C1 + C2$$

$$E(T) = E(D) + E(C1) + E(C2)$$

$$= 10 + 5 + 5$$

$$= 20$$

$$\text{var}(T) = \text{var}(D) + \text{var}(C1) + \text{var}(C2)$$

$$= \left(\frac{5}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2$$

$$= \frac{25}{4} + \frac{9}{2} = \frac{43}{4} = 10.75$$

$$E(T) = 20, \quad \text{var}(T) = 10.75$$

A1

b.i.  $H_0: \mu = 10$

$H_A: \mu > 10$  what we are trying to prove

A1

ii.  $\bar{x} = 11, \mu = 10, \sigma = 2.5, n = 25$

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{11 - 10}{\frac{2.5}{\sqrt{25}}} = \frac{1}{\frac{2.5}{5}} = 2$$

$$p = \Pr(Z \geq a) \text{ so } a = 2$$

A1

$$\Pr(Z \leq 1.65) = 0.95$$

A1

Yes reject the null hypothesis, accept the alternative hypothesis,  
his dogs appear over-weight.

**Question 6**

$$x^2 y^2 + \frac{4}{\pi} \arctan(2x) = 2 \text{ substitute } x = \frac{1}{2} \Rightarrow \frac{y^2}{4} + \frac{4}{\pi} \arctan(1) = 2, \frac{y^2}{4} + \frac{4}{\pi} \times \frac{\pi}{4} = 2$$

$$y^2 = 4 \text{ so } y = \pm 2 \text{ but in the fourth quadrant } y < 0, \text{ so } y = -2$$

A1

using implicit differentiation,  $\frac{d}{dx}(x^2 y^2) + \frac{4}{\pi} \frac{d}{dx}(\arctan(2x)) = 0$  product rule in the first term

$$2xy^2 + 2x^2 y \frac{dy}{dx} + \frac{4}{\pi} \times \frac{2}{1+4x^2} = 0 \text{ substitute } x = \frac{1}{2} \text{ and } y = -2$$

M1

$$1 \times 4 + 2 \times \frac{1}{4} \times -2 \frac{dy}{dx} + \frac{4}{\pi} \times \frac{2}{1+4 \times \frac{1}{4}} = 0 \Rightarrow 4 - \frac{dy}{dx} + \frac{4}{\pi} = 0$$

$$\frac{dy}{dx} = 4 + \frac{4}{\pi} = \frac{4(\pi + 1)}{\pi}$$

A1

**Question 7**

a.  $\underline{v}(t) = -2e^{-t} \underline{i} + 2e^{2t} \underline{j} \text{ ms}^{-1}$  integrating

$$\underline{r}(t) = \int -2e^{-t} dt \underline{i} + \int 2e^{2t} dt \underline{j}$$

$$\underline{r}(t) = 2e^{-t} \underline{i} + e^{2t} \underline{j} + \underline{c}$$

A1

Now  $\underline{r}(0) = 2\underline{i}$   $2\underline{i} = 2\underline{i} + \underline{j} + \underline{c} \Rightarrow \underline{c} = -\underline{j}$

$$\underline{r}(t) = 2e^{-t} \underline{i} + (e^{2t} - 1) \underline{j}$$

M1

The parametric equations are  $x = 2e^{-t}$ ,  $y = e^{2t} - 1$

$$e^t = \frac{2}{x} \Rightarrow y = \frac{4}{x^2} - 1$$

But  $t \geq 0$ , so  $0 < x \leq 2$  and  $y \geq 0$ , so the particle moves on

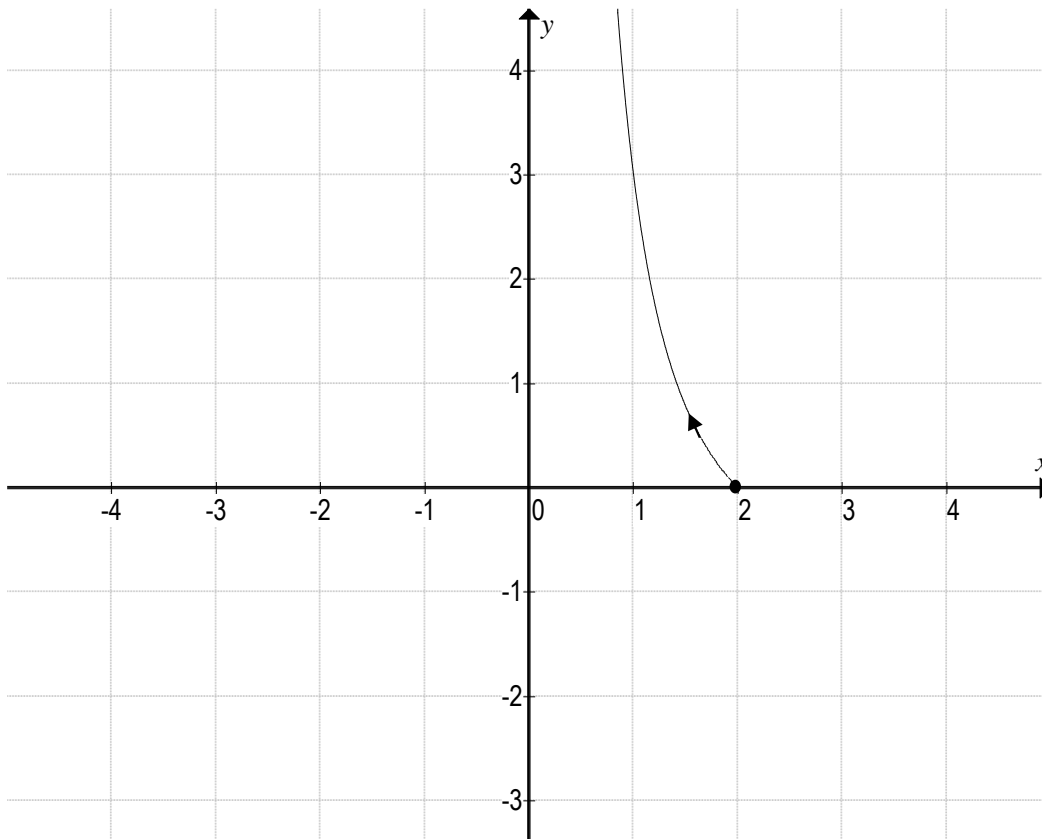
part of the curve  $y = \frac{4}{x^2} - 1$

A1

b. When  $t = 0$ ,  $x = 2$ ,  $y = 0$ ,

graph on restricted domain, with direction of motion

G1





**Question 8**

$$\text{when } 2\sec\left(\frac{x}{3}\right) = 4 \Rightarrow \cos\left(\frac{x}{3}\right) = \frac{1}{2} \Rightarrow \frac{x}{3} = \frac{\pi}{3} \text{ then } x = \pi \quad \text{A1}$$

$$V = \pi \int_0^{\pi} \left(16 - 4\sec^2\left(\frac{x}{3}\right)\right) dx$$

$$V = \pi \left[16x - 12 \tan\left(\frac{x}{3}\right)\right]_0^{\pi} \quad \text{A1}$$

$$V = \pi \left[\left(16\pi - 12 \tan\left(\frac{\pi}{3}\right)\right) - 0\right]$$

$$V = \pi(16\pi - 12\sqrt{3}) \quad \text{A1}$$

**Question 9**

$$\text{a. } s = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} dt$$

$$x = 5t^4 + 1, \quad y = 2t^5 + 3$$

$$\dot{x} = \frac{dx}{dt} = 20t^3 \quad \dot{y} = \frac{dy}{dt} = 10t^4$$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 400t^6 + 100t^8 = 100t^6(t^2 + 4)$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 10t^3\sqrt{t^2 + 4} \text{ since } t \geq 0$$

A1

$$s = \int_0^{\sqrt{5}} 10t^3\sqrt{t^2 + 4} dt$$

$$\text{b. } s = \int_0^{\sqrt{5}} 10t^3\sqrt{t^2 + 4} dt = \int_0^{\sqrt{5}} 10t^2\sqrt{t^2 + 4} \cdot t dt$$

$$\text{let } u = t^2 + 4 \quad \frac{du}{dt} = 2t \Rightarrow t dt = \frac{1}{2} du, \quad t^2 = u - 4$$

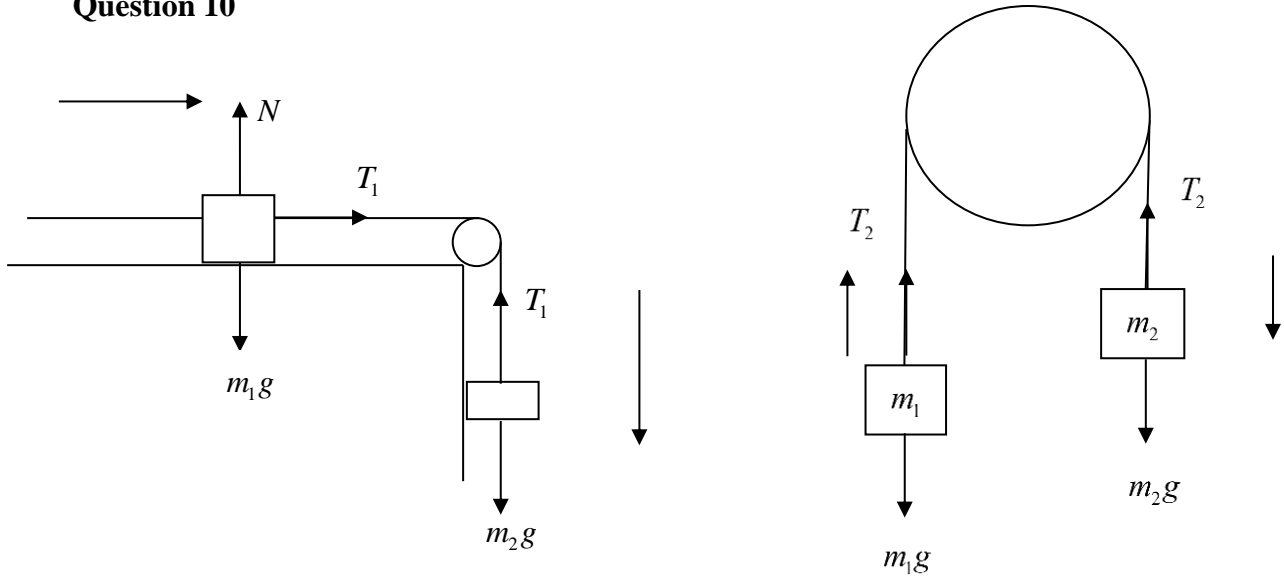
$$\text{terminals when } t = 0 \Rightarrow u = 4 \text{ when } t = \sqrt{5} \Rightarrow u = 9 \quad \text{M1}$$

$$s = 10 \int_4^9 (u - 4) \frac{\sqrt{u}}{2} du = 5 \int_4^9 \left(u^{\frac{3}{2}} - 4u^{\frac{1}{2}}\right) du \quad \text{A1}$$

$$= 5 \left[ \frac{2}{5} u^{\frac{5}{2}} - \frac{8}{3} u^{\frac{3}{2}} \right]_4^9 = \left[ 10u^{\frac{3}{2}} \left( \frac{3u - 20}{15} \right) \right]_4^9 = \frac{2}{3} \left( 7 \times 9^{\frac{3}{2}} + 8 \times 4^{\frac{3}{2}} \right) = \frac{2}{3} (7 \times 27 + 8 \times 8)$$

$$= \frac{506}{3} = 168\frac{2}{3} \quad \text{A1}$$

## Question 10



resolving parallel to the table

$$(1) T_1 = 3m_1a$$

resolving downwards around the  $m_2$  kg mass

$$(2) m_2g - T_1 = 3m_2a$$

$$m_2g - 3m_1a = 3m_2a$$

$$m_2g = 3m_1a + 3m_2a$$

$$m_2g = 3(m_1 + m_2)a$$

$$a = \frac{m_2g}{3(m_1 + m_2)}$$

equating the accelerations

$$\frac{m_2g}{3(m_1 + m_2)} = \frac{g(m_2 - m_1)}{m_1 + m_2}$$

$$m_2 = 3(m_2 - m_1) = 3m_2 - 3m_1$$

$$3m_1 = 2m_2$$

$$\frac{m_2}{m_1} = \frac{3}{2}$$

for the pulley

around  $m_2$  kg mass

$$(3) m_2g - T_2 = m_2a$$

around  $m_1$  kg mass

M2

$$(4) T_2 - m_1g = m_1a$$

adding to eliminate the tension  $T_1$

$$(3) + (4) m_2g - m_1g = a(m_1 + m_2)$$

$$a = \frac{g(m_2 - m_1)}{m_1 + m_2} \quad \text{A1}$$

**END OF SUGGESTED SOLUTIONS**