2018 Specialist Mathematics Trial Exam 2 Solutions

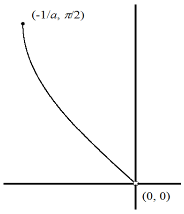
SECTION A – Multiple-choice questions

1	2	3	4	5	6	7	8	9	10
C	D	A	D	A	C	A	E	E	B

11	12	13	14	15	16	17	18	19	20
D	B	A	B	A	C	D	A	E	E

Q1 C

Q2 D

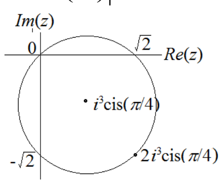


Q3 $x = \sin\left(\cos^{-1} t\right)$, .: $0 \le x \le 1$, $\sin^{-1} x = \cos^{-1} t = \frac{\pi}{2} - \sin^{-1} t$

$y = \cos\left(\sin^{-1} t\right)$, .: $\cos^{-1} y = \sin^{-1} t$, .: $\cos^{-1} y = \frac{\pi}{2} - \sin^{-1} x$,

.: $y = \cos\left(\frac{\pi}{2} - \sin^{-1} x\right) = \sin\left(\sin^{-1} x\right) = x$ A

Q4 $\left|z + i\,\text{cis}\left(\frac{\pi}{4}\right)\right| = \left|z - i^3\,\text{cis}\left(\frac{\pi}{4}\right)\right| = 1$ D



Q5 $z = \frac{a}{\sqrt{2}}(-1+i)$, $z + a\sqrt{2} = \frac{a}{\sqrt{2}}(1+i)$,

$z - a\sqrt{2}\,i = -\frac{a}{\sqrt{2}}(1+i)$, .: $\frac{z + a\sqrt{2}}{z - a\sqrt{2}\,i} = -1$ A

Q6 $\left(n + \frac{3\pi}{22}\right) - \left(n - \frac{4\pi}{11}\right) = \frac{\pi}{2}$, .: O, z_1, z_2 and w form a rectangle. $|w| =$ length of diagonal $= \sqrt{a^2 + b^2}$. C

Q7 $|a| = |b|$, .: $a = \pm b$, .: $a + b = 0$ or $a - b = 0$

Translate both to the right by $\frac{\alpha}{2}$, $f(x) = \left|a\,\text{cosec}\left(\frac{x}{2}\right)\right|$ and

$g(x) = \left|b\sec\left(\frac{x - (\alpha - \beta)}{2}\right)\right|$, $f(x) = g(x)$ when $\alpha - \beta = \pm\pi$. A

Q8 Let $\cos^{-1} x - \sin^{-1} y = \frac{\pi}{6}$. Try $\cos^{-1} x = 0$ and $-\sin^{-1} y = \frac{\pi}{6}$.

.: $x = 1$ and $y = -\frac{1}{2}$, .: $x + y = \frac{1}{2}$. E

Note: For an alternative solution, try $\cos^{-1} x = \frac{2\pi}{3}$ and

$\sin^{-1} y = \frac{\pi}{2}$. .: $x = -\frac{1}{2}$ and $y = 1$, .: $x + y = \frac{1}{2}$.

Q9 East of the origin, $\tan^{-1} t > 0$ and $\log_e t = 0$, .: $t = 1$

$\tilde{v} = \frac{d\tilde{r}}{dt} = \frac{1}{1+t^2}\tilde{i} + \frac{1}{t}\tilde{j} = \frac{1}{2}\tilde{i} + 1\tilde{j}$, speed $= \sqrt{\left(\frac{1}{2}\right)^2 + 1^2} = \frac{\sqrt{5}}{2}$ E

Q10 $\frac{dy}{dx} = \frac{1}{x} - x$, $y = -0.5$ when $x = 1$

.: $y = \log_e |x| - \frac{x^2}{2} = -1$, $x = \alpha = \pm 0.3982, \pm 1.775$

-0.40 is the closest. B

Q11 $-2 = 2u + 2a$, $4 = 4u + 8a$, $a = 2$ and $u = -3$

Let $-3t + t^2 = 0$, $t = 0$ or 3. D

Q12 $\theta = \cos^{-1} \frac{3+3}{\sqrt{10}\sqrt{11}}$, scalar resolute $= \sqrt{10}\sin\theta \approx 2.6$ B

Q13 $50a = 45g - 50g$, $a = -0.1g \approx -1$, acceleration is downward. The lift is either moving upwards but slowing down, or moving downwards and speeding up. A

Q14 $\tilde{v} = 10\tilde{i} - 9.8t\,\tilde{j}$, $\tan(-45°) = \frac{-9.8t}{10} = -1$, $t \approx 1.0$ B

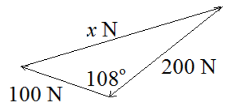
Q15 $\int_{-1}^{0} |f(x)|\,dx \ge 2$ and $\int_{0}^{1} |f(x)|\,dx \ge \frac{1}{2}$

.: $\int_{-1}^{1} |f(x)|\,dx = \int_{-1}^{0} |f(x)|\,dx + \int_{0}^{1} |f(x)|\,dx \ge \frac{5}{2}$ A

Q16 $\int 2v\,dv = \int \frac{1}{\sqrt{9 - x^2}}\,dx$, and $v = 0$ at $x = 0$, .: $v^2 = \sin^{-1}\left(\frac{x}{3}\right)$

$v = \frac{dx}{dt} = \sqrt{\sin^{-1}\left(\frac{x}{3}\right)}$, $\frac{dt}{dx} = \frac{1}{\sqrt{\sin^{-1}\left(\frac{x}{3}\right)}}$, $\Delta t = \int_{1}^{2} \frac{1}{\sin^{-1}\left(\frac{x}{3}\right)}\,dx \approx 1.4$ C

Q17 D



$x = \sqrt{100^2 + 200^2 - 2(100)(200)\cos 108°} \approx 250$

Q18 $E\left(\bar{X}\right) \approx \mu = 75$, $\text{sd}\left(\bar{X}\right) = \frac{12}{\sqrt{25}} = 2.4$

$\Pr\left(\bar{X} = 80\right) = \Pr\left(79.5 < \bar{X} < 80.5\right) \approx 0.01943$ A

Q19 $E\left(\bar{W}\right) \approx \mu = 75$, $\text{sd}\left(\bar{W}\right) = \frac{12}{\sqrt{100}} = 1.2$

p - value $= \Pr\left(\bar{W} \ge 78 \mid \mu = 75\right) \approx 0.0062$ E

Q20 $\Pr(-z < Z < z) = 0.8$, $z = 1.28155$

80% confidence interval for the mean life of the batteries is:

$\left(29.52 - 1.28155 \times \frac{0.45}{\sqrt{36}},\ 29.52 + 1.28155 \times \frac{0.45}{\sqrt{36}}\right)$

i.e. $(29.42, 29.62)$ E

SECTION B

Q1a $\overrightarrow{BP}=\frac{1}{2}\overrightarrow{BA}=\frac{1}{2}\left(\overrightarrow{OA}-\overrightarrow{OB}\right)$,

$\overrightarrow{OP}=\overrightarrow{OB}+\overrightarrow{BP}=\overrightarrow{OB}+\frac{1}{2}\left(\overrightarrow{OA}-\overrightarrow{OB}\right)=\frac{\overrightarrow{OA}+\overrightarrow{OB}}{2}$

Q1b $\overrightarrow{OP}=\overrightarrow{OM}+\overrightarrow{MP}=\overrightarrow{OM}+\frac{1}{x}\overrightarrow{OM}=\left(\frac{x+1}{x}\right)\overrightarrow{OM}$

$\therefore\ \overrightarrow{OM}=\left(\frac{x}{x+1}\right)\overrightarrow{OP}=\left(\frac{x}{x+1}\right)\left(\frac{\overrightarrow{OA}+\overrightarrow{OB}}{2}\right)=\frac{x}{x+1}\left(\frac{2\overrightarrow{OQ}+\overrightarrow{OB}}{2}\right)$

Q1c $\overrightarrow{BQ}=\overrightarrow{BM}+\overrightarrow{MQ}=\overrightarrow{BM}+\frac{1}{y}\overrightarrow{BM}=\left(\frac{y+1}{y}\right)\overrightarrow{BM}$

$\overrightarrow{OM}=\overrightarrow{OB}+\overrightarrow{BM}=\overrightarrow{OB}+\left(\frac{y}{y+1}\right)\overrightarrow{BQ}=OB+\left(\frac{y}{y+1}\right)\left(\overrightarrow{OQ}-\overrightarrow{OB}\right)$

$=OB+\left(\frac{y}{y+1}\right)\left(\overrightarrow{OQ}-\overrightarrow{OB}\right)=\frac{y\overrightarrow{OQ}+\overrightarrow{OB}}{y+1}$

Q1d Comparing the two expressions for $\overrightarrow{OM}$ in parts b and c,

$\frac{x}{x+1}=\frac{y}{y+1}$ and $\frac{x}{2(x+1)}=\frac{1}{y+1}$

$\therefore\ x=y=2$

Q2a $(-i)^{\frac{1}{3}}=(-1)^{\frac{1}{2}}=i$, $\therefore\ \left((-i)^{\frac{1}{3}}\right)^6=\left((-1)^{\frac{1}{2}}\right)^6$, $(-i)^2=(-1)^3$

Since $(-i)^4=(-1)^4$, $\therefore\ (-i)^2(-i)^4=(-1)^3(-1)^4$, $(-i)^6=(-1)^7$

$\therefore\ (-i)^{\frac{1}{7}}=(-1)^{\frac{1}{6}}$, $\therefore\ m=7$ and $n=6$ for both values to be higher.

Note: It is possible, for example $(-i)^2=(-1)^3(-1)^4$,

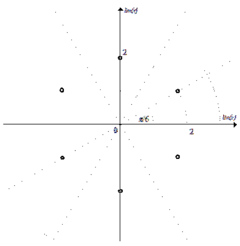
$\therefore\ (-i)^2=(-1)^7$, $(-i)^{\frac{1}{7}}=(-1)^{\frac{1}{2}}$, $\therefore\ m=7$ and $n=2$. In this case only one value is higher.

Q2b $(-8i)^{\frac{1}{3}}=(-64)^{\frac{1}{6}}$, $\left((-8i)^{\frac{1}{3}}\right)^6=\left((-64)^{\frac{1}{6}}\right)^6$,

$\therefore\ (-8i)^2=-64$ which is true.

Q2c $z=2i$

Q2d



Q2e One of the cube roots of $(-8i)^{\frac{1}{3}}$ is $2i$, i.e. $2\text{cis}\left(\frac{\pi}{2}\right)$. The other two are $2\text{cis}\left(\frac{7\pi}{6}\right)$ and $2\text{cis}\left(\frac{11\pi}{6}\right)$. Multiplying by i will rotate all by $\frac{\pi}{2}$ anticlockwise. They are $2\text{cis}(\pi)$, $2\text{cis}\left(\frac{5\pi}{3}\right)$ and $2\text{cis}\left(\frac{\pi}{3}\right)$, i.e. $2\text{cis}\left(-\frac{\pi}{3}\right)$, $2\text{cis}\left(\frac{\pi}{3}\right)$ and $2\text{cis}(\pi)$.

Q3a $\tilde{v}=-10\sin t\,\tilde{i}+10\cos t\,\tilde{j}-9.8t\,\tilde{k}$

$\tilde{a}=\frac{d\tilde{v}}{dt}=-10\cos t\,\tilde{i}-10\sin t\,\tilde{j}-9.8\,\tilde{k}$

Magnitude of vertical acceleration $=9.8$

Magnitude of horizontal acceleration $=10\sqrt{(-\cos t)^2+(-\sin t)^2}=10$

Q3b $\tilde{r}=\int\left(-10\sin t\,\tilde{i}+10\cos t\,\tilde{j}-9.8t\,\tilde{k}\right)dt$

and $\tilde{r}=10\,\tilde{i}+44.1\,\tilde{k}$ at $t=0$

$\therefore\ \tilde{r}=10\cos t\,\tilde{i}-10\sin t\,\tilde{j}+\left(44.1-4.9t^2\right)\tilde{k}$

Q3c $44.1-4.9t^2=0$, $t=3$

Q3d When $t=3$, $\tilde{r}=10\cos 3\,\tilde{i}-10\sin 3\,\tilde{j}+0\,\tilde{k}$

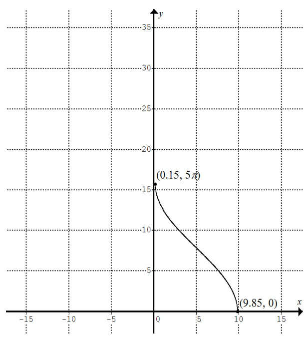
Landing point is $(10\cos 3,-10\sin 3,0)$, i.e. $(-9.9,-1.4,0)$ approx.

Q3e When $t=3$, $\tilde{v}=-10\sin 3\,\tilde{i}+10\cos 3\,\tilde{j}-29.4\,\tilde{k}$

Speed $=|\tilde{v}|=\sqrt{(-10\sin 3)^2+(10\cos 3)^2+(-29.4)^2}\approx 31.1$

Q3f $\tan\theta\approx\frac{29.4}{10}$, $\theta\approx 71.2°$

Q4a



Q4b Reflect in the x-axis, then translate upwards by 10π,

$y=10\pi-5\cos^{-1}\left(\frac{x-5}{4.85}\right)$

Q4c $V=\int_{5\pi-15}^{5\pi}\pi x^2\,dy=\int_{5\pi-15}^{5\pi}\pi\left(4.85\cos\left(\frac{y}{5}\right)+5\right)^2dy\approx 1599$

Q4d Flow rate $=\frac{1599}{60}\approx 26.65$

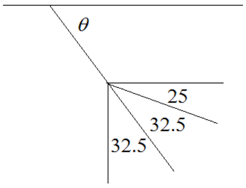
Q4e $\int_0^h\pi\left(4.85\cos\left(\frac{y}{5}\right)+5\right)^2dy=\frac{45}{60}\times 1599=1199.25$, $h\approx 4.46$

Q5a $a = 0.010\ \text{m s}^{-2}$

Q5b $F_{\text{pull}} - 3g = 3a$, $F_{\text{pull}} = 3g + 3\times 0.1 = 29.7\ \text{N}$

Q5c $T_2 = T_3 = F_{\text{pull}} = 29.7\ \text{N}$, $T_4 - 1g = 1\times 0.1$, $T_4 = 9.9\ \text{N}$

Q5d $\theta° = 25° + 32.5° = 57.5°$



Q5e $T_1 = T_2 \cos 32.5° + T_3 \cos 32.5° \approx 50.1\ \text{N}$

Q6ai $\text{E}(H) = 10\times\mu = 1700\ \text{cm}$

Q6aii $\text{Var}(H) = 10\times\text{Var}(X) = 10\times 20^2 = 4000$

$\text{sd}(H) = \sqrt{\text{Var}(H)} = \sqrt{4000} \approx 63.2\ \text{cm}$

Q6bi $\text{E}(\overline{X}) = \mu = 170\ \text{cm}$, $\text{sd}(\overline{X}) = \dfrac{\sigma}{\sqrt{n}} = \dfrac{20}{\sqrt{10}} \approx 6.3\ \text{cm}$

Q6bii $\Pr(\overline{X} > 175) \approx 0.2$

Q6ci $\bar{x} = 168.9\ \text{cm}$, $s_x = 16.0\ \text{cm}$

Q6cii Approximate 95% confidence interval for μ is

$$\left(168.9 - 1.96\times\frac{16.0}{\sqrt{10}}, 168.9 + 1.96\times\frac{16.0}{\sqrt{10}}\right) \approx (159.0, 178.8)\ \text{cm}$$

Q6ciii Let m be the sample size required to reduce the width of the interval by half.

$\dfrac{1}{\sqrt{m}} = \dfrac{1}{2}\times\dfrac{1}{\sqrt{10}}$, $m = 40$

Please inform mathline@itute.com re conceptual and/or mathematical errors

