Year 12 *Trial Exam Paper*

2018

SPECIALIST MATHEMATICS

Written examination 2

Reading time: 15 minutes
Writing time: 2 hours

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring blank sheets of paper and/or correction fluid/tape into the examination.

Materials provided
- Question and answer book of 23 pages.
- Formula sheet.
- Answer sheet for multiple-choice questions.

Instructions
- Write your **name** in the space provided above, **and** on the multiple-choice answer sheet.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

At the end of the examination
- Place the multiple-choice answer sheet inside the front cover of this book.
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination.

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1

The maximal domain of $f(x) = \sin^{-1}\left(\frac{1}{2x+1}\right)$ is

A. $x \in R$

B. $x \in R \setminus \left\{-\frac{1}{2}\right\}$

C. $x \in (-\infty, -1] \cup [0, \infty)$

D. $x \in [-1, 1]$

E. $x \in [-1, 1] \setminus \left\{-\frac{1}{2}\right\}$

Question 2

Consider the function $f(x) = a\cos^{-1}(bx + h) + k$, where $a > 0$, $b > 0$ and $h, k \in R$.

$f(x) < 0$ if

A. $k \geq -a\pi$

B. $x < -\frac{h}{b}$

C. $k > a$

D. $x \geq -\frac{h}{b}$

E. $k < -a\pi$

Question 3

If $\cos(\phi+\gamma)=-a$ and $\cos(\phi-\gamma)=3b$, then $\cos(\phi)\cos(\gamma)$ is equal to

A. $3ab$

B. $\dfrac{-a+3b}{2}$

C. $\sqrt{a^2+9b^2}$

D. $\dfrac{a+3b}{2}$

E. $\sqrt{a^2+3b^2}$

Question 4

If $z=r\text{cis}(\theta)$, then $\dfrac{z^3}{\overline{z}}$ is equivalent to

A. $r^2\text{cis}(4\theta)$

B. $3\text{cis}(4\theta)$

C. $r^3\text{cis}(4\theta)$

D. $r^2\text{cis}(2\theta)$

E. $r^2\text{cis}(\theta)$

Question 5

The relation $|z-2+3i|=2$ has the same Cartesian equation as

A. $|z-2+3i|=|z-2-3i|$

B. $(z-2+3i)(\overline{z}-2-3i)=2$

C. $|z-2|=|z+3i|$

D. $\text{Re}(z-2+3i)=0$

E. $(z-2+3i)(\overline{z}-2-3i)=4$

Question 6

The polynomial equation $P(z)=0$ has real coefficients. $P(z)=0$ has roots that include $z=1,\ z=i,\ z=2+i$.

The minimum possible degree of $P(z)$ is

A. 3

B. 4

C. 5

D. 6

E. 7

Question 7

The vectors $\underset{\sim}{a}=2\underset{\sim}{i}-\underset{\sim}{j}+2\underset{\sim}{k},\ \underset{\sim}{b}=-\underset{\sim}{i}+3\underset{\sim}{j}-2\underset{\sim}{k}$ and $\underset{\sim}{c}=3\underset{\sim}{i}+\underset{\sim}{j}+\lambda\underset{\sim}{k},$ where λ is a real constant, are linearly independent if

A. $\lambda=2$

B. $\lambda\in R\setminus\{-2\}$

C. $-2<\lambda\le 2$

D. $\lambda\in R\setminus\{2\}$

E. $\lambda\ge -2$

Question 8

The vector resolute of $\underset{\sim}{a}=\underset{\sim}{i}+4\underset{\sim}{j}+2\underset{\sim}{k}$ perpendicular to $\underset{\sim}{b}=2\underset{\sim}{i}+2\underset{\sim}{j}-\underset{\sim}{k}$ is

A. $\frac{8}{9}(2\underset{\sim}{i}+2\underset{\sim}{j}-\underset{\sim}{k})$

B. $\frac{1}{9}(-7\underset{\sim}{i}+20\underset{\sim}{j}+10\underset{\sim}{k})$

C. $\frac{1}{21}(34\underset{\sim}{i}+10\underset{\sim}{j}-5\underset{\sim}{k})$

D. $\frac{1}{21}(8\underset{\sim}{i}+32\underset{\sim}{j}+16\underset{\sim}{k})$

E. $\frac{1}{9}(-7\underset{\sim}{i}+20\underset{\sim}{j}+26\underset{\sim}{k})$

Question 9

The position vector $\underset{\sim}{r}_c = (7t^2 - 3)\underset{\sim}{i} + \left(\arcsin\left(\frac{\sqrt{3}}{3}t\right) + 2\right)\underset{\sim}{j}$ describes the position of a moving car, while the position vector $\underset{\sim}{r}_s = 3t^2\underset{\sim}{i} + (2\pi t^2 + b)\underset{\sim}{j}$ describes the position of a moving soccer ball.

Given that $t \geq 0$ and b is a real constant, the car and ball will collide when the value of b is

A. $2 + \frac{4\pi}{3}$

B. $-2 - \frac{4\pi}{3}$

C. $2 - \frac{7\pi}{6}$

D. $2 - \frac{4\pi}{3}$

E. $-2 - \frac{7\pi}{6}$

Question 10

An equivalent expression for $\int_{-5}^{-1} \frac{3-2x}{\sqrt{2-x}}\,dx$ is

A. $\int_7^3 \left(2u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du$

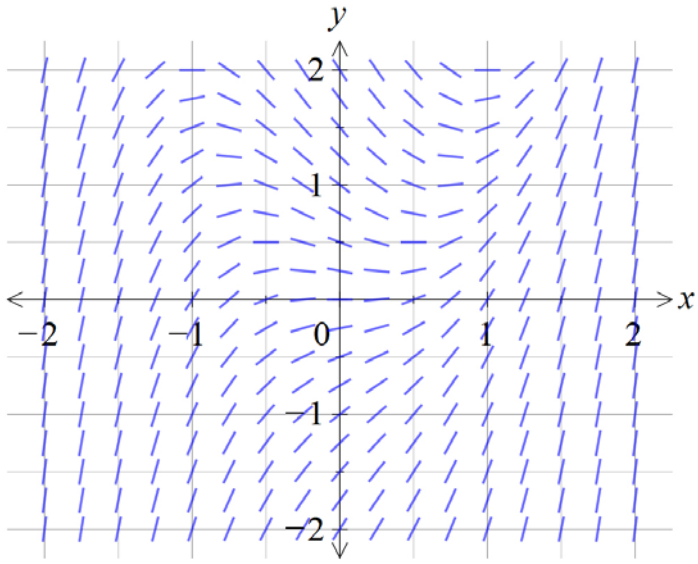
B. $\int_{-5}^{-1} \left(2u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du$

C. $\int_3^7 \left(2u - u^{\frac{1}{2}}\right) du$

D. $\int_3^7 \left(2u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du$

E. $\int_{-1}^{-5} \left(2u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right) du$

Question 11



The slope field shown above best represents the solution to the differential equation

A. $\frac{dy}{dx} = 2x - y$

B. $\frac{dy}{dx} = 2x^2 + y$

C. $\frac{dy}{dx} = y - 2x$

D. $\frac{dy}{dx} = y - 2x^2$

E. $\frac{dy}{dx} = 2x^2 - y$

Question 12

Given that $x\cos(y) + y\sin(x) = 2$, the value of $\frac{dy}{dx}$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{3}\right)$ is

A. $\frac{2}{\pi\sqrt{3} + 4}$

B. $\frac{-3\pi}{4\pi + 6\sqrt{3}}$

C. $\frac{2}{\pi\sqrt{3} - 4}$

D. $\frac{3\pi}{4\pi - 6\sqrt{3}}$

E. $\frac{2}{4 - \pi\sqrt{3}}$

Question 13

Let $\frac{dy}{dx} = (x+2)^2 - 1$, where $y_0 = y(2) = 1$.

Using Euler's method with a step size of 0.1, an approximation to y_3 is closest to

A. 2.3

B. 4.08

C. 5.75

D. 5.83

E. 7.94

Question 14

An object of mass 3 kg has an initial velocity of $2\underset{\sim}{i} - 3\underset{\sim}{j}$ ms^{-1}. After 3 seconds, the object's velocity is $-3\underset{\sim}{i} + 4\underset{\sim}{j}$ ms^{-1}. The change in momentum, in kg ms^{-1}, of the object is

A. $-5\underset{\sim}{i} + \underset{\sim}{j}$

B. $-15\underset{\sim}{i} + 3\underset{\sim}{j}$

C. $-15\underset{\sim}{i} + 21\underset{\sim}{j}$

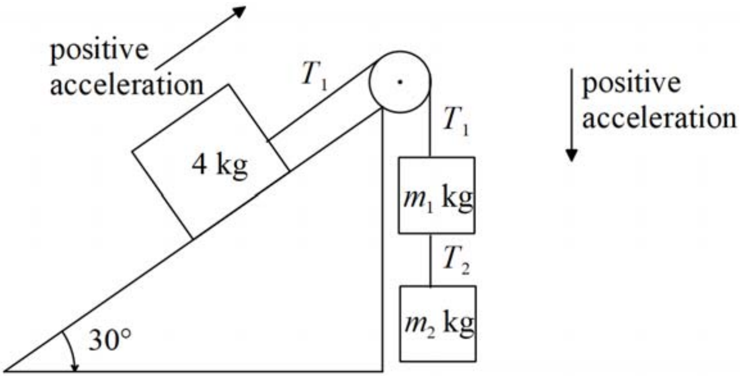
D. $-9\underset{\sim}{i} + 12\underset{\sim}{j}$

E. $5\underset{\sim}{i} - 7\underset{\sim}{j}$

Question 15

A mass of 4 kg is on a smooth plane that is inclined at $30°$ to the horizontal. The mass is connected by a light inextensible string passing over a smooth pulley to an m_1 kg mass, which in turn is connected to an m_2 kg mass.

The 4 kg mass is accelerating up the plane with an acceleration of size a, as shown in the diagram below.



The size of the tension T_1 in the string, in terms of m_1 and m_2, is

A. $T_1 = (m_1 + m_2)(g - a)$

B. $T_1 = m_1(a + g) - m_2(g - a)$

C. $T_1 = m_1(a + g) - m_2(a - g)$

D. $T_1 = (m_1 - m_2)(a - g)$

E. $T_1 = 4\left(a + \frac{g}{2}\right)$

Question 16

A moving go-kart has an acceleration of a ms^{-2}, given by $a = 2x + 2$, where x is the position of the go-kart, in metres, from the origin after $t \geq 0$ seconds.

Given that $v = 3\sqrt{2}$ ms^{-1} when $x = 2$ m, the velocity of the go-kart, v ms^{-1}, in terms of x, is

A. $v = \sqrt{2x^2 + 4x}$

B. $v = \sqrt{2}(x+1)$

C. $v = -\sqrt{2x^2 + 4x}$

D. $v = -\sqrt{2}(x+1)$

E. $v = \sqrt{2(x+1)}$

Question 17

A person of mass M kg carrying a package of mass m kg is standing in a lift that is accelerating upwards with an acceleration of a ms^{-2}.

The force of the lift floor acting on the person has a magnitude of

A. $Ma-(m+M)g$

B. $(m+M)g$

C. $(m+M)(a-g)$

D. $(m+M)(g-a)$

E. $(m+M)(a+g)$

Question 18

Let X be a random variable with a mean of 9 and a standard deviation of 4 and let Y be a random variable with a mean of 7 and a standard deviation of 2.

If X and Y are independent and $W = 2X - 3Y$, then the mean μ and standard deviation σ of W will be

A. $\mu = -3,\ \sigma = 28$

B. $\mu = -3,\ \sigma = 2\sqrt{7}$

C. $\mu = -13,\ \sigma = 10$

D. $\mu = -3,\ \sigma = 10$

E. $\mu = -13,\ \sigma = 8$

Question 19

The weight of a block of chocolate is normally distributed with a mean of 200 g and a standard deviation of 5 g.

The probability that the mean weight of 16 chocolate blocks does not exceed 202 g is closest to

A. 0.345

B. 0.945

C. 0.055

D. 0.655

E. 0.994

Question 20

Researchers test the hypothesis that crops given a special fertiliser will have a higher growth rate than average.

Which one of the following represents a type I error in this scenario?

A. Concluding that the growth rate is the same when in fact it is higher than average when using the special fertiliser.

B. Concluding that the growth rate is higher than average when using the special fertiliser when in fact it is not.

C. Concluding that the growth rate is lower than average when using the special fertiliser when in fact it is not.

D. Concluding that the growth rate is the same as the average when using the special fertiliser when there is no noticeable difference in the growth rate.

E. None of the above.

END OF SECTION A

CONTINUES OVER PAGE

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (10 marks)

a. Let $f: D \to R,\ f(x) = \dfrac{2x}{(x-1)^2} - 1$, where D is the maximal domain of f.

i. Find the equations of any asymptotes of the graph of f.

1 mark

ii. Find the coordinates of the stationary point of the graph of f.

2 marks

iii. Use the second derivative to show that the stationary point is a minimum turning point.

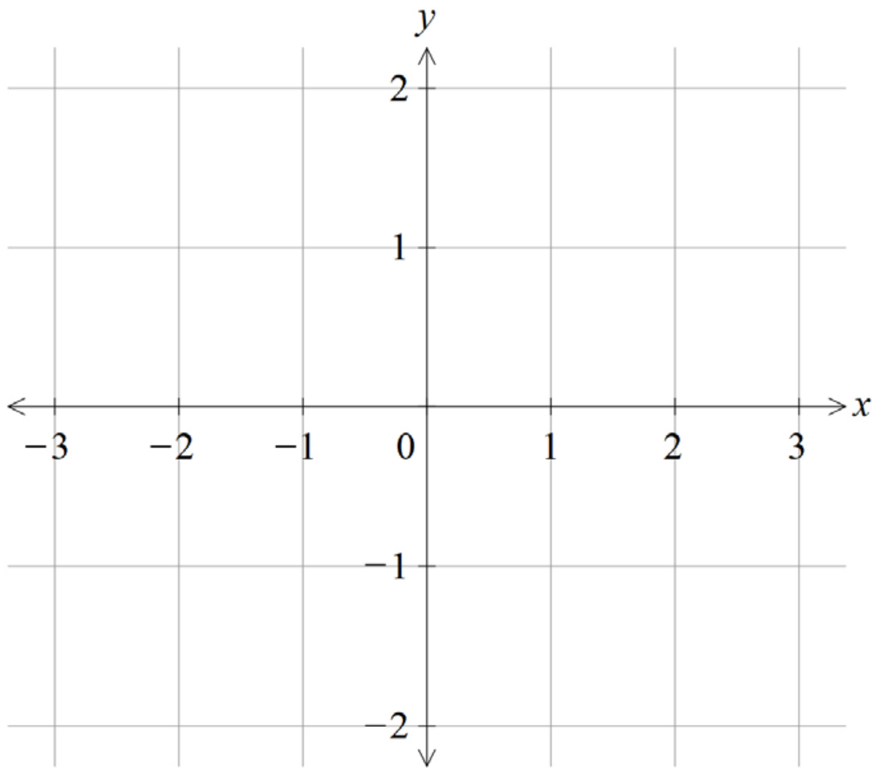
2 marks

iv. Find the coordinates of any points of inflection of the graph of f.

2 marks

b. Sketch the graph of $f(x)=\dfrac{2x}{(x-1)^2}-1$ from $x=-3$ to $x=3$ on the axes provided below, marking all stationary points, end points, points of inflection and intercepts with axes, labelling them with their coordinates. Show any asymptotes and label them with their equations.

3 marks



Question 2 (11 marks)

a. Consider the complex number $z_1 = 2\sqrt{3} + 2i$.

i. Express $z_1 = 2\sqrt{3} + 2i$ in polar form.

1 mark

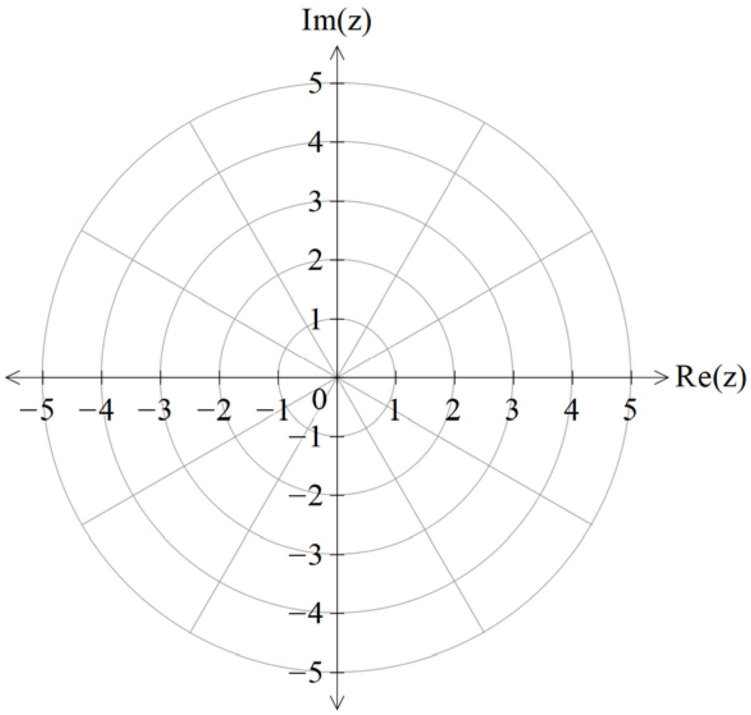
ii. Given that $z_1 = 2\sqrt{3} + 2i$ is a root of the equation

$z^3 + 4(1-\sqrt{3})z^2 + 16(1-\sqrt{3})z + 64 = 0$, find the other two roots.

2 marks

b. Plot and label the roots of $z^3 + 4(1-\sqrt{3})z^2 + 16(1-\sqrt{3})z + 64$ on the Argand diagram below.

3 marks



c. The relation $|z-4\sqrt{3}|=|z|$ represents the Cartesian equation of a line.

Determine the Cartesian equation of this line and sketch it on the Argand diagram in **part b.**

3 marks

d. The root $z_1 = 2\sqrt{3}+2i$ and its conjugate lie on the boundary of the circle given by $|z| = 4$.

Determine the area of the minor segment bounded by the line passing through $z_1 = 2\sqrt{3}+2i$ and its conjugate and the minor arc of the circle given by $|z| = 4$.

2 marks

Question 3 (11 marks)

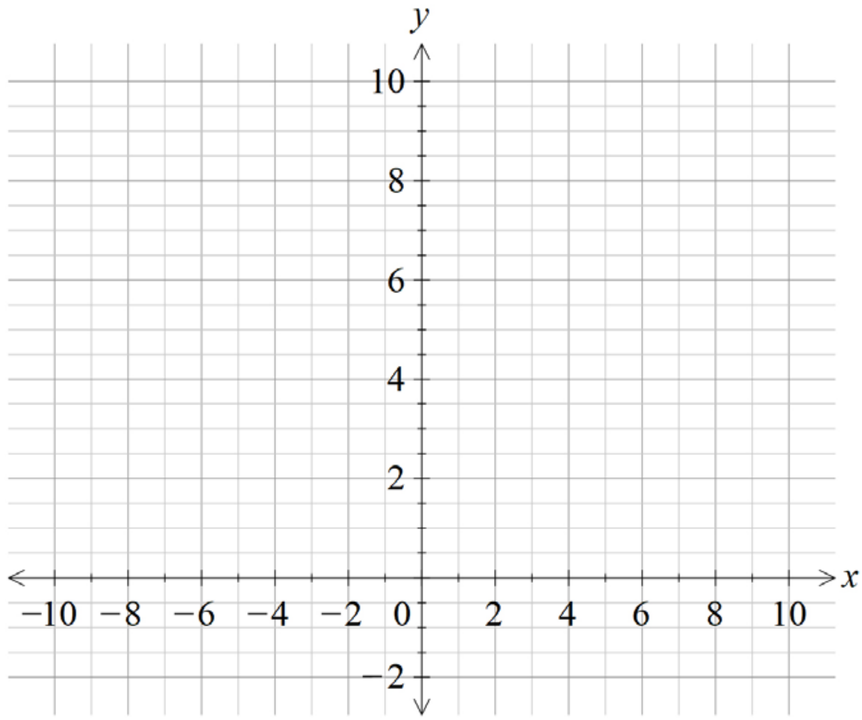
The side profile of a plastic coffee cup can be modelled by the equation

$$f(x) = 3\sin^{-1}\left(\frac{x-7}{2}\right) + \frac{3\pi}{2}, \quad x \in [5, 9]$$

where x is measured in centimetres.

a. Sketch a graph of the side profile of the coffee cup. Label all axes intercepts, end points and points of inflection with their coordinates.

3 marks



b. Show that the length of the side profile of the coffee cup is given by the definite integral $\int_5^9 \sqrt{1 - \frac{a}{(x+b)(x+c)}}\, dx$, where $a, b, c \in R$. State the values of a, b and c.

2 marks

c. Find the length of the side profile of the coffee cup. Give the answer in centimetres, correct to three decimal places.

1 mark

d. To determine the volume of the coffee cup the side profile is rotated around the *y*-axis.

i. Write a definite integral that gives the volume of the coffee cup.

1 mark

ii. Hence, find the volume of the coffee cup in cubic centimetres, correct to two decimal places.

1 mark

To test the insulation properties of the coffee cup, coffee with a temperature of 90°C is poured into the cup. The coffee then takes 15 minutes to cool to a temperature of 60°C. The coffee cools according to Newton's law of cooling.

e. The temperature of the room that the cup is in is 20°C .

Find an equation in the form $T = a + be^{-kt}$, where $a, b, k, t \geq 0$, that describes the temperature, *T*, of the coffee as a function of time, *t* minutes. Write *k* correct to two decimal places.

3 marks

Question 4 (11 marks)

Geoff is riding his bicycle in a park and his position can be described by the vector $\underset{\sim}{r}_G(t) = [1 - 2\cos(2t)]\underset{\sim}{i} + [2\sin(2t) + 3]\underset{\sim}{j}$, at any time for $t \geq 0$ minutes. All distances are measured in kilometres.

a. After riding for $\frac{3\pi}{4}$ minutes, Geoff passes a drinking fountain.

i. Find the distance, in kilometres, from his initial position to the drinking fountain.

1 mark

ii. Find the speed, in km/minute, at which Geoff passes the drinking fountain.

2 marks

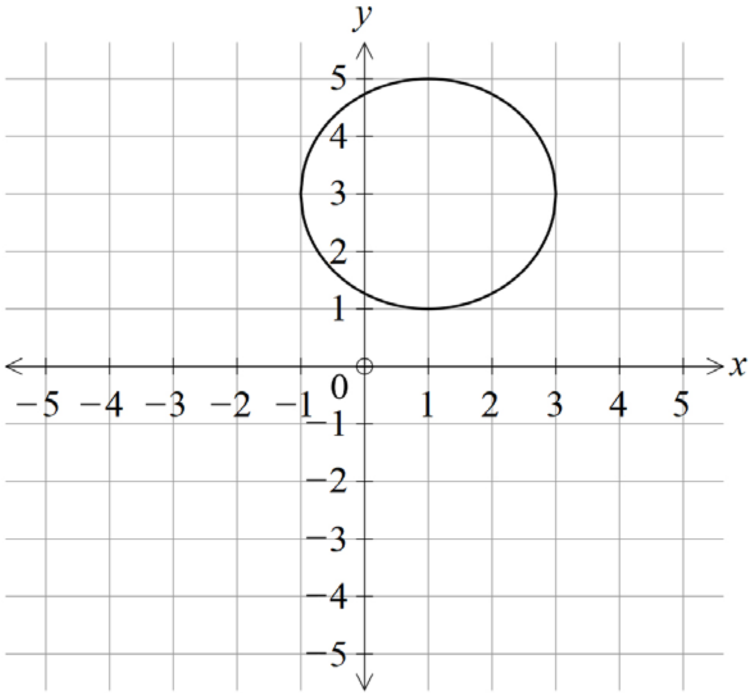
A second person, Dustin, is also riding his bicycle through the park. Dustin's position is described by the vector $\underset{\sim}{r}_D = (a + \sin(t))\underset{\sim}{i} + (3 - \cos(t))\underset{\sim}{j}$, where a is a real constant. After some time $t < \pi$, Geoff and Dustin will collide.

b. Find the value of a for which Geoff and Dustin collide and state the coordinates of the collision.

3 marks

c. The graph below shows the path of Geoff on his bicycle. On the same axes, sketch the path of Dustin on his bicycle for the value of a found in **part b.** Use arrows to indicate the direction of Dustin's movement.

2 marks

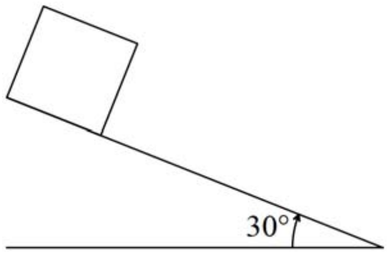


d. Determine the angle between Geoff and Dustin at the time of collision.

3 marks

Question 5 (9 marks)

A block with a mass of 5 kg, initially at rest, sits at the top of a 10 m long smooth ramp inclined at an angle of 30° to the horizontal, as shown in the diagram below.



A force of $28-40t$ N, parallel to the ramp, is applied to the block for a time period of $0<t\leq 0.7$ seconds. If the acceleration down the ramp is positive:

a. Show that the acceleration of the block down the ramp for the time period $0<t\leq 0.7$ is given by $10.5-8t\ \text{ms}^{-1}$.

2 marks

b. Determine the speed of the block, in ms^{-1}, at the moment the applied force is removed.

2 marks

c. Determine the distance travelled, in metres, along the ramp by the block in the first 0.7 seconds. Give the answer correct to two decimal places.

2 marks

d. After the applied force is removed, the block continues sliding down the ramp.

Using the distance found in **part c.**, determine the total time it takes for the block to reach the bottom of the ramp. Give the answer in seconds and correct to one decimal place.

3 marks

Question 6 (8 marks)

International tennis regulations require that tennis balls used in tournaments must have a minimum diameter of 66.5 mm.

To ensure that this requirement is met, a particular company produces tennis balls whose diameter is normally distributed with a mean of 66.9 mm and a standard deviation of 3 mm.

For quality control testing, samples of 100 balls are selected and their diameters measured.

a. Show that the standard deviation of the sample mean of the diameter of tennis balls that undergo quality control testing is 0.3 mm.

1 mark

b. The 90% confidence interval for the mean diameter of the tennis balls has the form $(66.9 - k_1,\ 66.9 + k_2)$. Find the values of k_1 and k_2, correct to one decimal place.

2 marks

The company's quality control tester claims that tennis balls with a mean that exceeds its production standards are being produced.

To test this claim, a sample of 100 tennis balls is selected at random and the mean diameter of the tennis balls is found to be 67 mm.

c. Write down the two hypotheses that would be used in a one-sided statistical test.

1 mark

d. Write down an expression for the *p*-value and evaluate it, correct to four decimal places.

2 marks

e. Explain whether or not the quality control tester's claim should be rejected at the 5% level of significance.

1 mark

f. For this test, determine the value of the sample mean that could be observed before the quality control tester's claim is rejected at the 1% level of significance. Give your answer correct to one decimal place.

1 mark

END OF QUESTION AND ANSWER BOOK