VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2017

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER

SPECIALIST MATHEMATICS

Written examination 1

Thursday 8 June 2017

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 3.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 10 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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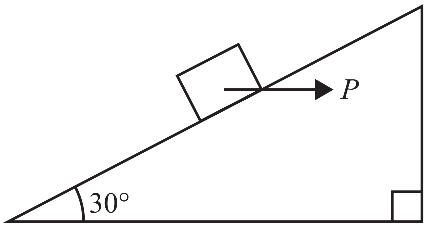
Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (3 marks)

A 5 kg mass on a smooth plane inclined at 30° is held in equilibrium by a horizontal force of magnitude P newtons, as shown in the diagram below.



a. On the diagram above, show all other forces acting on the mass and label them. 1 mark

b. Find P. 2 marks

Question 2 (3 marks)

Find a given that $\int_{-2}^{a} \frac{8}{16-x^2}\, dx = \log_e(6)$, $a \in (-2, 4)$.

Question 3 (3 marks)

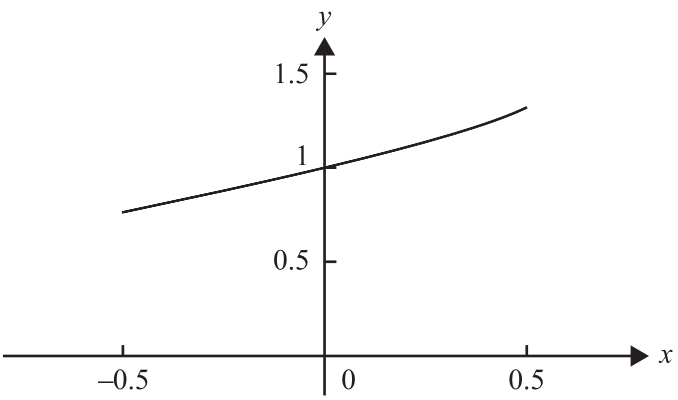
Find the gradient of the curve with equation $x = \sin\left(\frac{y}{15}\right)$ when $x = \frac{1}{4}$. Give your answer in the form $a\sqrt{b}$, where $a, b \in Z^+$.

Question 4 (4 marks)

Find the values of a and b given that $z - 1 - i$ is a factor of $z^3 + (a + b)z^2 + (b^2 - a)z - 4 = 0$, where a and b are real constants.

Question 5 (4 marks)

Part of the graph of $y = \dfrac{\sqrt{x+1}}{\sqrt[4]{1-x^2}}$ is shown below.



Find the volume generated if the region bounded by the graph of $y = \dfrac{\sqrt{x+1}}{\sqrt[4]{1-x^2}}$, the lines $x = -\dfrac{1}{2}$ and $x = \dfrac{1}{2}$, and the x-axis is rotated about the x-axis.

Question 6 (3 marks)

Find all real solutions of $\tan(2x) = -\tan(x)$.

Question 7 (5 marks)

Let $\frac{dy}{dx} = (4 - y)^2$.

a. Express y in terms of x, where $y(0) = 3$. 3 marks

b. Express $\frac{d^2y}{dx^2}$ in terms of y. 2 marks

Question 8 (3 marks)

A 3 kg mass has velocity v ms^{-1}, where $v = 2\arctan\sqrt{x}$ when it has a displacement x metres from the origin, $x > 0$.

Find the net force, F newtons, acting on the mass in terms of x.

Question 9 (4 marks)

The random variables X and Y are independent with $\mu_X = 4$, var(X) = 36 and $\mu_Y = 3$, var(Y) = 25.

a. The random variable Z is such that $Z = 2X + 3Y$.

i. Find E(Z). 1 mark

ii. Find the standard deviation of Z. 1 mark

b. Researchers have reason to believe that the mean of X has decreased. They collect a random sample of 64 observations of X and find that the sample mean is $\bar{X} = 3.8$

i. State the null hypothesis and the alternative hypothesis that should be used to test that the mean has decreased. 1 mark

ii. Calculate the mean and standard deviation for a distribution of sample means, $\bar{X}$, for samples of 64 observations. 1 mark

Question 10 (4 marks)

Consider the vectors $\underset{\sim}{a} = -\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + c\underset{\sim}{j} + \underset{\sim}{k}$.

Find the value of c, $c \in R$, if the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is $\frac{\pi}{3}$.

Question 11 (4 marks)

Find the length of the curve specified parametrically by $x = a\theta - a\sin(\theta)$, $y = a - a\cos(\theta)$ from $\theta = \frac{2\pi}{3}$ to $\theta = 2\pi$, where $a \in R^+$. Give your answer in terms of a.

END OF QUESTION AND ANSWER BOOK

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SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference. A question and answer book is provided with this formula sheet.

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Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\mathrm{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \mathrm{Arg}(z) \le \pi$
$z_1 z_2 = r_1 r_2\,\mathrm{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\mathrm{cis}\left(\theta_1 - \theta_2\right)$
$z^n = r^n\,\mathrm{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ $\mathrm{E}(aX + bY) = a\mathrm{E}(X) + b\mathrm{E}(Y)$ $\mathrm{var}(aX + b) = a^2\mathrm{var}(X)$
for independent random variables X and Y	$\mathrm{var}(aX + bY) = a^2\mathrm{var}(X) + b^2\mathrm{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\overline{X}$	mean $\mathrm{E}\left(\overline{X}\right) = \mu$ variance $\mathrm{var}\left(\overline{X}\right) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e \lvert x \rvert + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a}\log_e \lvert ax+b \rvert + c$
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1 + \left(f'(x)\right)^2}\, dx$ or $\int_{t_1}^{t_2} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert \underset{\sim}{r} \rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 . \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

END OF FORMULA SHEET