Neap:

Trial Examination 2017

VCE Specialist Mathematics Units 3&4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 22 pages.

Formula sheet.

Answer sheet for multiple-choice questions.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page, and on your answer sheet for multiple-choice questions.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

Place the answer sheet for multiple-choice questions inside the front cover of this booklet.

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2017 VCE Specialist Mathematics Units 3&4 Written Examination 2.

SECTION A – MULTIPLE-CHOICE QUESTIONS

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

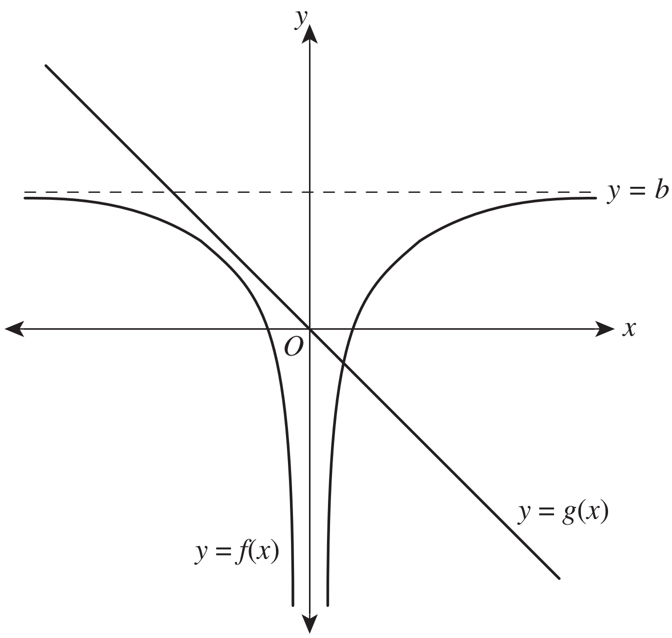
No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

The graphs of $y = f(x)$ and $y = g(x)$ are shown below.



The straight line asymptote(s) of the graph of $y = f(x) + g(x)$ is given by

A. $x = 0$ and $y = g(x) + b$

B. $x = 0$, $y = b$ and $y = g(x) + b$

C. $y = g(x) + b$ only

D. $x = 0$ and $y = g(x)$

E. $x = 0$ and $y = b$

Question 2

The implied domain of $y = -\sec(x + 2) - 3$, where k is an integer, is

A. $R\backslash\{k\pi - 2\}$

B. $R\backslash\left\{(k-1)\frac{\pi}{2} - 2\right\}$

C. $R\backslash\left\{(k-1)\frac{\pi}{2} - 3\right\}$

D. $R\backslash\left\{(2k-1)\frac{\pi}{2} - 2\right\}$

E. $R\backslash\left\{(2k-1)\frac{\pi}{2} - 3\right\}$

Question 3

The function f is defined by $f(x) = |2x\arccos(x)|,\ -1 \le x \le 1$.

Correct to three decimal places, the set of values for x such that $f(x) > 1$ is given by

A. $x < -0.271$ or $0.455 < x < 0.820$

B. $-1 \le x < -0.271$ or $0.455 < x < 0.820$

C. $0.455 \le x \le 6.283$ only

D. $0.455 < x < 0.820$ only

E. $x < -0.271$ only

Question 4

If $\sqrt{x + yi} = a + bi$ where $a, b \in R$ and $x, y \in R$, then

A. $x = a^2 - b^2$ and $y = ab$

B. $x = a^2 + b^2$ and $y = 2ab$

C. $x = a^2 + b^2$ and $y = ab$

D. $x = a^2 - b^2$ and $y = 2ab$

E. $x = a - b$ and $y = 2ab$

Question 5

If $z \in C$, where $\text{Re}(z) > 0$ and $\text{Arg}\left(\frac{z}{\overline{z}}\right) = \frac{\pi}{3}$, then Arg($z$) is equal to

A. 0

B. $-\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{3}$

E. $\frac{\pi}{6}$

Question 6

Which one of the following relations represents a graph of a straight line that passes through the origin?

A. $\text{Re}(z) + \text{Im}(z) = 1$

B. $z + \overline{z} = 1$

C. $\text{Re}(z) - \text{Im}(z) = 1$

D. $z\overline{z} = 1$

E. $\text{Re}(z)\text{Im}(z) = 1$

Question 7

A curve C is defined by the equation $x^2 - 4xy + 2y^2 = -2$.

Equations of all tangents that are parallel to the x-axis will satisfy the condition

A. $y = 2x$

B. $y = x$

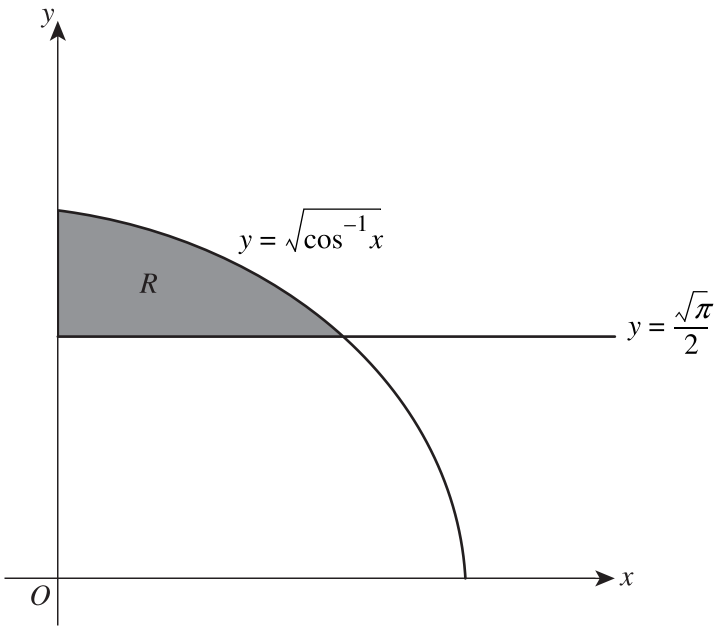
C. $y = \frac{x}{2}$

D. $y = -\frac{x}{2}$

E. $y = -x$

Question 8

The diagram below shows the curve with equation $y = \sqrt{\cos^{-1}(x)}$. The region in the first quadrant bounded by the curve, the line $y = \frac{\sqrt{\pi}}{2}$ and the y-axis is denoted by R.



The volume of revolution when R is rotated 360° about the x-axis is given by

A. $\pi\int_0^{\frac{1}{\sqrt{2}}}\left(\sqrt{\cos^{-1}(x)} - \frac{\sqrt{\pi}}{2}\right)^2 dx$

B. $\pi\int_0^{\frac{1}{2}} \cos^{-1}(x)dx - \frac{\pi^2}{8}$

C. $\int_0^{\frac{1}{\sqrt{2}}} \sqrt{\cos^{-1}(x)}dx - \frac{\sqrt{\pi}}{2\sqrt{2}}$

D. $\pi\int_0^{\frac{1}{\sqrt{2}}} \cos^{-1}(x)dx - \frac{\pi^2}{4\sqrt{2}}$

E. $\pi\int_0^{\frac{1}{\sqrt{2}}} \cos^{-1}(x)dx - \frac{\pi}{4\sqrt{2}}$

Question 9

The radius of a circle is decreasing at a constant rate of 0.2 metres per second.

In terms of the circumference C, the rate of change of the circle's area, A, in square metres per second, is

A. $0.04\pi C$

B. $0.04C$

C. $-\frac{0.1C}{\pi}$

D. $-0.2C$

E. $-0.4\pi C$

Question 10

The length of the path described by the parametric equations $x = \sin^3(t)$ and $y = \cos^3(t)$, for $0 \le t \le \frac{\pi}{6}$, is given by

A. $\int_0^{\frac{\pi}{6}} \sqrt{\sin^6(t) + \cos^6(t)}\,dt$

B. $\int_0^{\frac{\pi}{6}} \sqrt{3\sin^2(t)\cos(t) - 3\cos^2(t)\sin(t)}\,dt$

C. $\int_0^{\frac{\pi}{6}} \sqrt{3\sin^2(t) + 3\cos^2(t)}\,dt$

D. $\int_0^{\frac{\pi}{6}} \sqrt{\sin^3(t) + \cos^3(t)}\,dt$

E. $\int_0^{\frac{\pi}{6}} \sqrt{9\sin^4(t)\cos^2(t) + 9\cos^4(t)\sin^2(t)}\,dt$

Question 11

The direction field for the differential equation $\frac{dy}{dx} = \frac{2x^2y + y^2}{2x + y}$ will have vertical segments when

A. $y = 2x$ only

B. $y = -2x$ only

C. $y = -2x^2$ only

D. $y = 0$ only

E. $y = 0$ or $y = -2x^2$

Question 12

Let θ be the angle between the vectors $\underset{\sim}{i} + \underset{\sim}{j} + p\underset{\sim}{k}$ and $2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$.

Given that $\cos(\theta) = \frac{1}{3}$, the value of the constant p is

A. $\frac{9 - \sqrt{10}}{2}$

B. $-\frac{7}{4}$

C. $\frac{\sqrt{10} - 9}{2}$

D. 2

E. $\frac{7}{4}$

Question 13

The velocity vector of a particle at time t seconds, is given by $\underset{\sim}{v}(t) = 4e^{-t}\underset{\sim}{i} + \sin(1 + t)\underset{\sim}{j}$, $t \geq 0$. The particle's distance from a fixed origin O is measured in metres. The particle is initially at O.

Which one of the following statements is correct?

A. Correct to three decimal places, the total distance travelled by the particle between $t = 1$ and $t = 3$ is 1.642 metres.

B. The particle's initial velocity vector is $\sin(1)\underset{\sim}{j}$ m/s.

C. The particle's initial speed is $\sqrt{16e + (\sin(1))^2}$ m/s.

D. The particle's acceleration vector at $t = 1$ is $\underset{\sim}{a}(t) = -\frac{4}{e}\underset{\sim}{i} - \cos(2)\underset{\sim}{j}$ m/s^2.

E. The particle's initial acceleration vector is $4\underset{\sim}{i} - \cos(1)\underset{\sim}{j}$ m/s^2.

Question 14

In triangle AOB, $\underset{\sim}{a} = \overrightarrow{OA}$, $\underset{\sim}{b} = \overrightarrow{OB}$ and M is the midpoint of AB.

$OA^2 + OB^2$ is equal to

A. $2OM^2 + 2AM^2$

B. $OM^2 + AM^2$

C. $2OM^2 - 2AM^2$

D. $OM^2 - AM^2$

E. $2OM + 2AM$

Question 15

Two stones, A and B, are projected simultaneously from a vertical cliff. Stone A is projected horizontally from the top of the cliff with speed 28 m/s. Stone B is projected from the bottom of the cliff with speed 35 m/s at an angle α above the horizontal. The stones move freely under gravity in the same vertical plane and collide in mid-air.

By considering the horizontal motion of each stone, α is equal to

A. $\sin^{-1}\left(\frac{4}{5}\right)$

B. $\cos^{-1}\left(\frac{4}{5}\right)$

C. $\frac{\pi}{4}$

D. $\tan^{-1}\left(\frac{3}{5}\right)$

E. $\frac{\pi}{6}$

Question 16

A particle of mass 2 kg starts from rest at a fixed origin O and moves in a straight line with velocity v m/s under the action of a variable force, F newtons, where $F = 2 - 8v^2$. The displacement of the particle from O at time t seconds is x metres.

It follows that

A. $F = 2e^{-16x}$

B. $F = 2(2 + e^{-8x})$

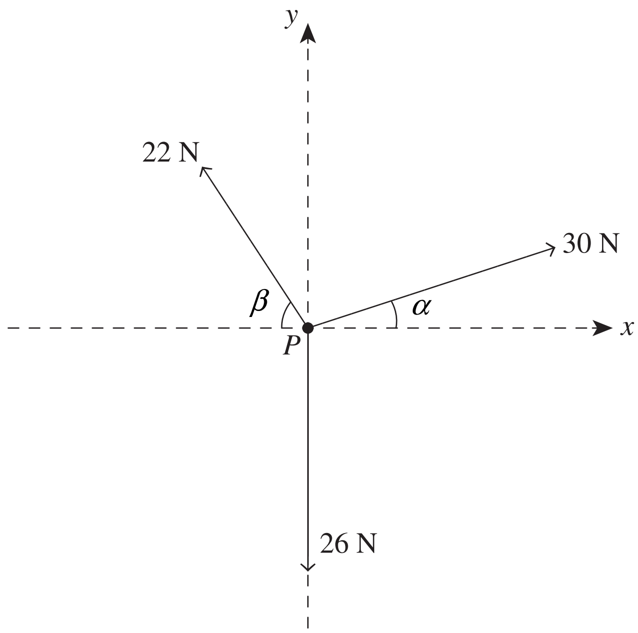
C. $F = 2e^{-8x}$

D. $F = -2e^{-8x}$

E. $F = -2e^{-16x}$

Question 17

Three horizontal forces of magnitude 30 N, 22 N and 26 N respectively act on a particle P in the directions shown in the diagram below. The angles α and β are such that $\sin(\alpha) = \frac{7}{25}$, $\cos(\alpha) = \frac{24}{25}$, $\sin(\beta) = \frac{4}{5}$ and $\cos(\beta) = \frac{3}{5}$.



The magnitude, in newtons, of the resultant of the three forces is

A. 15.6

B. 0

C. 52.6

D. 78

E. 18.5

Question 18

Which one of the following scenarios would require a one-tailed hypothesis test?

A. A test to determine whether the mean weight of a species of lizard is 1.5 kg.

B. A test to determine whether a new diet promotes weight loss.

C. A test to determine whether the number of visitors per day to an art gallery has changed recently due to the opening of a new exhibition.

D. A test to determine whether the mean lifetime of a battery has changed as a result of a modification in the manufacturing process.

E. A test to determine whether taking a new anti-snoring drug changes the mean amount of sleep an adult gets per night.

Question 19

The lengths of cucumbers are normally distributed with an average length of 21 cm and a standard deviation of 2.5 cm. A random sample of 20 cucumbers is selected.

Correct to four decimal places, the probability that the average length of this sample of 20 cucumbers exceeds 19.8 cm is

A. 0.0159
B. 0.1550
C. 0.6844
D. 0.8450
E. 0.9841

Question 20

Each training day a swimming coach measures the time taken for a swimmer to complete a 25-metre swim. The measurements obtained can be assumed to follow a normal distribution with mean μ seconds and standard deviation 0.8 seconds. After ten training days, the coach takes each independent measurement of the swimmer's time and correctly calculates a confidence interval of (12.4,13.3) for μ.

The confidence level of this interval, correct to one decimal place, is

A. 87.5%
B. 90.0%
C. 92.5%
D. 95.0%
E. 99.0%

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1 (12 marks)

The function f is defined by $f(x) = \dfrac{x^3 - a^3}{x^2 - 4}$ where $a > 2$. The graph of $y = f(x)$ has two vertical asymptotes and a non-vertical asymptote.

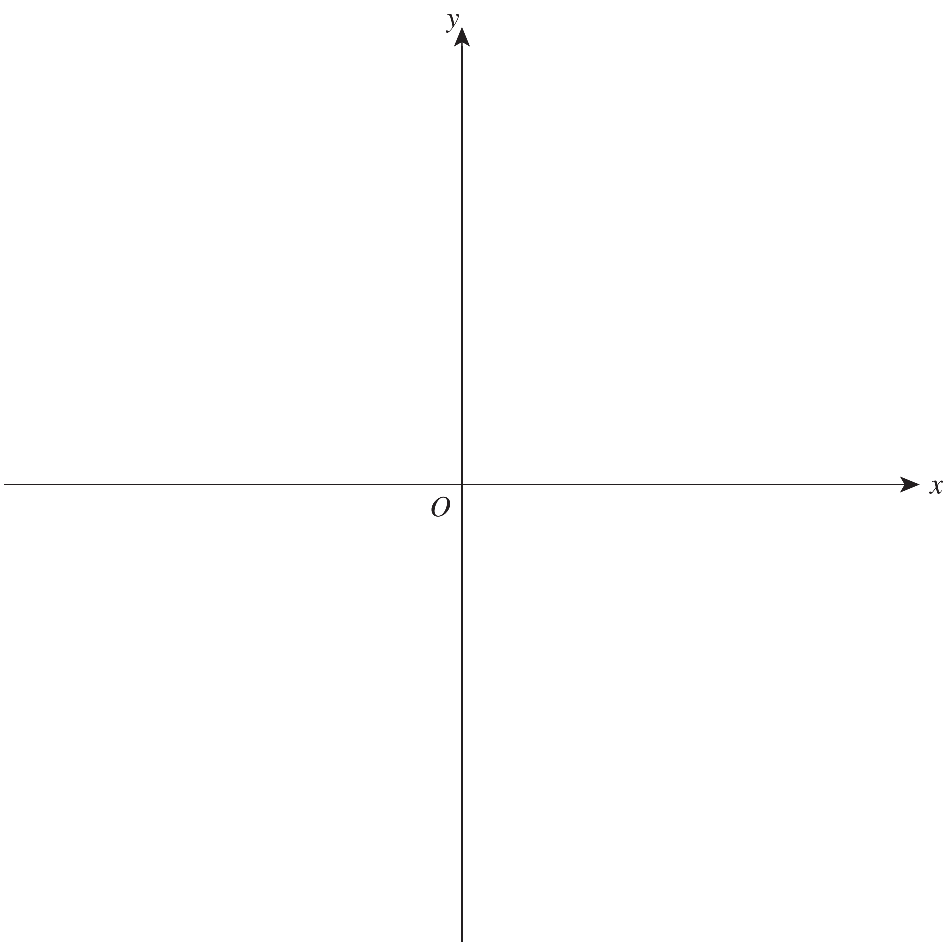
a. Find the equations of the three asymptotes. 3 marks

b. **i.** Show that the graph of f has a stationary point at $x = 0$. 2 marks

ii. Find $f''(0)$ and hence justify the nature of the stationary point at $x = 0$. 2 marks

c. Show algebraically that the graph of $y = f(x)$ crosses the non-vertical asymptote. 2 marks

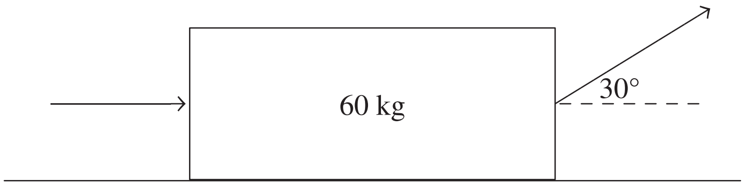
d. Sketch the graph of $y = f(x)$ on the axes below, showing the asymptotes with their equations, the coordinates of the stationary point at $x = 0$ and the coordinates of any axes intercepts. Do **not** attempt to determine the coordinates of any other stationary points. 3 marks



SMU34EX2_QA_2017.FM

Question 2 (9 marks)

Adrian and Peter are trying to slide a box of mass 60 kg along a rough, horizontal floor. Adrian pushes the box horizontally and Peter uses a light inextensible rope inclined at an angle of 30° to the horizontal to pull the box. Both Adrian and Peter apply forces to the box in the same vertical plane.



When Adrian applies a force of 75 N and Peter applies a force of 90 N, the box remains at rest.

a. Find, correct to the nearest newton, the magnitude of the resistance to motion of the box. 2 marks

b. Find the magnitude of the normal reaction of the floor on the box. 2 marks

When Adrian pushes the box with a force of 180 N and Peter does not apply a force to the box, the frictional resistance to the box sliding is 150 N. Adrian stops pushing the box when it has acquired a speed of 1.5 m/s.

c. From the instant Adrian stops pushing the box, find the distance travelled by the box before it comes to rest. 3 marks

Adrian and Peter make another attempt to slide the box along the floor. Adrian now pushes the box with a force of 190 N and Peter applies a force of P N. The frictional resistance to the box sliding is now 145 N and the acceleration of the box is 3 m/s^2.

d. Find the value of P. Give your answer correct to the nearest newton. 2 marks

Question 3 (12 marks)

a. Verify that $z = \cos(\theta) + i\sin(\theta)$ is a solution to the equation $z^2 - 2\cos(\theta)z + 1 = 0$. 3 marks

b. Use de Moivre's theorem to show that $z^n + \frac{1}{z^n} = 2\cos(n\theta)$. 2 marks

c. Show that $8\cos^3(\theta) + 4\cos^2(\theta) - 4\cos(\theta) - 2 = \left(z + \frac{1}{z}\right) + \left(z^2 + \frac{1}{z^2}\right) + \left(z^3 + \frac{1}{z^3}\right)$. 3 marks

d. Hence, or otherwise, solve the equation $\cos(\theta) + \cos(2\theta) + \cos(3\theta) = 0,\ 0 \le \theta \le 2\pi$.
Give your answers in exact form. 4 marks

Question 4 (9 marks)

Suppose that 50 rabbits are introduced to a small island which was previously free of rabbits. The rabbits have no natural enemies on the island but due to food constraints it is believed that the island can only support a population of 2000. The rate of growth of the rabbit population at *t* months (after their initial introduction) may be assumed to jointly vary with the number of rabbits *N* and the 'amount of room' (2000 – *N*) available for additional rabbits. (In this question, take 1 month to be exactly 30 days.)

a. If the initial rate of increase of the population is 25 rabbits per month, show that $\frac{dN}{dt} = \frac{N}{3900}(2000 - N)$. 2 marks

b. According to this model, determine to the nearest day when the population has reached 75% of its theoretical maximum. 4 marks

c. According to this model, determine to the nearest day when the population is increasing most rapidly. 3 marks

Question 5 (6 marks)

A taxi company wishes to study waiting times for customers at two regional taxi ranks. The waiting times, in minutes, at taxi rank *A* and taxi rank *B* follow independent normal distributions with means and standard deviations as shown in the following table.

	Mean waiting time (mins)	**Standard deviation (mins)**
Taxi rank *A*	12	2.7
Taxi rank *B*	20	4.4

a. 5 customers are randomly selected and their waiting times recorded. 3 customers waited at taxi rank *A* and 2 customers waited at taxi rank *B*.

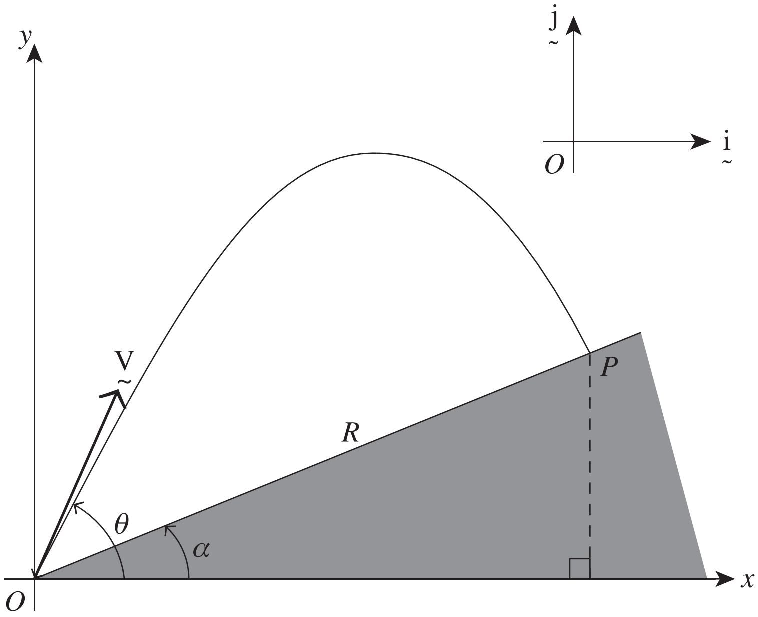
Find the probability that the mean waiting time for the 5 customers is less than 14 minutes. Give your answer correct to three decimal places. 3 marks

b. Christine arrives at taxi rank *A* at the same time that Daisy arrives at taxi rank *B*. Both catch a taxi from their respective taxi ranks.

Find the probability that they catch a taxi within 5 minutes of each other. Give your answer correct to three decimal places. 3 marks

Question 6 (12 marks)

The diagram below shows an inclined plane that makes an angle α with the horizontal. A projectile is fired from O at the bottom of the incline with velocity $V\cos(\theta)\underset{\sim}{i} + V\sin(\theta)\underset{\sim}{j}$ m/s at an angle θ degrees to the horizontal, where $0° < \theta < 90°$. The projectile hits the inclined plane at P. The acceleration of the projectile moving in the x-y plane is $-g\underset{\sim}{j}$. Let (x, y) be the projectile's position after t seconds of flight.



The cartesian equation describing the projectile's path is $y = \tan(\theta)x - \frac{g}{2V^2}(1 + \tan^2(\theta))x^2$.

a. Show that OP lies on the line with equation $y = \tan(\alpha)x$. 1 mark

b. Show that the projectile's range R metres up the inclined plane is given by

$$R = \frac{2V^2 \sin(\theta - \alpha)\cos(\theta)}{g\cos^2(\alpha)}.$$

3 marks

c. **i.** Find an expression for θ such that $\frac{dR}{d\theta} = 0$.

3 marks

ii. Find $\frac{d^2R}{d\theta^2}$ and hence verify that the expression for θ found in **part c.i.** gives the projectile's maximum range up the inclined plane. 3 marks

d. For the trajectory of maximum range, use the result that $\frac{dy}{dx} = \frac{-(\cos(2\theta) + 1)}{\sin(2\theta)}$ to show that the initial direction of the particle's trajectory is perpendicular to the direction at which the projectile hits the inclined plane. 2 marks

END OF QUESTION AND ANSWER BOOKLET