Neap:

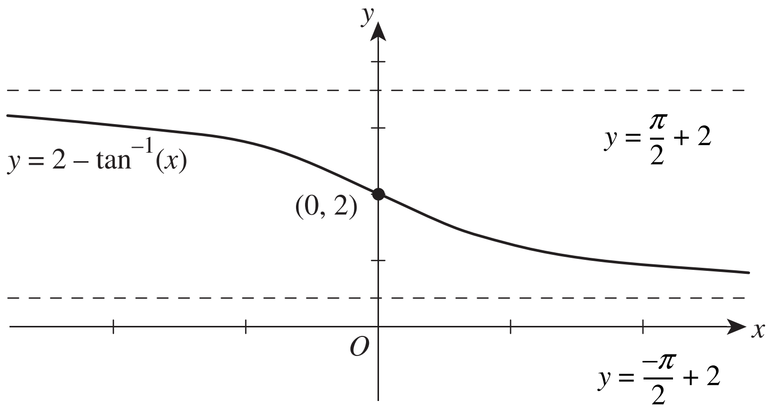
Trial Examination 2017

VCE Specialist Mathematics Units 3&4

Written Examination 1

Suggested Solutions

Question 1 (3 marks)



correct shape A1

The y-coordinate is (0, 2). A1

The horizontal asymptotes are $y = \pm\frac{\pi}{2} + 2$. A1

Question 2 (3 marks)

The equations of motion are $T - 2g = 2a$ and $4g - T = 4a$. A1

Attempting to solve for a and T, M1

$a = \frac{g}{3}$ and $T = \frac{8g}{3}$. A1

Question 3 (4 marks)

a.

$$\frac{1 + \tan^2(\theta)}{1 - \tan^2(\theta)} = \frac{1 + \frac{\sin^2(\theta)}{\cos^2(\theta)}}{1 - \frac{\sin^2(\theta)}{\cos^2(\theta)}}$$

$$= \frac{\cos^2(\theta) + \sin^2(\theta)}{\cos^2(\theta) - \sin^2(\theta)}$$ M1

$$= \frac{1}{\cos(2\theta)}$$

$$= \sec(2\theta)$$ A1

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b. **Method 1:**

$$\sec(2\theta) = 2 \Rightarrow \cos 2\theta = \frac{1}{2}$$

$$2\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$ M1

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$ A1

Method 2:

$$1 + \tan^2(\theta) = 2(1 - \tan^2(\theta))$$

$$\tan(\theta) = \pm\frac{1}{\sqrt{3}}$$ M1

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$ A1

Question 4 (4 marks)

a. $E(\bar{X}) = 30$ A1

b. $$\text{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{7}{\sqrt{25}}$$

$$= \frac{7}{5}$$ A1

c. $E(\bar{X}) = 30$ (unchanged as it is independent of n) A1

$$\text{sd}(\bar{X}) = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{7}{\sqrt{50}}$$

$$= \frac{7}{5\sqrt{2}}$$

$\text{sd}(\bar{X})$ is reduced by a factor of $\sqrt{2}$. A1

Question 5 (3 marks)

$$(a - bi)(x + yi) + (a + bi)(x - yi) = k$$ M1

$$(ax + by) + (ay - bx)i + (ax + by) + (bx - ay)i = k$$ A1

$$2ax + 2by = k$$ A1

Hence the equation $\bar{w}z + w\bar{z} = k$ represents a straight line.

Question 6 (5 marks)

a. Let $u = \frac{1}{x}$ and so $\frac{du}{dx} = -\frac{1}{x^2} = -u^2$. A1

$x = 1 \Rightarrow u = 1$ and $x = p \Rightarrow u = \frac{1}{p}$

$$\int_1^p \frac{1}{1+x^2}dx = \int_1^{\frac{1}{p}} \frac{1}{1+\left(\frac{1}{u}\right)^2}\left(-\frac{1}{u^2}\right)du$$ M1

$$= -\int_1^{\frac{1}{p}} \frac{1}{u^2+1}du$$

$$= \int_{\frac{1}{p}}^1 \frac{1}{1+u^2}du$$ A1

b. $$\int_1^p \frac{1}{1+x^2}dx = \int_{\frac{1}{p}}^1 \frac{1}{1+u^2}du$$

$$\left[\arctan(x)\right]_1^p = \left[\arctan(u)\right]_{\frac{1}{p}}^1$$ M1

$$\arctan(p) - \arctan(1) = \arctan(1) - \arctan\left(\frac{1}{p}\right)$$ A1

$\arctan(p) + \arctan\left(\frac{1}{p}\right) = 2\left(\frac{\pi}{4}\right)$ (or equivalent)

And so $\arctan(p) + \arctan\left(\frac{1}{p}\right) = \frac{\pi}{2}$.

Question 7 (6 marks)

a. $$P\left(\frac{i}{2}\right) = 0 \Rightarrow 2\left(\frac{i}{2}\right)^4 + bi\left(\frac{i}{2}\right)^3 + 2\left(\frac{i}{2}\right) + bi = 0$$ M1

$$\frac{1}{8} + \frac{b}{8} + i + bi = 0 \Rightarrow b = -1$$ A1

b. $2z^4 - iz^3 + 2z - i = (2z - i)(z^3 + 1) = 0$

$2z - i = 0 \Rightarrow z = \frac{i}{2}$ and so $z = \frac{1}{2}\text{cis}\left(\frac{\pi}{2}\right)$ A1

EITHER

$z^3 + 1 = (z + 1)(z^2 - z + 1) = 0$ M1

$\Rightarrow z = -1$ or $z = \frac{1 \pm \sqrt{3}i}{2}$ A1

OR

$z^3 = -1$ M1

$= \text{cis}(\pi) \Rightarrow z = \text{cis}\left(\frac{\pi + 2k\pi}{3}\right), k = 0, \pm 1$ A1

THEN

$\Rightarrow z = \text{cis}\left(-\frac{\pi}{3}\right), \text{cis}\left(\frac{\pi}{3}\right), \text{cis}(\pi)$ A1

So $z = \frac{1}{2}\text{cis}\left(\frac{\pi}{2}\right), \text{cis}\left(-\frac{\pi}{3}\right), \text{cis}\left(\frac{\pi}{3}\right), \text{cis}(\pi)$.

Question 8 (4 marks)

The denominator has an irreducible quadratic factor and a single linear factor. Hence there are two partial fractions.

$$\frac{x^2 + x}{(x^2 + 2)(x - 2)} = \frac{Ax + B}{x^2 + 2} + \frac{C}{x - 2}$$

$$x^2 + x = (Ax + B)(x - 2) + C(x^2 + 2) \qquad \text{A1}$$

EITHER

Attempting to find the values of A, B and C. M1

Substituting $x = 2$ gives:

$6 = 6C \Rightarrow C = 1$

Substituting $x = 0$ for example with $C = 1$ gives:

$0 = -2B + 2 \Rightarrow B = 1$

Substituting $x = 1$ for example with $B = 1$ and $C = 1$ gives:

$2 = -1 - A + 3 \Rightarrow A = 0$

So $A = 0$, $B = 1$ and $C = 1$.

OR

$x^2 + x = (A + C)x^2 + (-2A + B)x + (-2B + 2C)$

Attempting to equate coefficients: M1

$$A + C = 1$$
$$-2A + B = 1$$
$$-2B + 2C = 0$$

Solving gives $A = 0$, $B = 1$ and $C = 1$.

THEN

$$\int \frac{x^2 + x}{(x^2 + 2)(x - 2)} dx = \int \left(\frac{1}{x^2 + 2} + \frac{1}{x - 2} \right) dx$$

$$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}} \right) + \log_e|x - 2| + c \qquad \text{M1 A1}$$

Question 9 (3 marks)

$$(\underset{\sim}{a} \cdot \underset{\sim}{b})^2 = |\underset{\sim}{a}|^2 |\underset{\sim}{b}|^2 \cos^2(\theta) \qquad \text{A1}$$

$$\leq |\underset{\sim}{a}|^2 |\underset{\sim}{b}|^2 \text{ since } 0 \leq \cos^2(\theta) \leq 1 \qquad \text{A1}$$

$(\underset{\sim}{a} \cdot \underset{\sim}{b})^2 = (a_1 b_1 + a_2 b_2 + a_3 b_3)^2$, $|\underset{\sim}{a}|^2 = a_1^2 + a_2^2 + a_3^2$ and $|\underset{\sim}{b}|^2 = b_1^2 + b_2^2 + b_3^2$ leading to

$$(a_1 b_1 + a_2 b_2 + a_3 b_3)^2 \leq (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2) \qquad \text{A1}$$

Question 10 (5 marks)

$$\frac{dx}{dt} = \frac{-2t(1+t^2) - 2t(1-t^2)}{(1+t^2)^2}$$

$$= \frac{-4t}{(1+t^2)^2}$$ A1

$$\frac{dy}{dt} = \frac{4(1+t^2) - 2t(4t)}{(1+t^2)^2}$$

$$= \frac{4-4t^2}{(1+t^2)^2}$$ A1

$$\frac{dy}{dx} = \left(\frac{4-4t^2}{(1+t^2)^2}\right)\left(-\frac{(1+t^2)^2}{4t}\right)$$

$$= \frac{t^2-1}{t}$$ M1

At $t = \frac{1}{2}, x = \frac{3}{5}, y = \frac{8}{5}$ and $\frac{dy}{dx} = -\frac{3}{2}$. M1

The equation of the tangent is $y - \frac{8}{5} = -\frac{3}{2}\left(x - \frac{3}{5}\right)$ (or equivalent). A1