2017 VCE Specialist Mathematics 1 examination report

General comments

In Specialist Mathematics examination 1 students were required to answer ten questions worth a total of 40 marks. Students were not permitted to bring technology or notes into the examination.

The clarity of students' responses and the manner in which they set out their working were of concern. Students should be reminded that if an assessor is not certain as to what a response is conveying or cannot follow a student's working, that assessor cannot award marks. Furthermore, students are expected to set out their work properly. Equals signs should be placed between terms that are equal – working should not appear to be a collection of disjointed statements. If there are inconsistencies in the student's working, full marks will not be awarded. For example, if an equals sign is placed between terms that are not equal, full marks will not be awarded.

It should also be emphasised that students should as a general practice consider the reasonableness of their answers. It was apparent that this was not the case in Question 4, in which several of the answers given were not feasible.

Areas of strength included:

- recognising the need to use the chain rule when differentiating implicitly (Question 1)
- recognising the need to use the product rule when differentiating implicitly (Question 1)
- knowing to separate variables in a differential equation (Question 8b.)
- showing a given result. This was required in Question 10a. In such questions, the onus is on students to include sufficient relevant working to demonstrate that they know how to derive the result. Students should be reminded that they can use a given value in the remaining part(s) of the question whether they were able to derive it or not.

Areas of weakness included:

- not reading the question carefully enough – this included not answering the question, proceeding further than required or not giving the answer in the specified form. These were common and particularly evident in Questions 1, 2, 7, 8b., 9a., 9b. and 10c. Many students would benefit from highlighting key words in the question. Students should also be reminded that good examination technique includes re-reading the question after it has been answered to ensure that they have answered what was required and that they have given their answer in the correct form
- algebraic skills. The inability to simplify expressions often prevented students from completing the question. Incorrect attempts to factorise, expand and simplify were common. Poor use of brackets was also common
- arithmetic skills. The inability to evaluate expressions, especially those involving fractions or surds, was common
- notation, especially the omission of the *dx* or equivalent in integration, and showing the dot in the dot product (Question 5)
- recognising the method of integration required (Questions 2, 7 and 10)
- knowing the exact values for circular functions (Questions 5, 7 and 10c.)
- recognising the correct form of partial fractions (Question 2)

- checking whether each of the answers found algebraically is valid by considering the original equation being solved (Question 10c.)

In this examination, students are expected to be able to apply techniques, routines and processes, involving rational, real and complex arithmetic, without the use of technology. Students are expected to be able to simplify simple arithmetic expressions. Many students found this difficult and missed out on marks as a consequence. For several questions, students who drew carefully labelled diagrams and graphs were advantaged over those who did not.

Many students made algebraic or numerical slips at the end of an answer so that full marks could not be awarded. This sometimes occurred when they had a correct answer and there was no need for further simplification.

There were several cases where incorrect working led to a correct answer. Students should be reminded that in such cases full marks will not be awarded if the answer is not supported by relevant and correct working.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Question 1

Marks	0	1	2	3	Average
%	13	18	11	57	2.1

$y = \frac{x}{2} - \frac{3}{2}$

This question was answered well by most students. Typical errors included not being able to use the product rule and/or chain rule on the first term, finding the correct derivative and not continuing to find the equation of the tangent. There were several arithmetic errors made that gave the derivative as $-\frac{1}{2}, -2 \text{ or } 2$. There were also notational errors. A few students found the equation of the normal instead. Some tried to make *x* or *y* the subject before differentiating, with these attempts usually leading to difficulties. Many realised that substitution could occur without isolating the derivative first, but in both cases some errors occurred in substituting numbers into their equation to find the gradient. A number of students gave the derivative of *x* to be zero. A few found the derivative in terms of *x* and *y* and used that in their equation.

Question 2

Marks	0	1	2	3	4	Average
%	38	11	9	7	35	1.9

$\log_e\left(\sqrt{\frac{3}{2}}\right)$

This question tended to be answered well by students who knew that partial fractions were required and which form of partial fractions to use. A large number of students did not use partial fractions, which meant that no progress could be made. Students gave answers such as $\log_e(x(1+x^2))$ and $\log_e(x \times \tan^{-1}(x))$ using this approach. Several students used partial fractions of the form $\frac{A}{x}+\frac{B}{1+x^2}$, often getting correct partial fractions with incorrect working, or $\frac{A}{x}+\frac{Bx}{1+x^2}$, which led to correct partial fractions since the value of *C* was zero. Some used a substitution such as $u=x^2$ or $u=1+x^2$, which led to an alternative partial fractions form that was sometimes handled successfully but often terminals were not adjusted. Occasionally $x=\tan(u)$ was used but this was rarely followed through correctly. A number of students found the correct antiderivative but made errors in final arithmetic simplification work, which frequently gave the incorrect answer $\log_e\left(\sqrt{\frac{3}{4}}\right)$.

Others did not put the answer in the correct form, often giving $\log_e\left(\frac{\sqrt{6}}{2}\right)$.

Question 3

Marks	0	1	2	3	Average
%	8	23	27	43	**2.1**

$1+i,\ 2$

Students generally performed well on this question, with most students able to obtain at least two marks. Typical errors included:

- giving a second solution as –1 – *i*
- correctly giving 1 + *i* as a second solution then multiplying this by the given solution to get 2 and stating 2 as the third solution, which was a correct answer but incorrect reasoning
- not being able to correctly determine *G*. Students could correctly find *a* = –4 but were unable to get the third solution.

A small number of students expressed the real solution in terms of *a*. Some students quoted the answer as factors rather than solutions. Those who attempted to use polar form were unsuccessful.

Question 4

Marks	0	1	2	3	Average
%	42	9	12	37	**1.5**

0.025 or 0.023

This question was answered well by students who found the standard deviation of the sample, but many used the standard deviation of the population. Students' notation was often not clear and did not distinguish between the standard deviation of *X* and *the standard deviation of* $\bar{X}$. Some arithmetic errors were made when dividing by $\frac{3}{2}$. Other typical errors included:

- not working with the mean leading to finding Pr(*X* < 295)
- using the total volume and taking the standard deviation to be 12 rather than 6
- working with the mean but using $\frac{3}{4}$ as the standard deviation
- finding the probability that the mean was greater than 295
- finding *z* = +2 by incorrect standardisation using $\Pr\left(Z<\frac{298-295}{1.5}\right)$
- an inability to obtain the value of Pr(*Z* < –2) due to arithmetic mistakes.

Some used the 68% or 99.7% approximation instead of 95%, while others made attempts to find a confidence interval. Others used the 0.16 as a *p*-value in a binomial distribution.

Question 5

Marks	0	1	2	3	4	Average
%	25	5	45	13	11	1.8

$a = -2$

A broad spread of levels of achievement was seen for this question. Most students were able to make some progress but many had some difficulties. The majority knew that they needed to find to vectors involving *C* and attempt to use the dot product to find the unknown, though some algebra when finding the dot product was poor. The most common errors involved finding the dot product of two (or sometimes all three) of the given vectors, not understanding that when finding the angle between vectors they need to be tail to tail and therefore working with $\overrightarrow{BC}$ and $\overrightarrow{CD}$. Some used the correct application of the dot product or cosine rule but poor algebra led to an incorrect equation for a, others correctly found a = ±2 from the surd equation but did not eliminate a = 2 or incorrectly eliminated a = –2. Many students did not know their exact values. Notation was often poor, with students not showing the dot or using another symbol. A large number of students struggled with the algebra. A number of students incorrectly solved $x^2 = 4$ to get $x = \pm\sqrt{2}$ or similar.

Question 6

Marks	0	1	2	3	Average
%	33	30	19	18	1.2

$f'(x) = \dfrac{-1}{(\arcsin(x))^2\sqrt{1-x^2}}$, $(-1,0)\cup(0,1)$

This question was not answered well. Some students confused the inverse function with the reciprocal function. The most common incorrect derivatives were $\sqrt{1-x^2}$ and $\dfrac{\log_e(\sin^{-1} x)}{\sqrt{1-x^2}}$, while some had $\dfrac{d(\arcsin x)}{dx} = \log_e(\arcsin x)$. Common errors for the domain included *R*, *R* \ {–1, 0, 1}, [–1, 1], (–1, 1) and [–1, 1] \ {0}. Many students did not exclude zero. The incorrect answer $(-\infty,0)\cup(0,\infty)$ was also relatively common.

Question 7

Marks	0	1	2	3	4	Average
%	27	16	18	9	30	2

$\dfrac{3}{4}$

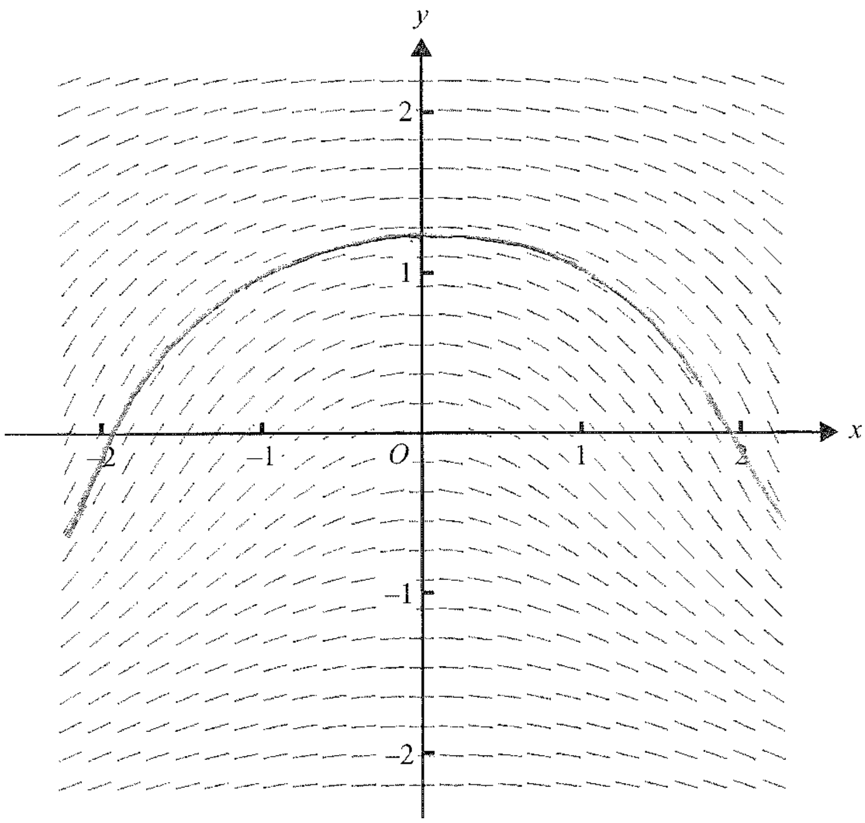
Students had varied success with this question. A number of students were unable to find the necessary derivatives, neglecting to use the chain rule. Of those who did use the chain rule, the question was reasonably well answered, although there were many who did not recognise the appropriate form of the arc length formula. Some incorrect answers involved:

- finding $\left|r\dfrac{\pi}{4} - r(0)\right|$
- using the formula with *dy*/*dx* (sometimes with correct working, except for using *dt* rather than *dx*)
- errors in derivatives
- an inability to correctly simplify the expression under the square root

- taking the square root of individual terms
- correct simplification but an error at the end with terminals or substitution and missing *dt* in lines of working.

Question 8a.

Marks	0	1	2	Average
%	54	29	17	0.7



$1.7 \le x \le 1.9$

This question was not answered well. Several curves crossed the slope ticks rather than following them. Errors included:

- the final curve not being symmetrical
- the curve not passing through (–1, 1), giving the value for x as around 1.2 (the value of the *y* intercept)
- finding an approximate value from the solution in part b. even though this was inconsistent with the student's graph (part a. used the word 'hence').

Many graphs were almost flat between $x = -0.5$ and $x = 0.5$, resulting in missing the desired *y*-intercept. Some drew the graph just to the x-intercepts rather than for the whole domain. Several graphs were not drawn smoothly with sufficient care.

Question 8b.

Marks	0	1	2	Average
%	15	49	36	1.2

$2y^3 + 6y + 3x^2 - 11 = 0$

This question was answered reasonably well. Most students were able to separate the variables (though some algebraic errors occurred) but several arrived at an incorrect value of the constant of integration *c*, of which $\frac{5}{6}$ was most common. Most students had the correct integration after separating variables but made no attempt to express the answer with integers as required. Some students who attempted to express the answer in the form requested made arithmetic errors, finishing with +11 on the left side.

Question 9a.

Marks	0	1	2	Average
%	37	18	45	1.1

$|\underset{\sim}{F}| = \sqrt{80} = 4\sqrt{5}$

Most students were able to make a reasonable attempt at this question. Some of the typical errors included leaving the force as a vector and not finding its magnitude, using velocity rather than acceleration, and an inability to find the acceleration in vector form first, usually leading to an incorrect value for the magnitude of the acceleration. There were several errors with the negatives when subtracting vectors. A small number of students tried to average the velocities.

Question 9b.

Marks	0	1	2	3	Average
%	53	9	8	30	1.2

$230\underset{\sim}{i} - 80\underset{\sim}{j}$

The most common error was using scalars throughout (finding distance rather than displacement). Other errors included attempting to use one of the constant acceleration formulas without vectors, poor recall of the constant acceleration formulas where attempts were made to use these, integrating the acceleration vector but either without a constant of integration or getting an incorrect value for the constant. Some students attempted to square vectors.

Question 10a.

Marks	0	1	Average
%	15	85	0.9

Answer given

The majority of students answered this question well. A number of students showed insufficient working to enable the mark to be awarded; some students simply wrote the answer as given. Some students gave a different answer from that given.

Question 10b.

Marks	0	1	2	Average
%	35	25	40	1.1

Dom: $x \in [-2, 2]$ Ran: $y \in \left[0, \sqrt{\pi}\right]$

This question was not particularly well answered. The most common errors were to state the domain as (–2, 2) or [0, 2] or other variations; the range was frequently given as [0, π].

Question 10c.

Marks	0	1	2	3	4	Average
%	38	21	19	8	14	1.4

$2\pi^2$

Many students found this question quite challenging. Typical errors included:

- trying to find the area rather than the volume of revolution
- forgetting the π
- sign errors when attempting to use the result given in part a.
- integrating from –2 to 0 was common or from –2 to π, which was less common.

There were many poor attempts to integrate $\dfrac{x}{\sqrt{4-x^2}}$ where arcsin expressions and incorrect constants or incorrect signs were common. Some students attempted to integrate the correct integral expression by turning it into the integration of a cos function, finding the area to the *y*-axis and subtracting from the surrounding rectangle. This was occasionally done successfully.