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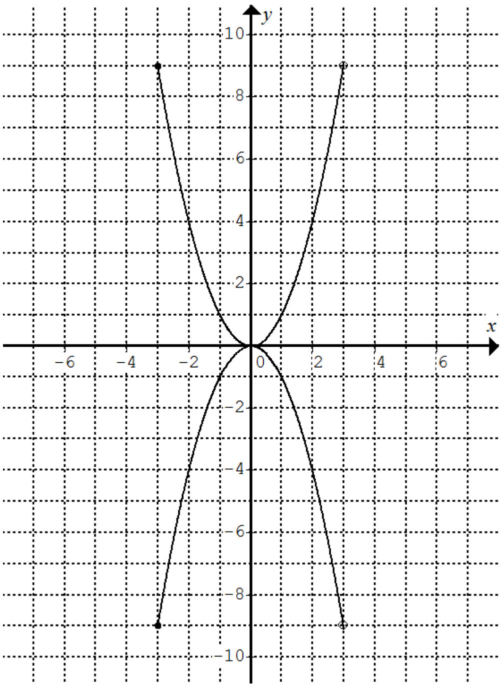
# 2018 Specialist Mathematics Trial Exam 1 Solutions



Q1a $|y| = x^2$, $y = \pm x^2$




Q1b $y = \pm x^2$, $\frac{dy}{dx} = \pm 2x$

Q2 $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^3 x\, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x\left(\sec^2 x - 1\right)dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(\tan x \sec^2 x - \tan x\right)dx$

$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x \sec^2 x\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{-\sin x}{\cos x} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x\, d(\tan x) + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\cos x} d(\cos x)$

$= \left[\frac{\tan^2 x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} + \left[\log_e(\cos x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = 1 - \frac{1}{2}\log_e 2$

Q3a $\overrightarrow{PQ} = \tilde{b} - \tilde{a}$, $\overrightarrow{AR} = 2\tilde{b} - 2\tilde{a} = 2\left(\tilde{b} - \tilde{a}\right)$, $\overrightarrow{AR} = 2\overrightarrow{PQ}$

.: $AR \,//\, PQ$ and $AR = 2PQ$

Q3b From part a, $PQ \,//\, SR$, .: $\overrightarrow{PQ} = n\overrightarrow{SR}$.

Since $R$ is the midpoint of $CQ$, and $\overrightarrow{SR} = \tilde{d} - \tilde{c}$, .: $\overrightarrow{PQ} = 2\left(\tilde{d} - \tilde{c}\right)$

Q3c From parts a and b, $\overrightarrow{AR} = 2\overrightarrow{PQ}$ and $\overrightarrow{PQ} = 2\overrightarrow{SR}$, .: $\overrightarrow{AR} = 4\overrightarrow{SR}$,

$AR = 4SR$, .: $AS = 3SR$, hence $AS : SR = \frac{AS}{SR} = 3$

Q4a A vector dependent of $\tilde{p}$ is $m\tilde{p} = m\left(\tilde{i} - 2\tilde{j} + 3\tilde{k}\right)$ where $m \neq 0$.

By choosing $m = 1.5$, a possible vector is $1.5\tilde{i} - 3\tilde{j} + 4.5\tilde{k}$.

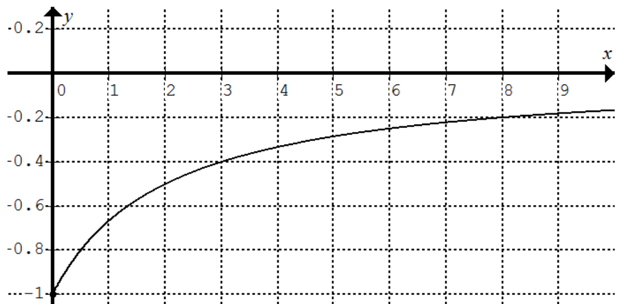
Q4b Let $x\tilde{i} + y\tilde{j} + z\tilde{k}$ be a vector independent of $\tilde{p}$.

.: $\left(x\tilde{i} + y\tilde{j} + z\tilde{k}\right).\left(\tilde{i} - 2\tilde{j} + 3\tilde{k}\right) = 0$, $x - 2y + 3z = 0$

Choose $y = 1$ and $z = 1$, .: $x = -1$, and the vector is $-\tilde{i} + \tilde{j} + \tilde{k}$.

.: a unit vector independent of $\tilde{p}$ is $\frac{1}{\sqrt{3}}\left(-\tilde{i} + \tilde{j} + \tilde{k}\right)$. There are infinitely many other possible unit vectors.

Q5a $x = 2t$, $y = -\frac{1}{t+1}$, eliminating $t$, $y = -\frac{2}{x+2}$ for $x \geq 0$




$x$-intercept $(0, -1)$, asymptote $y = 0$

Q5b Length $= \int_5^{10} \sqrt{(x')^2 + (y')^2}\, dt = \int_5^{10} \sqrt{4 + \frac{1}{(t+1)^4}}\, dt \approx \int_5^{10} 2\, dt = 10$

Note: $\frac{1}{(t+1)^4} << 4$ for large $t$.

Q6a $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{4+v^2}{2}$, $\frac{dx}{d\left(v^2\right)} = \frac{1}{4+v^2}$, $x = \log_e\left(4+v^2\right) + c$

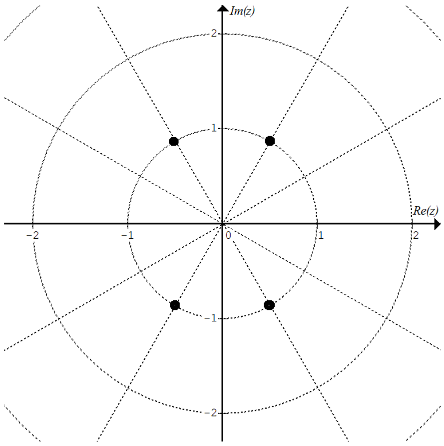
$v = 0$ when $x = 0$, $x = \log_e\left(\frac{4+v^2}{4}\right)$, $v = \pm 2\sqrt{e^x - 1}$, $|v| = 2\sqrt{e^x - 1}$

Q6b $e^x - 1 \geq 0$, i.e. $x \geq 0$. The UFO travels from $x > 0$ towards $x = 0$ with decreasing speed. It stops momentarily at $x = 0$, reverses its direction and travels away from $x = 0$ with increasing speed.

Q7a $f(z)=z^4+z^2+1=z^4+2z^2+1-z^2=\left(z^2+1\right)^2-z^2$

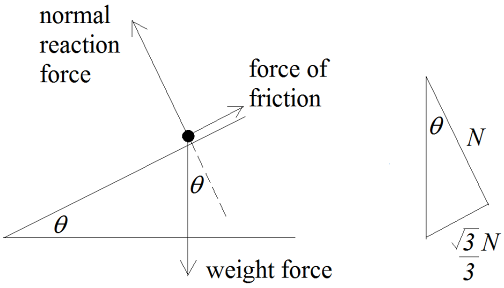
$=\left(z^2+z+1\right)\left(z^2-z+1\right)=0$ .: $z=\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$, $z=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$

Q7b $|z|=1$, $\theta=\pm\frac{\pi}{3}$, $\pm\frac{2\pi}{3}$




Q8 Net force is zero at constant speed and direction.

Refer to the following diagrams, $\tan\theta° =\frac{\sqrt{3}}{3}$, $\theta=30$




Q9a $\mathrm{E}(X+2Y)=\mathrm{E}(X+X-1)=\mathrm{E}(2X-1)=2\mathrm{E}(X)-1=1.5$

Q9b $\mathrm{Var}(X+2Y)=\mathrm{Var}(2X-1)=2^2\times\mathrm{Var}(X)=2$

Q10a $\mu=32$, $\sigma=15$, $\mathrm{E}(\overline{X})=\mu=32$, $\mathrm{sd}(\overline{X})=\frac{\sigma}{\sqrt{n}}=\frac{15}{10}=1.5$

Q10b The distribution of $\overline{X}$ is approximately normal, and
$(29, 35)=\left(\mathrm{E}(\overline{X})-2\mathrm{sd}(\overline{X}),\ \mathrm{E}(\overline{X})+2\mathrm{sd}(\overline{X})\right)$
.: $\Pr(29<\overline{X}<35)\approx 0.95$

*Please inform mathline@itute.com re conceptual and/or mathematical errors*