

2017 Trial Examination

STUDENT
NUMBER

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Letter

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SPECIALIST MATHEMATICS

Units 3&4 – Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION & ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.
- No calculator is permitted in this examination.

Materials supplied

- Question and answer book of 13 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 (2 marks)

α is an angle with $\cos\left(\alpha - \frac{\pi}{4}\right) = \frac{4\sqrt{2}}{7}$. Express $\sin(2\alpha) = \frac{p}{q}$, where p and q are integers.

TURN OVER

Question 2 (5 marks)

Let $P(z) = z^5 + 4z^4 + 5z^3 + 8z^2 + 32z + 40$.

a. Show that $(z^3 + 8)$ is a factor of $P(z)$.

1 mark

b. Find all the solutions to $P(z) = 0$ in Cartesian form.

4 marks

Question 3 (3 marks)

Two farms A and B produce eggs. The weight of the eggs produced by Farm A has a mean of 70 grams and with variance of 4 grams squared. The weight of the eggs produced by Farm B has a mean of 60 grams and with variance of 3 grams squared.

4 eggs are randomly selected from Farm A and 3 eggs are randomly selected from Farm B.

a. Find the expected value of the weight of the 7 eggs.

1 mark

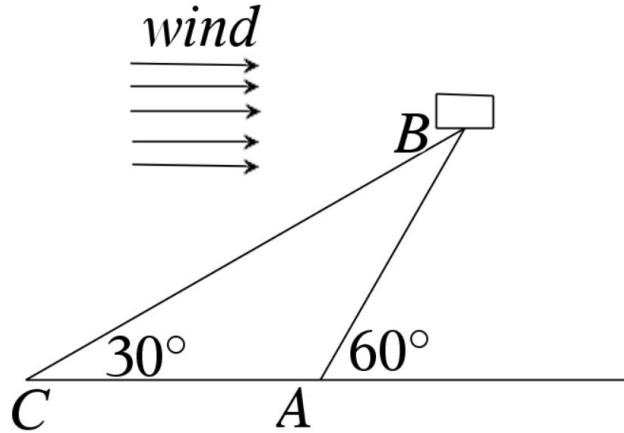
b. Find the standard deviation of the weight of the 7 eggs.

2 marks

TURN OVER

Question 4 (4 marks)

An advertisement board of 30 kg is supported by a beam AB which has an angle of 60° with the ground. The advertisement board is also held by a string BC which has an angle of 30° with the ground. The advertisement board receives a force of $10\sqrt{3}g$ newtons from a westerly wind.



- a. Label all the forces acting on the board in the diagram.

1 mark

- b. Find the supporting force from the beam AB and the tension force in the string BC.

3 marks

Question 5 (4 marks)

Points A, B and C are the vertices of a triangle. Let $\vec{AB} = \vec{b}$ and $\vec{AC} = \vec{c}$.

a. Show that the area of the triangle ΔABC is equal to $\frac{1}{2} \sqrt{|\vec{b}|^2 |\vec{c}|^2 - (\vec{b} \cdot \vec{c})^2}$.

4 marks

b. The coordinates of A, B and C are given as $A(1, 1)$, $B(3, 3)$ and $C(5, -1)$ respectively. Find the area of ΔABC .

2 marks

TURN OVER

Question 6 (4 marks)

Find the equation of the normal of the graph defined by the equation

$$y^4 + xy^2 - 6x^2 + 4 = 0$$

at the point (1, 1).

Question 7 (4 marks)

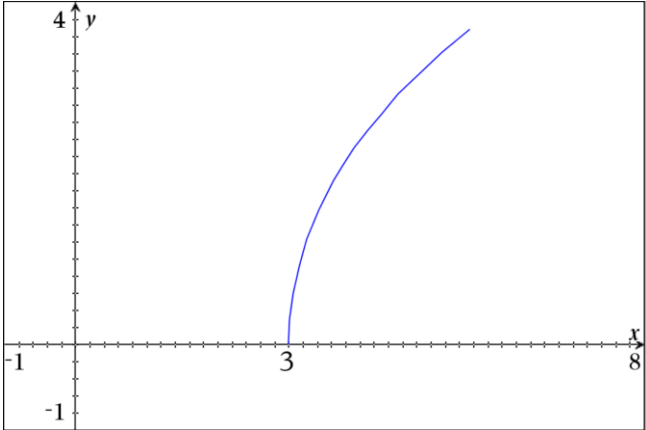
Solve the differential equation:

$$(2x + 1) \frac{dy}{dx} - \frac{4xy}{2x + 3} = 0$$

expressing y in terms of x , given that $y = 1$ when $x = -1$.

TURN OVER

Question 8 (5 marks)



The equation of the arc shown above is given by

$$\begin{cases} x = 3 \sec(t) \\ y = 3 \ln\left(\frac{\cos(t)}{1 - \sin(t)}\right) \end{cases} , \quad 0 \leq t \leq \frac{\pi}{3}$$

a. Show that the equation of the tangent to the curve at $t = \frac{\pi}{4}$ is

$$y = x - 3\sqrt{2} + 3 \ln(1 + \sqrt{2})$$

3 marks

b. Find the length of the arc for $0 \leq t \leq \frac{\pi}{3}$.

2 marks

TURN OVER

Question 9 (4 marks)

A sphere-shaped air balloon is getting flat at the rate of $9r^{\frac{4}{3}} \text{ cm}^3/\text{s}$, where r , in cm, is the radius of the balloon t seconds after starting leaking. Find the time it takes for the radius to be reduced from 27 cm to 8 cm.

Question 10 (4 marks)

A body starts to move from rest in a line with acceleration of $\frac{v(v^2+3)}{36}$ m/s², where v is the speed after t seconds. Find the distance travelled by the body when $v = 3$ m/s.

END OF QUESTION AND ANSWER BOOK