Neap:

Trial Examination 2017

VCE Specialist Mathematics Units 3&4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.

Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

Question and answer booklet of 9 pages.

Formula sheet.

Working space is provided throughout the booklet.

Instructions

Write your **name** and your **teacher's name** in the space provided above on this page.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

All written responses must be in English.

At the end of the examination

You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2017 VCE Specialist Mathematics Units 3&4 Written Examination 1.

Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

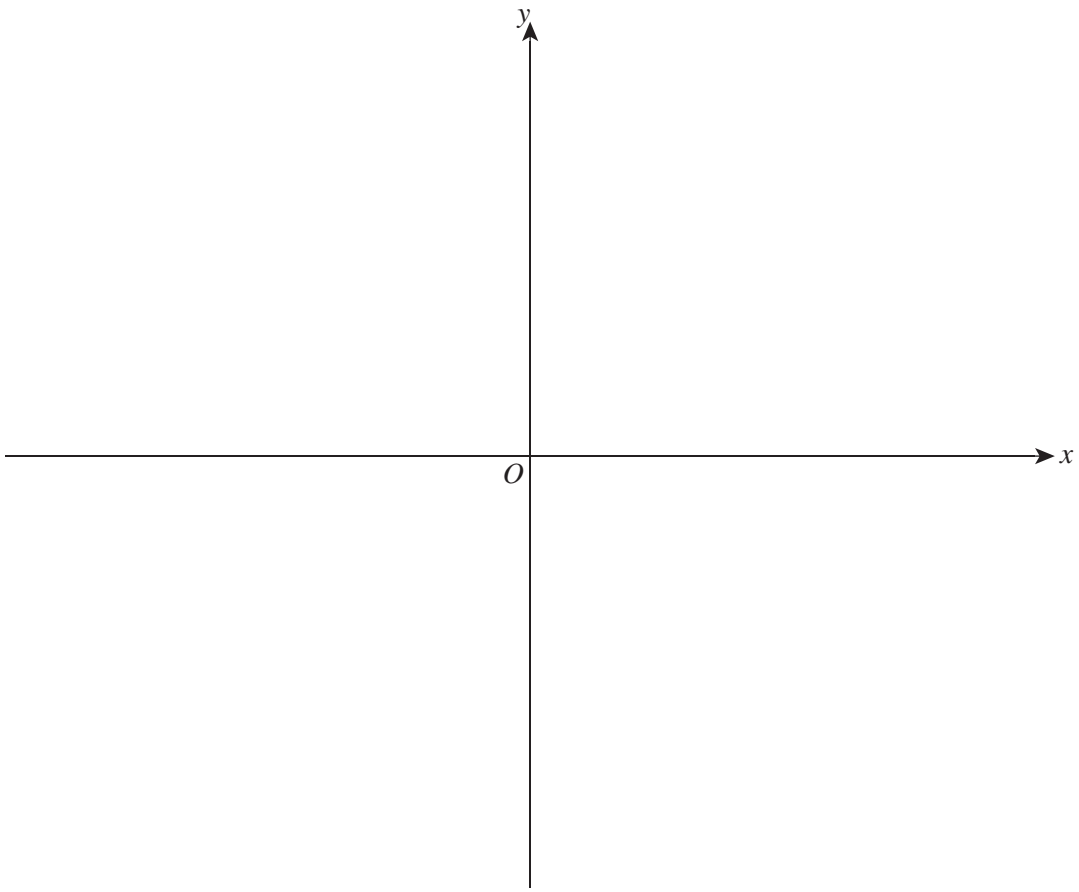
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^2, where $g = 9.8$.

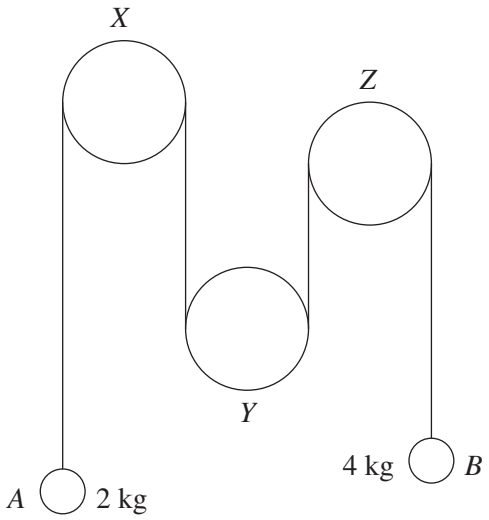
Question 1 (3 marks)

Sketch the graph of $y = 2 - \tan^{-1}(x)$ on the set of axes below. Label any asymptotes with their equations and label any intercepts with the axes, expressing them as coordinates.



Question 2 (3 marks)

Two particles of mass 2 kg and 4 kg are attached to the ends A and B of a light, inextensible string. The string passes around fixed smooth pulleys at X, Y and Z. The system is released from rest with the string taut.



The tension in the string is T N and the magnitude of the acceleration of the particles is a ms^{-2}.

Find the values of T and a.

Question 3 (4 marks)

a. Show that $\frac{1+\tan^2(\theta)}{1-\tan^2(\theta)} = \sec(2\theta)$. 2 marks

b. Hence, or otherwise, solve the equation $\frac{1+\tan^2(\theta)}{1-\tan^2(\theta)} = 2, 0 \leq \theta \leq \pi$. 2 marks

Question 4 (4 marks)

The study scores of a large population of VCE Specialist Mathematics students have a mean of 30 and a standard deviation of 7. A random sample of 25 students is to be selected from the population. The sample mean is denoted by $\overline{X}$.

a. State $E(\overline{X})$. 1 mark

b. Find $sd(\overline{X})$. 1 mark

The sample size is now increased by a factor of 2.

c. Determine how $E(\overline{X})$ and $sd(\overline{X})$ would change as a result of this increase in sample size. 2 marks

Question 5 (3 marks)

Let $z = x + yi$ and $w = a + bi$.

Show that the equation $\overline{w}z + w\overline{z} = k$ where $k \in R$ represents a straight line in the complex plane.

Question 6 (5 marks)

a. Use a suitable substitution to show that $\int_1^p \frac{1}{1+x^2}dx = \int_{\frac{1}{p}}^1 \frac{1}{1+u^2}du$ where $p > 1$. 3 marks

b. Hence show that $\arctan(p) + \arctan\left(\frac{1}{p}\right) = \frac{\pi}{2}$. 2 marks

Question 7 (6 marks)

Consider the polynomial $P(z) = 2z^4 + biz^3 + 2z + bi$, $z \in C$ and $b \in R$.

a. Given that $P\left(\frac{i}{2}\right) = 0$, show that $b = -1$. 2 marks

b. Solve the equation $P(z) = 0$, expressing your answers in the form $r\text{cis}(\theta)$ where $r > 0$ and $-\pi < \theta \leq \pi$. 4 marks

Question 8 (4 marks)

Find $\int \frac{x^2 + x}{(x^2 + 2)(x - 2)} dx$.

Question 9 (3 marks)

Relative to an origin O, the position vectors of the points A and B are $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively where $\underset{\sim}{a} = a_1\underset{\sim}{i} + a_2\underset{\sim}{j} + a_3\underset{\sim}{k}$ and $\underset{\sim}{b} = b_1\underset{\sim}{i} + b_2\underset{\sim}{j} + b_3\underset{\sim}{k}$.

Show that $(a_1b_1 + a_2b_2 + a_3b_3)^2 \leq (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$.

Question 10 (5 marks)

The curve C is defined by the parametric equations $x = \frac{1 - t^2}{1 + t^2}$, $y = \frac{4t}{1 + t^2}$, $0 \le t \le 1$.

Find the equation of the tangent to C where $t = \frac{1}{2}$.

END OF QUESTION AND ANSWER BOOKLET