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Year 12 *Trial Exam Paper*

2018

SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

STUDENT NAME:

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided above.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (4 marks)

Consider the complex numbers $z = 2 + i$ and $v = 1 + 3i$.

a. Show that $\frac{v}{z} = \sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$.

2 marks

b. Find $\text{Arg}\left(\frac{\overline{v}}{\overline{z}}\right)$ and $\text{Arg}\left(\frac{v}{zi}\right)$.

2 marks

Question 2 (4 marks)

Consider the relation defined below.

$$\log_e\left(\frac{x}{y}\right) + x^2 - y = 0, \quad x,\ y > 0$$

a. Show that (1, 1) is a point on the curve defined by this relation.

1 mark

b. Find the equation of the tangent to the curve at the point (1, 1) in the form $ax + by + c = 0$.

3 marks

Question 3 (6 marks)

The waiting time, in minutes, for customers entering the Royal Melbourne Show is normally distributed with mean μ.

In 2016, a random sample of 16 customers entering the Royal Melbourne Show gave an approximate 95% confidence interval for μ as $5.02 < \mu < 6.98$.

a. Find the mean $\bar{x}$ and hence show that the sample standard deviation is 2.

2 marks

In 2017, the number of inspectors at entry points to the Royal Melbourne Show was increased and the waiting time, in minutes, for customers was claimed to be normally distributed with a mean waiting time of 4.5 minutes and a standard deviation of 1.5 minutes.

A random sample of 36 customers entering the Royal Melbourne Show had a mean waiting time of 4 minutes. Assume that the population standard deviation remains at 1.5 minutes.

b. Test the hypothesis that the mean waiting time is less than that claimed by finding an approximate p value for this test. Hence explain whether or not the null hypothesis should be rejected at the 5% level of significance.

4 marks

Question 4 (3 marks)

Evaluate $\int_0^{\sqrt{e^2-1}} \frac{4x \log_e(x^2+1)}{x^2+1}\, dx.$

Question 5 (3 marks)

A parallelogram $OABC$ is defined by the vectors $\overrightarrow{OA} = \underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$ and $\overrightarrow{OC} = 2\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$.

N is the point on OA that is closest to the point C.

a. Find the scalar resolute of $\overrightarrow{OC}$ in the direction of $\overrightarrow{OA}$.

1 mark

b. Hence find the length of CN.

2 marks

Question 6 (4 marks)

A lift is travelling downwards at a constant speed of v ms^{-1}. A woman standing in the lift drops her cup of coffee from a height of h metres above the floor of the lift. The coffee cup hits the floor of the lift after 0.5 seconds.

Find the value of h.

Question 7 (3 marks)

Solve the differential equation $\frac{dy}{dx} = \frac{e^{0.5x}}{3\sqrt{y}}$ for y given that $y = 1$ when $x = 0$.

Question 8 (4 marks)

The acceleration of a particle at time t seconds is given by $\ddot{\underset{\sim}{r}}(t) = 2\underset{\sim}{i} + 4\underset{\sim}{j}$ ms^{-2}.

Find the distance of the particle from its initial position when $t = 2$, given that $\dot{\underset{\sim}{r}}(0) = -2\underset{\sim}{i} - 3\underset{\sim}{k}$ ms^{-1} and $\underset{\sim}{r}(0) = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ m.

Question 9 (4 marks)

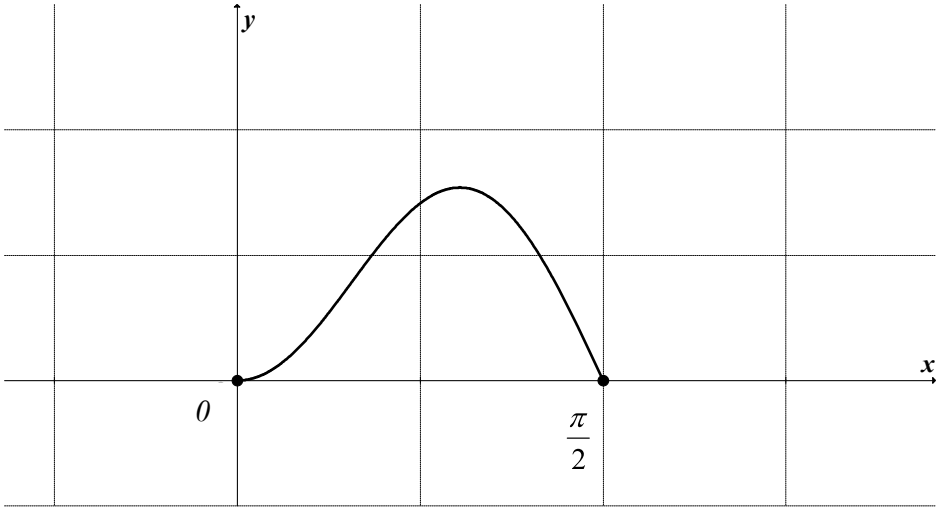
Given that $\sin(x+y)=\frac{1}{2}$ and $\frac{\tan(x)}{\tan(y)}=3$, find $\sin(x-y)$.

Question 10 (5 marks)

The curve shown below can be defined parametrically by

$$x = \arcsin(t)$$

$$y = 2t^2\sqrt{1-t^2}, \text{ where } 0 \le t \le 1.$$



a. Find the cartesian equation of the curve in the form $y = f(x)$.

1 mark

The area bounded by the curve and the x-axis is rotated about the x-axis.

b. Find the volume of the solid generated.

4 marks

END OF QUESTION AND ANSWER BOOK