2017 VCE Specialist Mathematics 2 examination report

General comments

The 2017 Specialist Mathematics examination 2 comprised 20 multiple-choice questions (worth a total of 20 marks) and six extended-answer questions (worth a total of 60 marks).

There were five questions (Questions 2b., 3e., 4b., 4d. and 6b.) where students needed to establish a given result, with instructions to 'show that' a given result was reached. In these cases all steps that led to the given result needed to be clearly and logically set out. Students needed to provide a convincing and clear sequence of steps to obtain full marks.

Answers were generally given in the required forms, except for the following instances where errors in form occurred:

- Coordinates of stationary points and the points of inflection in Questions 1aii. and 1aiii. were frequently given in exact form rather than the required decimal approximation.
- The time of collision and the value of *a* in Question 5d. were occasionally given as decimal approximations rather than the required (by default) exact form.

The examination revealed areas of strength and weakness in student performance.

Areas of strength included:

- the use of CAS technology to evaluate definite integrals
- determining conditions for linear dependence of vectors
- finding vector resolutes.

Areas of weakness included:

- understanding that $f''(x)=0$ does not necessarily imply a point of inflection
- careful attention to details specified in questions such as identifying points and directions in Question 5a.
- careful use of notation in the statistics area of study.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Section A

The table below indicates the percentage of students who chose each option. The correct answer is indicated by shading.

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	1	8	75	4	12	0	$-1 \le \frac{1}{x} \le 1 \Rightarrow x \in (-\infty, -1] \cup [1, \infty)$
2	9	30	11	12	37	1	The solve and graphing capabilities of a CAS could have been used to find the correct answer.
3	4	4	9	47	35	0	Use of complex solve gives five solutions.
4	14	7	11	15	53	0	
5	75	6	5	10	4	0	
6	6	46	10	30	8	1	$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(e^x \arctan(y)\right) = e^x \arctan(y) + \frac{e^x}{1+y^2}\frac{dy}{dx}$
7	4	6	20	60	9	1	
8	4	29	7	52	7	0	$f''(x) = 6x - 2m \ge 0$ when $x \ge \frac{m}{3}$
9	4	45	12	9	30	0	
10	31	9	45	7	6	1	$f''(x)$ does not change sign at *a*.
11	6	9	76	7	2	0	
12	49	21	10	11	7	1	
13	2	16	78	2	2	0	
14	51	21	14	10	4	1	$m_1 g \sin(2\theta) = m_2 g \sin(\theta)$ $\frac{m_1}{m_2} = \frac{\sin(\theta)}{\sin(2\theta)} = \frac{\sec(\theta)}{2}$
15	4	71	13	7	4	0	
16	5	9	22	58	5	1	
17	37	17	20	8	17	1	
18	25	9	14	8	42	1	$E(W) = -4, \text{sd}(W) = 5$
19	9	18	19	44	9	1	The new width is 25% of the old width.
20	5	9	77	5	3	0	

Section B

Question 1ai.

Marks	0	1	Average
%	64	36	0.4

$x = -1, y = 0$

The majority of students stated the vertical asymptote but significantly fewer stated the horizontal asymptote. Various incorrect attempts at partial fraction forms were made.

Question 1aii.

Marks	0	1	2	Average
%	2	17	81	1.8

$f'(x)=\dfrac{1-2x^3}{\left(1+x^3\right)^2}$, $(0.79, 0.53)$

This question was generally answered well. Some students did not give the coordinates of the stationary point in the required form.

Question 1aiii.

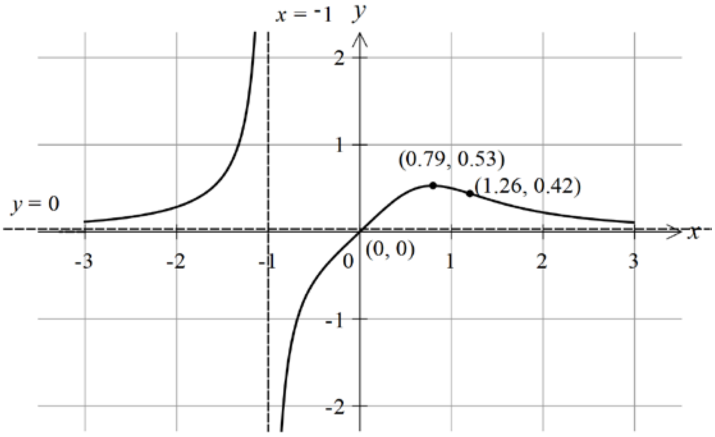
Marks	0	1	2	Average
%	10	77	13	1

$(1.26, 0.42)$

The majority of students provided the correct inflection point. A common error was to erroneously include the point $(0,0)$, which is another point where $f''(x)=0$, but it is not a point of inflection as there is no change of concavity; $f''(x)$ does not change sign.

Question 1b.

Marks	0	1	2	3	Average
%	3	9	24	64	2.5



Graphing was generally completed to a reasonable standard. In some cases the shape of the graph was poor and the required points were not marked clearly or were not placed in the correct position.

Question 1ci.

Marks	0	1	2	Average
%	17	13	71	1.6

$\pi\int_0^a \left(f(x)\right)^2 dx = \pi\int_a^3 \left(f(x)\right)^2 dx$

This question was answered well. Other equivalent correct forms were presented. A common error was a failure to square $f(x)$ or including π on only one side of the equation above.

Question 1cii.

Marks	0	1	Average
%	35	65	0.7

0.98

The majority of students who answered Question 1ci. correctly were also able to answer this question correctly.

Question 2a.

Marks	0	1	2	Average
%	7	5	88	1.8

19.6

Most students correctly used constant acceleration formulas. A smaller number of students started with an expression for acceleration and integrated correctly, either using definite integrals or evaluating the constant separately.

Question 2b.

Marks	0	1	Average
%	7	93	1

$v = 9.8 \times 2 = 19.6$

The majority of students were able to demonstrate the key steps required to show the correct value of v.

Question 2c.

Marks	0	1	Average
%	52	48	0.5

$14\sqrt{5}, 10\sqrt{g}$

Correct solutions were generally obtained by setting $a = 0$. Some answers were not given in exact form.

Question 2di.

Marks	0	1	2	Average
%	38	44	17	0.8

$$\int_{19.6}^{30} \frac{1}{9.8 - 0.01v^2}\, dv + 2$$

This question was often misinterpreted by students, either by assuming that the model applied from the start of the skydiver's fall (integrating from 0 to 30) or by giving an answer that only gave the time after 2 seconds. Many students did not attempt this question.

Question 2dii.

Marks	0	1	Average
%	75	25	0.3

5.8

Question 2e.

Marks	0	1	2	3	Average
%	52	26	6	16	0.9

$$\int_{19.6}^{30} \frac{v}{9.8-0.01v^2}\,dv + 19.6 \approx 120$$

While a variety of solutions was given, the errors apparent in Question 2di., as a result of not taking the first 2 seconds of motion into account, also appeared in responses to this question.

Question 3a.

Marks	0	1	Average
%	26	74	0.8

$$\left(\sqrt{2}, \frac{3\pi}{4}\right)$$

This question was answered well. Some students who knew the correct *x*-coordinate value did not correctly apply the dilation factor of 3 to obtain the correct *y*-coordinate value.

Question 3b.

Marks	0	1	Average
%	51	49	0.5

$$g(x) = \begin{cases} -3\arccos\left(-\frac{x}{2}\right), & -2 \le x < -\sqrt{2} \\ -3\arcsin\left(-\frac{x}{2}\right), & -\sqrt{2} \le x \le 0 \end{cases}$$

Many students did not use a hybrid function. Of those who did, domains were frequently incorrect.

Question 3c.

Marks	0	1	2	3	Average
%	16	7	15	61	2.2

$$4\left(\int_0^{\sqrt{2}} 3\arcsin\left(\frac{x}{2}\right)dx + \int_{\sqrt{2}}^{2} 3\arccos\left(\frac{x}{2}\right)dx\right) \approx 9.9$$

The majority of students stated definite integrals over the correct intervals. Some of these did not include the factor of 3 in their expressions.

Question 3d.

Marks	0	1	2	3	Average
%	62	3	8	26	1

$67.4°$

Many students did not attempt this question. Where attempts were made, some students did not use a derivative to find the gradient of the function. In some cases the obtuse angle, rather than the acute angle, was given.

Question 3e.

Marks	0	1	2	Average
%	63	21	16	0.5

$$4\left(\int_0^{\sqrt{2}}\sqrt{1+\left(\frac{3}{\sqrt{4-x^2}}\right)^2}\,dx+4\int_{\sqrt{2}}^{2}\sqrt{1+\left(\frac{-3}{\sqrt{4-x^2}}\right)^2}\,dx\right)=4\int_0^2\sqrt{1+\frac{9}{4-x^2}}\,dx=\int_0^2\sqrt{16+\frac{144}{4-x^2}}\,dx$$

$a=16, b=144$

Many students stated the formula for arc length but did not apply it. The instruction to ‘show that’ was not always followed; this would have required the use of the derivatives of the branches of $f(x)$ over the corresponding domains to reach the form required by the question.

Question 4a.

Marks	0	1	Average
%	26	74	0.8

$4\operatorname{cis}\left(-\frac{2\pi}{3}\right)$

A number of incorrect answers had arguments outside the third quadrant, which should have alerted students to an error, given the signs of the real and imaginary parts. A diagram could have reminded students that the answer needed to be in the third quadrant.

Question 4b.

Marks	0	1	Average
%	45	55	0.5

$$z=\frac{-4\pm\sqrt{4^2-4\times1\times16}}{2}=\frac{-4\pm\sqrt{-48}}{2}=\frac{-4\pm4\sqrt{3}i}{2}=-2\pm2\sqrt{3}i$$

Correct solutions were obtained by using the quadratic formula or completing the square. Some students did not correctly follow the ‘show that’ instruction either by not showing key steps in their solution or by solely verifying the solutions given by substitution. Some students confused factors with solutions or did not proceed beyond factorising the quadratic.

Question 4c.

Marks	0	1	Average
%	69	31	0.3

$-2+2\sqrt{3}i=-\left(2-2\sqrt{3}i\right)$

$-2-2\sqrt{3}i=-\overline{\left(2-2\sqrt{3}i\right)}$

Misunderstanding of the question was apparent in student responses to this question. Many attempts at solutions were not expressed in terms of $2-2\sqrt{3}i$ as required.

Question 4d.

Marks	0	1	2	Average
%	20	15	65	1.5

$|z|=\left|z-\left(2-2\sqrt{3}i\right)\right|$

$x^2+y^2=(x-2)^2+\left(y+2\sqrt{3}\right)^2$

$x^2+y^2=x^2-4x+4+y^2+4\sqrt{3}y+12$

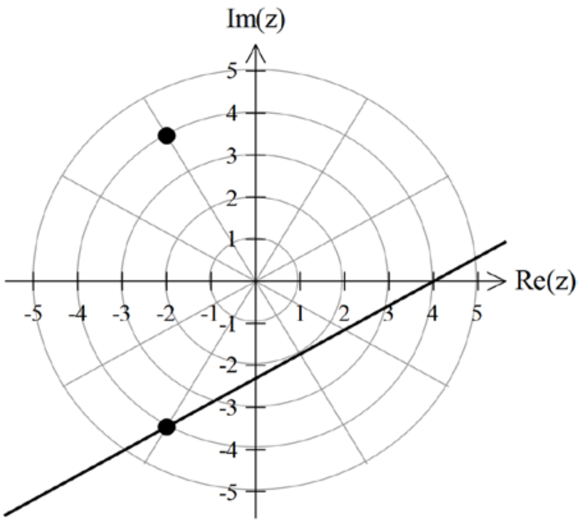
$0=-4x+4\sqrt{3}y+16$

$x-\sqrt{3y}-4=0$

The solution shown above was the most common correct approach. A smaller proportion of students correctly applied a perpendicular bisector approach. The ‘show that’ instruction was generally followed.

Question 4e.

Marks	0	1	2	Average
%	16	31	53	1.4



Incorrect plotting of the line was the most common error. Students should ensure that points plotted are accurately placed and can be clearly seen.

Question 4f.

Marks	0	1	Average
%	99	1	0

$b = -4 - \bar{a}$

This question caused significant difficulty for students. While a variety of correct forms was accepted, very few correct answers were given.

Question 4g.

Marks	0	1	2	Average
%	63	14	24	0.6

$A = \frac{4^2}{2}\left(\frac{4\pi}{3} - \sin\left(\frac{4\pi}{3}\right)\right) = 4\sqrt{3} + \frac{32\pi}{3}$

A significant number of students incorrectly used a sector angle of $\frac{2\pi}{3}$. Solutions using definite integrals were also seen; these solutions were usually completed correctly.

Question 5a.

Marks	0	1	2	Average
%	19	29	52	1.4

- Jet ski marked at $(1, 1)$ and an indication of clockwise direction.
- Boat marked at $(-1, 3)$ and an indication of clockwise direction.

Students plotted the initial positions correctly but significant numbers of students did not label the direction of motion or clearly identify the jet ski and the boat. Both requirements were explicitly stated in the question.

Question 5bi.

Marks	0	1	2	Average
%	31	13	56	1.3

$\left|\dot{\underset{\sim}{r}}_{\text{boat}}\right| = \left|\dot{\underset{\sim}{r}}_{\text{jet ski}}\right|, t = \pi$

Most students found correct expressions for velocity vectors. The most common error was to equate these velocity vectors rather than equating speeds.

Question 5bii.

Marks	0	1	Average
%	52	48	0.5

$(3, 3)$

Some answers were not given in coordinate form.

Question 5ci.

Marks	0	1	Average
%	45	55	0.6

$\sqrt{(\sin t - 2\cos t)^2 + (1 + \sin t + \cos t)^2}$

A variety of correct forms was given by students; many of these were likely produced by CAS technology, including expressions involving double angles. Students should take care when transcribing expressions from technology output as errors frequently occur, particularly regarding the number and placement of brackets. Some incorrect answers retained vectors in the expression.

Question 5cii.

Marks	0	1	Average
%	85	15	0.2

0.33

Many students found this question difficult. Incorrect answers involving other locally minimum values were frequent.

Question 5d.

Marks	0	1	2	3	Average
%	39	25	9	27	1.3

$1 - \sin(t) = 1 - 2\cos(t),\ a - \cos(t) = 3 + \sin(t) \Rightarrow t = \arctan 2,\ a = 3 + \frac{3}{\sqrt{5}}$

Most students correctly equated the vector components and solved for t. Many went on to give decimal approximations rather than supplying the exact forms. Students are reminded of the instruction saying that an exact answer is required unless otherwise specified.

Question 6a.

Marks	0	1	Average
%	43	57	0.6

79.8

While responses indicated that this question was understood, many students either did not express their answer as a percentage or did not give the required level of accuracy.

Question 6b.

Marks	0	1	2	Average
%	18	28	54	1.4

$\text{Mean} = 2005 \times 10 = 20\ 050,\ \text{variance} = 6^2 \times 10,\ \text{standard deviation} = \sqrt{36 \times 10} = 6\sqrt{10}$

The mean was calculated correctly by most students. Errors frequently occurred when calculating the variance, leading to students being unable to show the given standard deviation.

Question 6c.

Marks	0	1	Average
%	39	61	**0.6**

99.6

A significant proportion of otherwise correct answers were not given in the required form.

Question 6d.

Marks	0	1	2	3	Average
%	46	14	24	16	**1.1**

$z = -3.090, \sigma = 16.18, s = 5.1$

While some students were able to find $z = -3.090$, many did not account for the sample size in subsequent calculations.

Question 6e.

Marks	0	1	2	Average
%	35	20	45	**1.1**

$p = 0.0569,\ p > 0.05$, accept the dairy's claim

A majority of students obtained a correct *p*-value and correctly completed the question. Some students not continue to explicitly answer the question or state a correct conclusion.