The Mathematical Association of Victoria

Trial Examination 2017

SPECIALIST MATHEMATICS

Written Examination 2

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of Book

Section	*Number of Questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 23 pages.
- Formula sheet
- Answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the space provided above on this page.
- Write your **name** on the multiple-choice answer sheet
- Unless otherwise indicated, the diagrams are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1

If $z = \sqrt{3} - i$, $w = 4(1+i)$ and $\text{Arg}\left(\frac{z^k}{\overline{w}}\right) = \frac{11\pi}{12}$ then a possible value of k is

A. 2
B. 3
C. 6
D. 8
E. 10

Question 2

If $2-i$ is a root of the equation $z^3 + bz^2 + 17z - 15 = 0$ where $b \in R$, then b is

A. -1
B. 7
C. $-\frac{19}{2}$
D. 1
E. -7

Question 3

If $(a+i)(1-ai) = 6 + (b+2)i$ where $a, b \in R$ then

A. $ab = 30$
B. $b + a = 7$
C. $\frac{b}{a} = 4$
D. $a + b = -7$
E. $\frac{b}{a} = -4$

Question 4

If $\text{cosec}(x) = \frac{4\sqrt{7}}{7}, \quad \frac{\pi}{2} < x < \pi$ then $\sec(x) + \tan(x)$ is

A. $-\frac{4}{3} + \frac{3\sqrt{7}}{16}$

B. $-\left(\frac{12 - 3\sqrt{7}}{12}\right)$

C. $-\frac{1}{3}(4 + \sqrt{7})$

D. $-\frac{4}{3}(1 - \sqrt{7})$

E. $-\frac{1}{3}(4 - \sqrt{7})$

Question 5

If $3\cos(2\theta) + 5\sin(\theta) - 2 = 0$ then the following are solutions

A. $\sin(\theta) = 1$ and $\sin(\theta) = -\frac{1}{6}$

B. $\cos(\theta) = 1$ and $\cos(\theta) = -\frac{1}{6}$

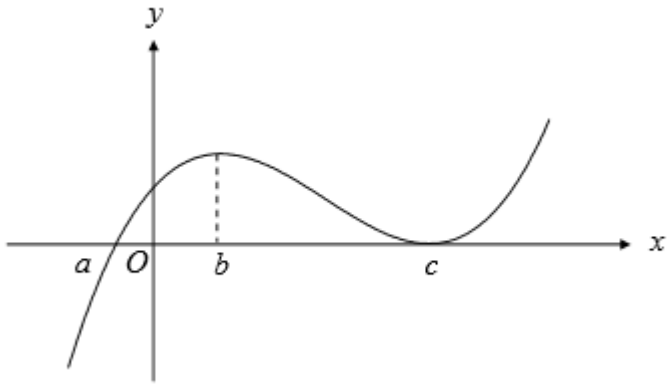
C. $\sin(\theta) = -1$ and $\sin(\theta) = -\frac{1}{6}$

D. $\sin(\theta) = -1$ and $\cos(\theta) = \frac{1}{6}$

E. $\sin(\theta) = 1$ and $\sin(\theta) = \frac{1}{6}$

Question 6

The graph of $y = f(x)$ is shown below.



If $F(x)$ is an antiderivative of $f(x)$, then the graph of $y = F(x)$ will have a

A. local maximum at $x = b$
B. local maximum at $x = a$
C. point of inflexion at $x = a$
D. zero gradient at $x = b$
E. point of inflexion at $x = c$

Question 7

If $f'(t) = \ln(2t+1)$ and $f(0) = 2$ then the value of $f(1)$ can be found by evaluating

A. $\int_0^1 (\ln(2t+1) - 2)dt$

B. $\int_0^2 (\ln(2t+1)dt - 1$

C. $\int_0^1 (2 + \ln(2t+1))dt$

D. $\int_1^0 (2 - \ln(2t+1))dt$

E. $\int_0^1 \ln(2t+1)dt + 2$

Question 8

The gradient of the curve $y^2 + 2yx^4 = 33$ when $y = 1$ in the first quadrant is equal to

A. $-\frac{31}{34}$

B. $-\frac{32}{17}$

C. $\frac{31}{34}$

D. -32

E. $\frac{31}{17}$

Question 9

$\int_{\pi/6}^{2\pi/3} (\tan^3(x) + \tan(x))dx$ is equivalent to

A. $\int_{-\sqrt{3}}^{1/\sqrt{3}} u\,du$

B. $\int_{1/\sqrt{3}}^{-\sqrt{3}} (u^3 + u)du$

C. $\int_{\pi/6}^{\pi/3} (u^3 + u)du$

D. $\int_{1/\sqrt{3}}^{-\sqrt{3}} u^3\,du$

E. $-\int_{-\sqrt{3}}^{1/\sqrt{3}} u\,du$

Question 10

The graph of the function $y = \sqrt{\frac{16x^4 - 1}{x^4}}$, between the lines $y = 2$ and $y = 3$ is rotated about the *y*-axis to form a solid of revolution. The volume formed is given by

A. $\cos^{-1}\left(\frac{3}{4}\right) - \frac{\pi}{3}$

B. $\pi \sin^{-1}\left(\frac{3}{4}\right) - \frac{\pi^2}{6}$

C. $\frac{\ln 7}{8} - \frac{\ln 3}{8}$

D. $\pi \sin^{-1}\left(\frac{3}{4}\right) + \frac{\pi^2}{6}$

E. $\frac{\pi}{4}\left(\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right)$

Question 11

$\underset{\sim}{a} = 8\underset{\sim}{i} - \underset{\sim}{j} + 13\underset{\sim}{k}$, $\underset{\sim}{b} = m\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{c} = \underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$ are linearly dependent when

A. $m = \frac{1}{3}$

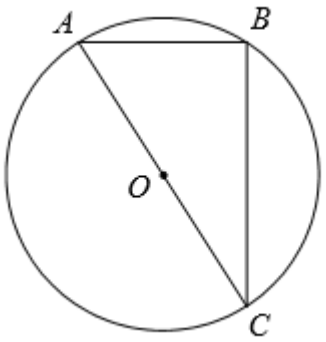
B. $m = -\frac{2}{3}$

C. $m = -2$

D. $m = -\frac{1}{3}$

E. $m = 2$

Question 12



If the points A,B and C lie on the circumference of the circle whose centre is *O*, which of the following statements is **not** true?

A. $\overrightarrow{AB} = \overrightarrow{AC} - \overrightarrow{BC}$

B. $\overrightarrow{AB}.\overrightarrow{AC} = \left|\overrightarrow{BA}\right|\left|\overrightarrow{AC}\right|\cos(A)$

C. $\left(\overrightarrow{CA} + \overrightarrow{AB}\right).\overrightarrow{CB} = \left|\overrightarrow{CB}\right|^2$

D. $\left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right| = \left|\overrightarrow{AC}\right|$

E. $\overrightarrow{BA}.\overrightarrow{BC} = 0$

Question 13

The acceleration $a\ \text{ms}^{-2}$ of a body moving in a straight line when it is *x* m from the origin is given by $a = \dfrac{1}{\sqrt{(9-x^2)}}$. If the velocity, *v*, of the body is 2ms^{-1} when it passes through the origin, then the function for *x* is given by

A. $x = 3\sin\left(\dfrac{v^2-4}{2}\right)$

B. $x = 2\sin\left(\dfrac{2-v^2}{3}\right)$

C. $x = 3\cos\left(\dfrac{v^2-2}{2}\right)$

D. $x = 3\sin(v^2-2)$

E. $x = 2\sin^{-1}\left(\dfrac{v}{3}\right)$

Question 14

The position vector of a particle at time t, $t \geq 0$, is given by $\underset{\sim}{\mathrm{r}} = 2\sqrt{t}\underset{\sim}{\mathrm{i}} + (5-t)\underset{\sim}{\mathrm{j}}$. The particle is closest to the origin when

A. $t = 0$

B. $t = 1$

C. $t = \frac{3}{2}$

D. $t = 3$

E. $t = 5$

Question 15

A smooth inclined plane makes an angle of θ^o to the horizontal. An object of mass *m* kg is held at rest on this plane by a force F newtons inclined at α^o upwards to the plane. If the normal reaction of the mass on the plane is R newtons, which of the following statements is true?

A. $\mathrm{R} = mg\cos\theta + \mathrm{F}\sin\alpha$

B. $\mathrm{F}\sin\alpha - mg\cos\theta = 0$

C. $\tan\alpha = \frac{mg\cos\theta - \mathrm{R}}{mg\sin\theta}$

D. $\mathrm{F} = \frac{m\sin\theta}{\cos\alpha}$

E. $\tan\alpha = \frac{\mathrm{F}\cos\theta}{\mathrm{R} - \mathrm{F}\sin\theta}$

Question 16

A body of mass 10 kg is acted upon by three forces $\underset{\sim}{\mathrm{F}}_1$, $\underset{\sim}{\mathrm{F}}_2$ and $\underset{\sim}{\mathrm{F}}_3$ all measured in Newtons. If, at time *t* seconds, $\underset{\sim}{\mathrm{F}}_1 = 2\underset{\sim}{\mathrm{i}} + (t-3)\underset{\sim}{\mathrm{j}}$, $\underset{\sim}{\mathrm{F}}_2 = -t^2\underset{\sim}{\mathrm{i}} - \underset{\sim}{\mathrm{j}}$ and $\underset{\sim}{\mathrm{F}}_3 = 5t\underset{\sim}{\mathrm{i}} + 2\underset{\sim}{\mathrm{j}}$ then the magnitude of the acceleration of the body in ms^{-2} when $t = 3$ is

A. $\frac{\sqrt{65}}{10}$

B. $\frac{73}{10}$

C. 65

D. 9

E. $\frac{\sqrt{65}}{5}$

Question 17

A confidence interval estimate for a population mean can be used to test a null hypothesis about the population mean only if

A. The distribution is symmetrical
B. A one-tailed test is used
C. A two-tailed test is used
D. All of the above statements are true
E. None of the above statements are true

Question 18

X is a random variable with $\mathrm{E}(X) = 14.2$ and $\mathrm{Var}(X) = 2.3$. Y is another random variable with $\mathrm{E}(Y) = 5.5$ and $\mathrm{Var}(Y) = 0.8$. If X and Y are independent variables and $W = 2X - Y + 3$ then

A. $\mathrm{E}(W) = 22.9$ and $\mathrm{SD}(W) = \sqrt{13}$
B. $\mathrm{E}(W) = 22.9$ and $\mathrm{SD}(W) = \sqrt{10}$
C. $\mathrm{E}(W) = 33.9$ and $\mathrm{SD}(W) = 10$
D. $\mathrm{E}(W) = 25.9$ and $\mathrm{SD}(W) = \sqrt{10}$
E. $\mathrm{E}(W) = 25.9$ and $\mathrm{SD}(W) = \sqrt{13}$

Question 19

A Type II error is made when the

A. alternative hypothesis is accepted when it is true
B. null hypothesis is accepted when it is false
C. null hypothesis is true and we accept it
D. null hypothesis is rejected when it is true
E. alternative hypothesis is accepted when it is false

Question 20

The distribution of serum vitamin E in a certain population is approximately normal with mean 800μg/dL and standard deviation 200μg/dL, (μg/dL stands for micrograms per decilitre). In a random sample of 20 people, the mean vitamin E level would lie within which range approximately 95% of the time

A. Between 780μg/dL and 820μg/dL
B. Between 408μg/dL and 1192μg/dL
C. Between 712μg/dL and 888μg/dL
D. Between 760μg/dL and 839μg/dL
E. Between 625μg/dL and 975μg/dL

END OF SECTION A

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (11 marks)

A particle travels in a closed circuit. At time t secs it has a position vector, measured in metres, given by

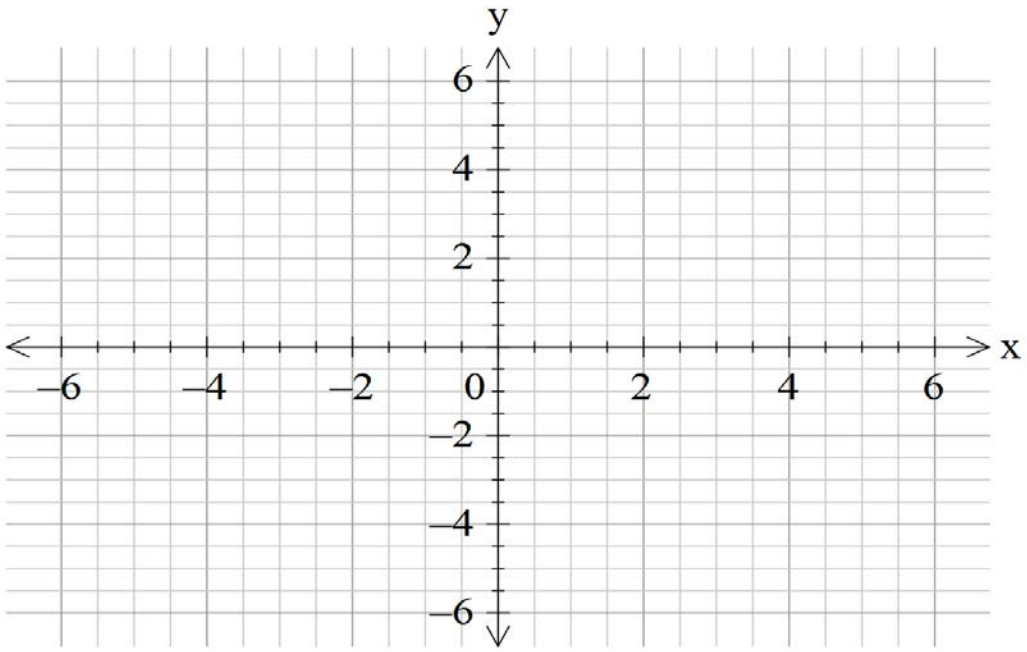
$$\underset{\sim}{r}(t) = 2\cos(t)\underset{\sim}{i} + (2 + 3\sin(2t))\underset{\sim}{j}, t \geq 0$$

a. Show that the Cartesian equation of the path of the particle can be written as

$$9(x^2 - 2)^2 + 4(y - 2)^2 = 36$$ 1 mark

b. **i** Sketch the circuit of the particle on the set of axes below. There is no need to find the x-intercepts.

1 mark



ii. Mark the starting point and the direction**s** of motion of the particle on the diagram above. 1 mark

c. How long does it take the particle to complete one full circuit? 1 mark

d. Find an expression for the speed of the particle at time t secs. 1 mark

e. For $0 < t \leq 2\pi$, what are the maximum and minimum speeds of the particle? 2 marks

f.

i. Write down a definite integral for the length that the particle has travelled in the first 2 seconds.

1 mark

ii. Find this length. Give your answer to two decimal places. 1mark

g. Find the first time at which the particle will be travelling in a direction parallel to the line $y = 3x$. Give your answer correct to two decimal places.

2 marks

SECTION B — continued
TURN OVER

Question 2 (9 marks)

Points A, B, C and D have position vectors $\underset{\sim}{a} = \underset{\sim}{i} + m\underset{\sim}{j}$, $\underset{\sim}{b} = -2\underset{\sim}{i} + 2\underset{\sim}{j}$, $\underset{\sim}{c} = n\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{d} = 6\underset{\sim}{i} + 6\underset{\sim}{j}$ relative to the origin O, where *m* and *n* are real numbers.

a. Find $\overrightarrow{AB}$ and $\overrightarrow{AD}$ in terms of *m.* 1 mark

b. If ABCD is a rhombus, find the values of *m* and *n*. 2 marks

c. Find $\overrightarrow{AE}$ the vector resolute of $\overrightarrow{AB}$ parallel to $\overrightarrow{AD}$. 2 marks

d. Find the area of triangle ABE. 2 marks

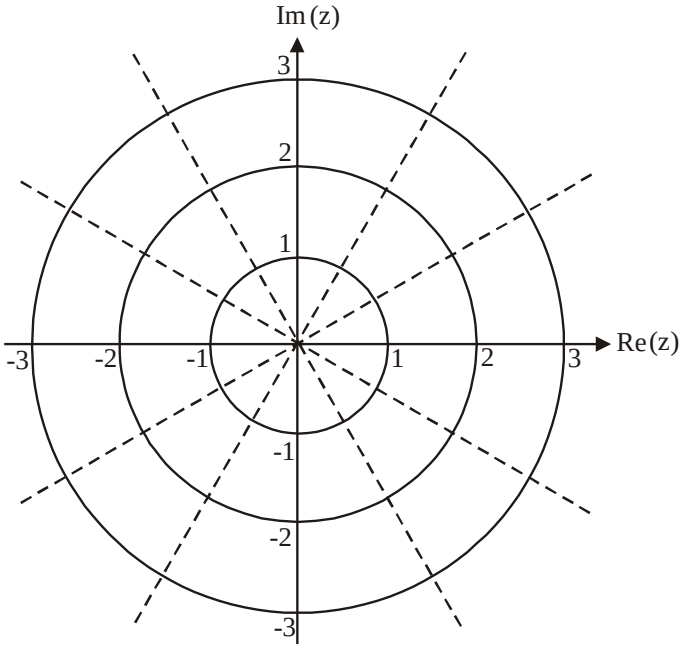
SECTION B – Question 2 – continued

e. If F divides $\overrightarrow{CD}$ in the ratio 2:1, find the angle θ^o between vector $\overrightarrow{BD}$ and $\overrightarrow{BF}$. Give your answer correct to two decimal places. 2 marks

Question 3 (11 marks)

$w, \overline{w}$ and v are the solutions of the equation $\{z : z^3 = k, z \in C\}$ where $w = 3\text{cis}\left(\frac{\pi}{3}\right)$

a. Plot $w, \overline{w}$ and v on the Argand diagram below. 1 mark



b. Find the value of *k*. 1 mark

c. Show that $w \in \{z : \sqrt{3}\,\text{Im}(z) - \text{Re}(z) = 3\}$. 2 marks

d. On the Argand diagram given in part a.

i. Sketch $\{z : \sqrt{3}\,\text{Im}(z) - \text{Re}(z) = 3\}$ 1 mark

SECTION B – Question 3 – continued

ii. Sketch the ray given by $\{z : \text{Arg}(z) = \text{Arg}(\overline{w}^{\frac{1}{2}})\}$ 2 marks

e. If $u = 1 + i$

i. Express $\frac{4w}{u}$ in Cartesian form 1 mark

ii. Express $\frac{4w}{u}$ in Polar form 1 mark

iii. Hence, express $\tan\left(\frac{\pi}{12}\right)$ in the form $a - \sqrt{b}$ where a and b are positive integers.

2 marks

SECTION B – continued
TURN OVER

Question 4 (9 marks)

A tank initially contains 100 litres of water with 5kg of salt dissolved uniformly into it. A salt solution with a concentration of 0.1kg/litre flows into the tank at a rate of 2litres/min. The mixture is kept uniform by stirring and flows out at the rate of 2litres/min. If x kilograms is the amount of salt in the tank after t minutes

a. Show that the differential equation for x in terms of t is

$$\frac{dx}{dt} = \frac{10 - x}{50}$$

1 mark

b. Solve this differential equation to give x as a function of t. 2 marks

c. Calculate the amount of salt in the tank after 2 minutes. Give your answer correct to 2 decimal places.

1 mark

d. Sketch a graph of x against t

1 mark

SECTION B – Question 4 – continued

e. The outflow is now adjusted to 3litres/min instead of 2litres/min

i. Set up a new differential equation for x in terms of t. Do not attempt to solve it.

1 mark

ii. Use Euler's method with increments of 1 minute to predict the amount of salt in the tank after 2 minutes. Give your answer correct to 3 decimal places.

2 marks

iii. Verify that the following is a solution for this differential equation

$$x = \frac{1}{10}(100 - t) - \frac{5}{1000000}(100 - t)^3$$

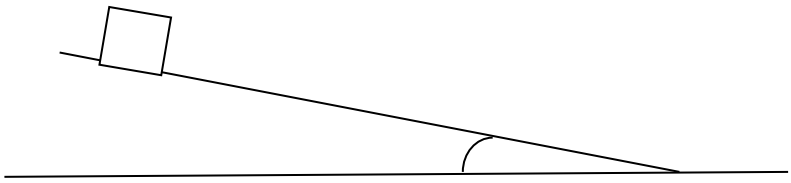
2 marks

TURN OVER

Question 5 (11 marks)

A baggage ramp in an airport is 6 metres in length and is inclined at 25 degrees to the horizontal. A suitcase of mass 15kg is initially at rest at the top of the ramp. The suitcase slides down the ramp under the force of gravity with a constant frictional force of 40N acting upon it.

a. Mark in all of the forces acting on the suitcase. 1 mark



b. Find the acceleration of the suitcase down the ramp. Give your answer correct to 2 decimal places.

2 marks

c. Find the time taken for the suitcase to reach the bottom of the ramp. Give your answer correct to 2 decimal places.

2 marks

d. An identical suitcase, also initially at rest, is pushed down the ramp with a force of $30-50t$ for the first 0.6 seconds. Show that the acceleration of this suitcase at time t where $0 < t \leq 0.6$ is approximately $3.47-\frac{10}{3}t$

1 mark

e. Using this approximate acceleration, find the speed of this second suitcase when $t = 0.6$. Give your answer correct to 2 decimal places.

2 marks

f. How long does it take this suitcase to reach the bottom of the ramp? Give your answer correct to 2 decimal places.

3 marks

TURN OVER

Question 6 (9 marks)

The monthly turnover of a retail shop is normally distributed with an average of $40500 per month and a standard deviation of $4500

a. What is the probability that in a three monthly period of time the mean turnover per month would be more than $42000? Give your answer correct to three decimal places.

2 marks

b. What is the probability that the shop will turnover more than half a million dollars in a year? Give your answer correct to three decimal places.

2 marks

c. What is the probability of them turning over more than half a million dollars in a year, given that they are guaranteed to turn over at least $480000? Give your answer correct to three decimal places.

2 marks

d. Following an advertising campaign the next three month period produced an average monthly turnover of \$46500. At the 5% level, test whether the advertising campaign has lead to a significant increase in the monthly turnover.

3 marks

END OF QUESTION AND ANSWER BOOK