The Mathematical Association of Victoria

Trial Examination 2017

SPECIALIST MATHEMATICS

Written Examination 2 - SOLUTIONS

SECTION A

Question	Answer	Question	Answer
1	D	**11**	E
2	E	**12**	D
3	D	**13**	A
4	C	**14**	D
5	A	**15**	C
6	E	**16**	A
7	E	**17**	C
8	B	**18**	D
9	E	**19**	B
10	B	**20**	C

Question 1

$\text{Arg}(z) = -\frac{\pi}{6}$ and $\text{Arg}(\overline{w}) = -\frac{\pi}{4}$

$$\arg\left(\frac{z^k}{\overline{w}}\right) = -\frac{k\pi}{6} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{12}(3-2k)$$

When $k = 8$, $\text{Arg}\left(\frac{z^k}{\overline{w}}\right) = -\frac{13\pi}{12} + 2\pi = \frac{11\pi}{12}$

Alternatively, students can use substitution to find the values of k

Answer is D

Question 2

The conjugate $2+i$ is also a root as the coefficients of the polynomial are real. If we let a denote the remaining root, the polynomial can be written as

$$(z-2+i)(z-2-i)(z-a) = z^3-(a+4)z^2+(4a+5)z-5a$$

Therefore $4a+5=17$, $a=3$

and $b=-a-4=-7$

Answer is E

Question 3

If $(a+i)(1-ai) = 6+(b+2)i$

$a+i-a^2i+a = 6+(b+2)i$

Real part: $2a=6 \;\therefore\; a=3$

Imag part: $(1-3^2) = b+2 \;\therefore\; b=-10$

Therefore $a+b=-7$

Answer is D

Question 4

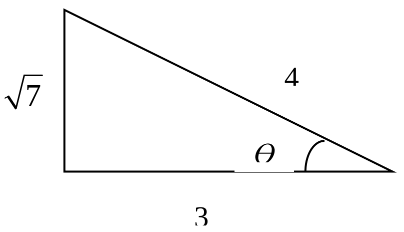
$$\operatorname{cosec}(x) = \frac{4\sqrt{7}}{7} \qquad \therefore \sin(x) = \frac{\sqrt{7}}{4}$$

$$\cos(x) = -1\sqrt{\left(1-\frac{7}{4^2}\right)} = -\sqrt{\frac{9}{16}} = -\frac{3}{4}$$ negative as $\frac{\pi}{2} < x < \pi$

$$\therefore \quad \sec(x) = -\frac{4}{3} \quad \text{and } \tan(x) = \frac{\frac{7}{4\sqrt{7}}}{-\frac{3}{4}} = -\frac{\sqrt{7}}{3}$$

(or use the right-angled triangle below to see that $\tan(x) = -\tan(\theta) = -\frac{\sqrt{7}}{3}$)

$$\sec(x)+\tan(x) = -\frac{4}{3}-\frac{\sqrt{7}}{3} = -\frac{1}{3}(4+\sqrt{7}))$$



Answer is C

Question 5

$$3\cos(2\theta)+5\sin(\theta)-2=0$$
$$3(1-2\sin^2(\theta))+5\sin(\theta)-2=0$$
$$6\sin^2(\theta)-5\sin(\theta)-1=0$$
$$(6\sin(\theta)+1)(\sin(\theta)-1)=0$$
$$\therefore \quad \sin(\theta)=1,\ \sin(\theta)=-\frac{1}{6}$$

Answer is A

Question 6

At $x=a$ the gradient is zero, and it is negative to the left and positive to the right. Therefore it is a minimum this rules out answers B and C.

At $x=b$ the gradient is positive, not zero which rules out answer D and A.

At $x=c$ the gradient is zero and it is positive to the left and right of this point, therefore it is a point of inflexion.

Answer is E

Question 7

$$f(b)=f(a)+\int_a^b f'(t)dt$$
$$f(1)=f(0)+\int_0^1 f'(t)dt$$
$$f(1)=2+\int_0^1 \ln(2t+1)dt$$

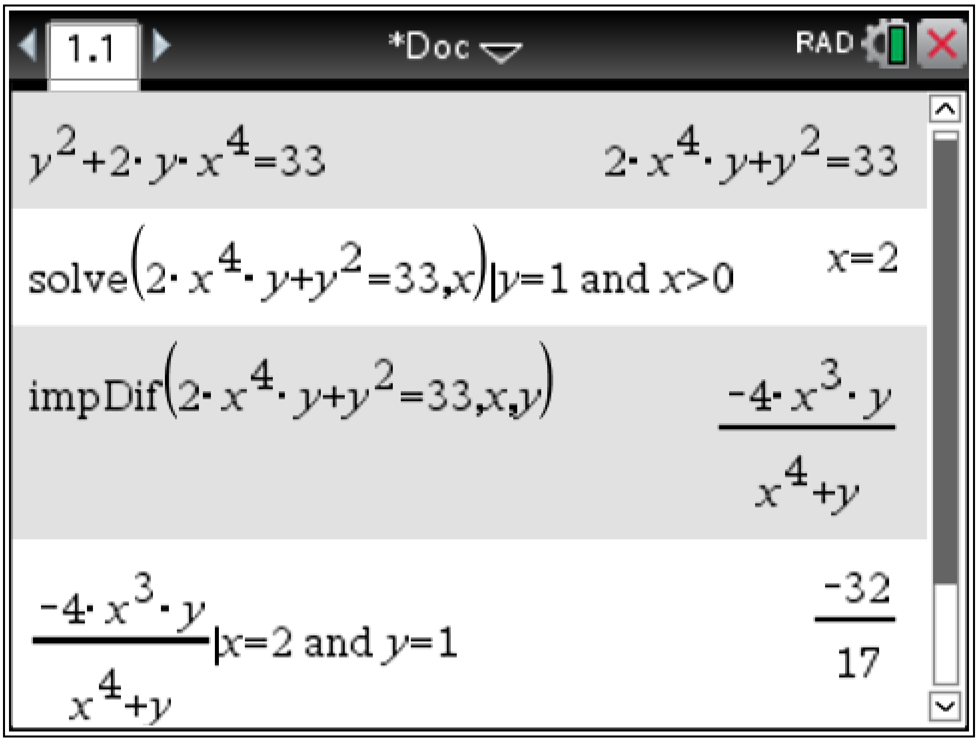
Answer is E

Question 8

$$y^2+2yx^4=33$$
$$\therefore \quad 2y\frac{dy}{dx}+2\frac{dy}{dx}x^4+8yx^3=0$$
$$\frac{dy}{dx}=-\frac{8yx^3}{2(y+x^4)}=-\frac{4yx^3}{(y+x^4)}$$

when $y=1,\ \ 1+2x^4=33 \quad \therefore x=2$ (as x is in the first quadrant)

thus $\dfrac{dy}{dx}=-\dfrac{4\times1\times2^3}{(1+2^4)}=-\dfrac{32}{17}$



Answer is B

Question 9

$$\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} (\tan^3(x)+\tan(x))dx = \int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \tan(x)(\tan^2(x)+1)dx = \int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \tan(x)\sec^2(x)dx$$

$u = \tan(x), \quad du = \sec^2(x)dx$

$x = \frac{\pi}{6},\ u = \frac{1}{\sqrt{3}}$ and $x = \frac{2\pi}{3},\ u = -\sqrt{3}$

$$\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \tan(x)\sec^2(x)dx = \int_{\frac{1}{\sqrt{3}}}^{-\sqrt{3}} u\,du = -\int_{-\sqrt{3}}^{\frac{1}{\sqrt{3}}} u\,du$$

Answer is E

Question 10

$$V = \pi\int_2^3 x^2 dy$$

$y^2 = \frac{16x^4-1}{x^4}$ rearranging gives $x^2 = \frac{1}{\sqrt{16-y^2}}$

$$\therefore V = \pi\int_2^3 \frac{1}{\sqrt{16-y^2}}dy = \pi\left(\sin^{-1}\left(\frac{3}{4}\right)-\frac{\pi}{6}\right)$$

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solve(y^2=(16x^4-1)/x^4,x)
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$$\left\{x=\left(\frac{-1}{y^2-16}\right)^{\frac{1}{4}}\right\}$$

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ans^2
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$$\left\{x^2=\sqrt{\frac{-1}{y^2-16}}\right\}$$

$$\pi\int_2^3 1/(16-y^\wedge 2)^\wedge 0.5\,dy$$

$$\left(\sin^{-1}\left(\frac{3}{4}\right)-\frac{\pi}{6}\right)\cdot\pi$$

Answer is B

Question 11

For linear dependence we require $\underset{\sim}{a} = s\underset{\sim}{b} + t\underset{\sim}{c}$ where $s, t \in R$

$\therefore \quad 8 = sm + t$

$-1 = -s + t$

$13 = 3s + 2t$ solve using CAS

$$\begin{cases}8=sm+t\\-1=-s+t\\13=3s+2t\end{cases}\Bigg|\, s, m, t$$

$$\{m=2, s=3, t=2\}$$

Answer is E

Question 12

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ answer A is just a rearrangement of this, therefore A is correct.

The dot product gives $\overrightarrow{AB}.\overrightarrow{AC} = \left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|\cos(A)$, but $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BA}\right|$ therefore B is also correct.

$\left(\overrightarrow{CA} + \overrightarrow{AB}\right).\overrightarrow{CB} = \overrightarrow{CB}.\overrightarrow{CB} = \left|\overrightarrow{CB}\right|^2$ therefore C is correct

Angle B is a right angle, therefore $\overrightarrow{BA}.\overrightarrow{BC} = 0$ and E is correct.

Whereas for a right angled triangle $\left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{BC}\right|^2 = \left|\overrightarrow{AC}\right|^2$ which is NOT the same as $\left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right| = \left|\overrightarrow{AC}\right|$ therefore D is not correct.

Answer is D

Question 13

$$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{1}{\sqrt{\left(9 - x^2\right)}}$$

$$\therefore \ \frac{v^2}{2} = \int \frac{1}{\sqrt{(9 - x^2)}}\,dx = \sin^{-1}\frac{x}{3} + c \quad x = 0, v = 2 \therefore c = 2$$

$$\therefore \ \frac{v^2 - 4}{2} = \sin^{-1}\frac{x}{3}$$

$$\therefore \ x = 3\sin\left(\frac{v^2 - 4}{2}\right)$$

Answer is A

Question 14

If $\underset{\sim}{r} = 2\sqrt{t}\,\underset{\sim}{i} + (5 - t)\,\underset{\sim}{j}$

distance from the origin $= \sqrt{((2\sqrt{t})^2 + (5 - t)^2)}$

$= \sqrt{(t^2 - 6t + 25)}$

this is a minimum when $2t - 6 = 0, \quad t = 3$

Alternative method:

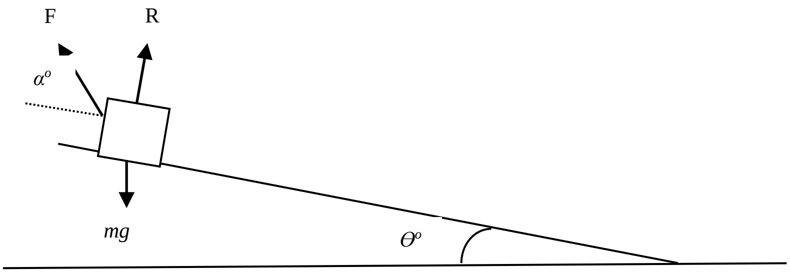
If $\underset{\sim}{r} = 2\sqrt{t}\,\underset{\sim}{i} + (5 - t)\,\underset{\sim}{j}$ then $\underset{\sim}{v} = \frac{1}{\sqrt{t}}\underset{\sim}{i} - \underset{\sim}{j}$

Now the dot product of these two vectors is zero when closest to the origin.

Hence $2 - (5 - t) = 0$ and so $t = 3$

Answer is D

Question 15



Resolving perpendicular to the plane:
$\text{R} = mg\cos\theta - \text{F}\sin\alpha$

Resolving parallel to the plane:
$\text{F}\cos\alpha - mg\sin\theta = 0$

Thus, A is wrong because the sign is wrong. B is wrong because the sine and cosine are in the wrong positions. D is wrong because the g is missing. E is wrong because the angles are in the wrong position. The equations above give

$$\frac{\text{F}\sin\alpha}{\text{F}\cos\alpha} = \frac{mg\cos\theta - R}{mg\sin\theta}$$

$$\tan\alpha = \frac{mg\cos\theta - R}{mg\sin\theta}$$

Answer is C

Question 16

$\underset{\sim}{F} = \underset{\sim}{F}_1 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3 = (2 - t^2 + 5t)\underset{\sim}{i} + (t - 3 - 1 + 2)\underset{\sim}{j}$

When $t = 3$ $\underset{\sim}{F} = 8\underset{\sim}{i} + \underset{\sim}{j}$

$\underset{\sim}{F} = m\underset{\sim}{a} \quad \therefore \quad \underset{\sim}{a} = \frac{1}{10}(8\underset{\sim}{i} + \underset{\sim}{j})$

$|\underset{\sim}{a}| = \frac{1}{10}\sqrt{(8^2 + 1^2)} = \frac{\sqrt{65}}{10}$

Answer is A

Question 17

For a 95% confidence interval

$$\bar{x}-1.96\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x}-1.96\frac{\sigma}{\sqrt{n}}$$

$$-1.96 \le \frac{\mu-\bar{x}}{\left(\frac{\sigma}{\sqrt{n}}\right)} \le 1.96$$

In a two-tailed hypothesis test at the 5% we would reject the null hypothesis if

$\left|\frac{\mu-\bar{x}}{\frac{\sigma}{\sqrt{n}}}\right| \ge 1.96$ this would be the same as saying, reject the null hypothesis if

the confidence interval does not contain μ_0.This is true for any confidence level.

Answer is C

Question 18

$$\text{E}(W) = 2\times\text{E}(X)-\text{E}(Y)+3$$
$$=2\times 14.2-5.5+3 = 25.9$$

$$\text{SD}(W) = \sqrt{(2^2\,\text{Var}(X)+(-1)^2\,\text{Var}(Y))}$$
$$=\sqrt{(4\times 2.3+1\times 0.8)} = \sqrt{10}$$

Answer is D

Question 19

Both A and C are not errors because the correct conclusion is reached. Both D and E are Type I errors because they both lead to a rejection of the null hypothesis in favour of the alternative hypothesis, when the null hypothesis is true.

B is the only Type II error.

Answer is B

Question 20

95% confidence interval is given by

$$800-1.96\times\frac{200}{\sqrt{20}} \le \bar{x} \le 800+1.96\times\frac{200}{\sqrt{20}}$$

$712.346 \le \bar{x} \le 887.653$

$712 \le \bar{x} \le 888$ to the nearest μg/dL

Answer is C

SECTION B

Question 1 (11 marks)

a. $\underset{\sim}{r}(t) = 2\cos(t)\underset{\sim}{i} + (2+3\sin(2t))\underset{\sim}{j}, t \geq 0$

$x = 2\cos(t)$, $y = 2+3\sin(2t)$

$\cos(t) = \dfrac{x}{2}$, $\sin(2t) = \dfrac{y-2}{3}$

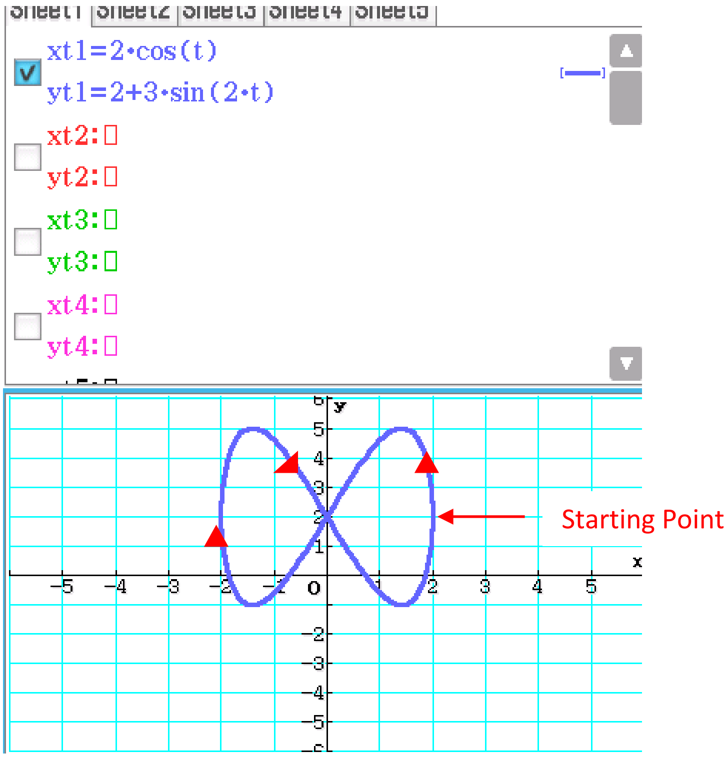
$\cos(2t) = 2\cos^2(t) - 1 = \dfrac{x^2}{2} - 1$

$\cos^2(2t) + \sin^2(2t) = \dfrac{(x^2-2)^2}{4} + \dfrac{(y-2)^2}{9} = 1$

$\therefore \; 9(x^2-2)^2 + 4(y-2)^2 = 36$

[A1]

b. i.



[A1]

ii. Starting point is (2,2). It starts moving away from this point in an anticlockwise direction. [A1]

c. The period of $\cos(t)$ is 2π and the period of $\sin(2t)$ is π, therefore it takes 2π seconds to complete one circuit [A1]

d.

$\dot{\underset{\sim}{r}}(t) = -2\sin(t)\underset{\sim}{i} + 6\cos(2t)\underset{\sim}{j}$

$\therefore \left|\dot{\underset{\sim}{r}}(t)\right| = \sqrt{(4\sin^2(t) + 36\cos^2(2t))}$ [A1]

$4\sin^2(t) = 2 - 2\cos(2t)$

$$\begin{aligned}\therefore \left|\dot{\underset{\sim}{r}}(t)\right| &= \sqrt{(2 - 2\cos(2t) + 36\cos^2(2t))} \\ &= \sqrt{2(1 - \cos(2t) + 18\cos^2(2t))}\end{aligned}$$

e. To find the maximum and minimum points solve

$$\frac{d}{dt}(1 - \cos(2t) + 18\cos^2(2t)) = 0$$

$$\therefore \quad 2\sin(2t) - 72\cos(2t)\sin(2t) = 0$$

$$2\sin(2t)(1 - 36\cos(2t)) = 0$$

$$\sin(2t) = 0, \quad \cos(2t) = \frac{1}{36}$$ [M1]

$\sin(2t) = 0 \quad t = \frac{\pi}{2}, \pi$

$\left|\dot{\underset{\sim}{r}}\left(\frac{\pi}{2}\right)\right| = \sqrt{(2 + 2 + 36)} = \sqrt{40} = 2\sqrt{10}$

$\left|\dot{\underset{\sim}{r}}(\pi)\right| = \sqrt{(2 - 2 + 36)} = \sqrt{36} = 6$

$\cos(2t) = \frac{1}{36} \qquad \left|\dot{\underset{\sim}{r}}(t)\right| = \sqrt{\left(2 - 2\times\frac{1}{36} + 36\times\frac{1}{36^2}\right)} = \frac{\sqrt{71}}{6}$ [A1]

Therefore the maximum is $2\sqrt{10}$ ms^{-1} and the minimum is $\frac{\sqrt{71}}{6} ms^{-1}$

f. **i.**

Length travelled $L = \int_0^2 \sqrt{\left(\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right)}\,dt$

$$= \int_0^2 \sqrt{\left((-2\sin(t))^2 + (6\cos(2t))^2\right)}\,dt$$

$$= \int_0^2 \sqrt{2\left(1 - \cos(2t) + 18\cos^2(2t)\right)}\,dt$$ [A1]

ii. L = 8.97 [A1]

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∫_0^2 (2(1−cos(2x)+18(cos(2x))^2))^0.5dx
                                   8.968044233
```

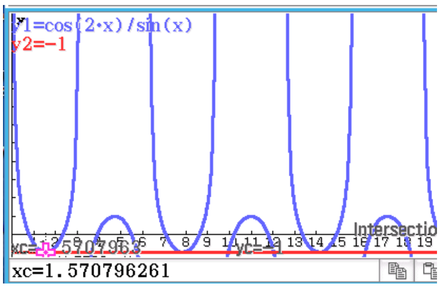
g. Need to solve $\frac{dy}{dx} = 3$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6\cos(2t)}{-2\sin(t)} =$$ [M1]

$$\therefore \frac{\cos(2t)}{\sin(t)} = -1$$

$t = 1.57$ secs [A1]

The value of *t* can be found graphically, or by solving with CAS. If using CAS solve, students need to ensure they select the correct root as there are multiple solutions.

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solve(cos(2x)/sin(x)=−1,x)
{x=6.283185307·constn(1)−2.61799387▶
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solve(cos(2x)/sin(x)=−1,x)
◀=6.283185307·constn(3)+1.570796327}
```

Question 2

a.

$$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a} = -3\underset{\sim}{i} + (2-m)\underset{\sim}{j}$$

$$\overrightarrow{AD} = \underset{\sim}{d} - \underset{\sim}{a} = 5\underset{\sim}{i} + (6-m)\underset{\sim}{j}$$ [A1]

b. For a rhombus the opposite sides must be identical vectors

$\overrightarrow{AB} = \overrightarrow{DC}$

$-3\underset{\sim}{i} + (2-m)\underset{\sim}{j} = (n-6)\underset{\sim}{i} - 4\underset{\sim}{j}$ [M1]

$\therefore n = 3$ and $m = 6$ [A1]

OR use $\left|\overrightarrow{AB}\right|^2 = \left|\overrightarrow{AD}\right|^2$

$\therefore 9 + (2-m)^2 = 25 + (6-m)^2$

$m = 6$ etc.

c.

$$\overrightarrow{AE} = (\overrightarrow{AB}.\widehat{\overrightarrow{AD}})\widehat{\overrightarrow{AD}}$$ [M1]

$$\overrightarrow{AD} = 5\underset{\sim}{i} \quad \therefore \quad \widehat{\overrightarrow{AD}} = \underset{\sim}{i}$$ [A1]

$$\overrightarrow{AE} = ((-3\underset{\sim}{i} - 4\underset{\sim}{j}).\underset{\sim}{i})\underset{\sim}{i} = -3\underset{\sim}{i}$$

d.

Area of triangle ABE $= \frac{1}{2}\left|\overrightarrow{AE}\right|\left|\overrightarrow{EB}\right|$ [M1]

$\left|\overrightarrow{AE}\right| = 3,\quad \overrightarrow{EB} = \overrightarrow{AB} - \overrightarrow{AE} = -4\underset{\sim}{j}\ ,\ \left|\overrightarrow{EB}\right| = 4$

Area $= \frac{1}{2} \times 3 \times 4 = 6$ units^2 [A1]

e.

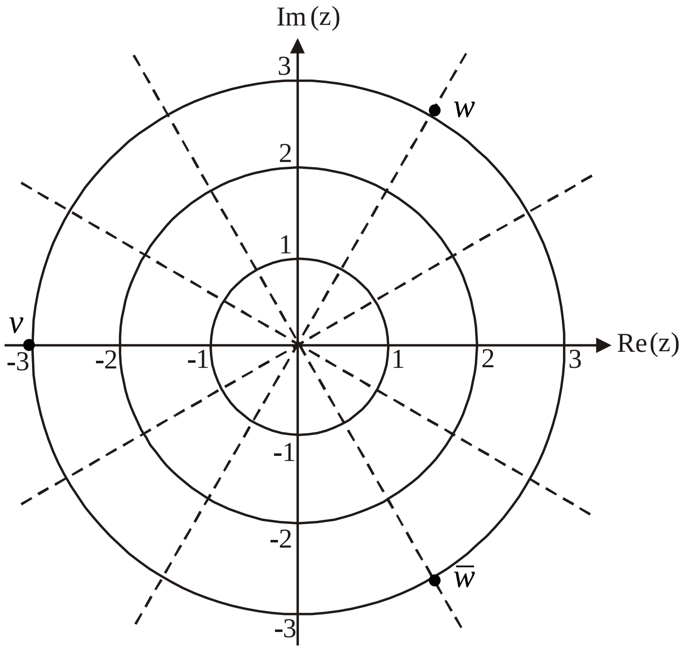
$$\theta = \cos^{-1}\left(\frac{\overrightarrow{BD}.\overrightarrow{BF}}{\left|\overrightarrow{BD}\right|\left|\overrightarrow{BF}\right|}\right)$$

$\overrightarrow{BD} = 8\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\overrightarrow{BF} = \overrightarrow{BC} + \frac{2}{3}\overrightarrow{CD} = 5\underset{\sim}{i} + \frac{2}{3}(3\underset{\sim}{i} + 4\underset{\sim}{j}) = \frac{21}{3}\underset{\sim}{i} + \frac{8}{3}\underset{\sim}{j}$ [M1]

$$\theta = \cos^{-1}\left(\frac{\frac{1}{3}(21\times 8 + 8\times 4)}{\frac{1}{3}\sqrt{(21^2+8^2)}\sqrt{(8^2+4^2)}}\right) = 5.71^o$$ [A1]

Question 3

a.



All roots correctly plotted and labelled. Equally spaced by $\dfrac{2\pi}{3}$ around a circle radius 3 [A1]

b.

$k = w^3 = 3^3 \operatorname{cis}\left(3 \times \dfrac{\pi}{3}\right) = 27\operatorname{cis}(\pi)$ [A1]

$k = -27$

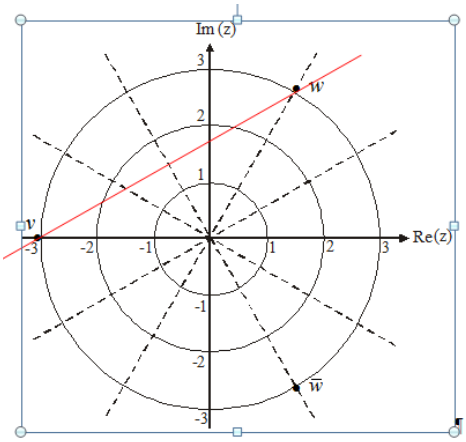
c.

$\operatorname{Im}(w) = 3\sin\left(\dfrac{\pi}{3}\right) = \dfrac{3\sqrt{3}}{2}$, $\operatorname{Re}(w) = 3\cos\left(\dfrac{\pi}{3}\right) = \dfrac{3}{2}$ [M1]

$\sqrt{3}\operatorname{Im}(w) - \operatorname{Re}(w) = \dfrac{3 \times 3}{2} - \dfrac{3}{2} = \dfrac{6}{2} = 3$ [A1]

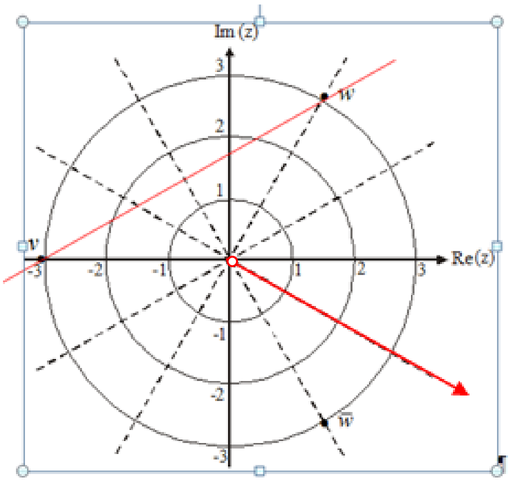
d.

i. The line should go through the intercepts $x = -3$ and $y = \sqrt{3}$ and it should have an angle of $\dfrac{\pi}{6}$ to the horizontal (it goes through *w* and *v*, if they have been plotted correctly) [A1]



ii. $Arg(\bar{w}) = -\dfrac{\pi}{3} \quad \therefore \quad Arg(\bar{w}^{\frac{1}{2}}) = -\dfrac{\pi}{6}$

Sketch $\{z : \text{Arg}(z) = -\dfrac{\pi}{6}\}$ straight line extended from the origin, on an angle of $-\dfrac{\pi}{6}$ to the horizontal. (The origin should be excluded).

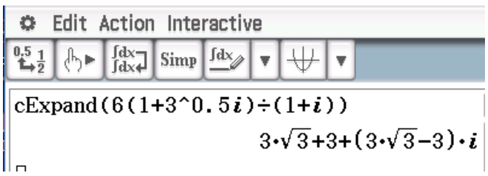


[A1]

e. i.

$$\frac{4w}{u} = \frac{6(1+\sqrt{3}i)}{(1+i)} = 3(\sqrt{3}+1+(\sqrt{3}-1)i)$$

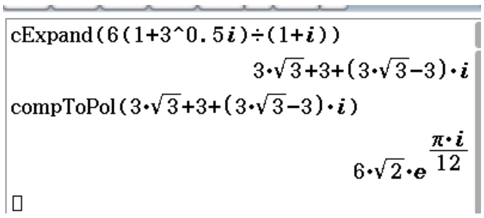
[A1]



ii.

$$w = 3\text{cis}\left(\frac{\pi}{3}\right)\ ,\ \ u = \sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$$

$$\frac{4w}{u} = \frac{4\times 3}{\sqrt{2}}\text{cis}\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = 6\sqrt{2}\text{cis}\left(\frac{\pi}{12}\right)$$



[A1]

iii.

$$\tan\left(\frac{\pi}{12}\right) = \frac{3\sqrt{3}-3}{3\sqrt{3}+3}$$ [M1]

$$= \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}\times\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$\tan\left(\frac{\pi}{12}\right) = 2-\sqrt{3}$$ [A1]

Question 4 (9 marks)

a. $\frac{dx}{dt} =$ Inflow - Outflow

Inflow $= 2 \times 0.1 = 0.2$

Outflow $= \frac{2x}{100}$

$\therefore \frac{dx}{dt} = 0.2 - \frac{2x}{100} = \frac{20-2x}{100} = \frac{10-x}{50}$ [A1]

b.

$\int \frac{1}{10-x} dx = \int \frac{1}{50} dt$ [A1]

$-\log_e |10-x| = \frac{t}{50} + C$

$t = 0, \quad x = 5 \quad \therefore \quad C = -\log_e 5$

$\log_e(10-x) - \log_e 5 = -\frac{t}{50}$

$\frac{10-x}{5} = e^{-t/50}$

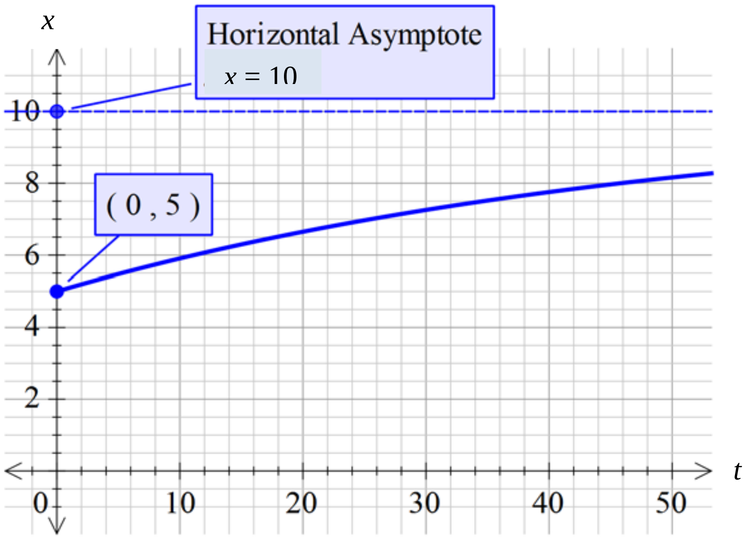
$x = 10 - 5e^{-t/50}$ [M1]

c.

$t = 2, \quad x = 10 - 5e^{-0.02 \times 2} = 5.196$ [A1]

$x = 5.20\text{kg}$

d. Sketch over domain $t \geq 0$, y intercept is 5 and there is an asymptote at $y = 10$



[A1]

e.

i.

Inflow $= 2\times 0.1 = 0.2$ (same)

Outflow $= \dfrac{3x}{100+2t-3t}$

$\dfrac{dx}{dt} = 0.2 - \dfrac{3x}{100-t}$ [A1]

ii. Using Euler's method with $h=1$ when $t=2, x=5.097$

t	x	$\dfrac{dx}{dt}$
0	5	0.05
1	$5+1\times 0.05$ $=5.05$	0.04696
2	$5.05+1\times 0.04696$ $=5.097$	

[M1]

[A1]

iii. Differential equation is

$$\frac{dx}{dt} = 0.2 - \frac{3x}{100-t}$$

$$\frac{dx}{dt} + \frac{3x}{100-t} = 0.2$$

If $x = \dfrac{1}{10}(100-t) - \dfrac{5}{1000000}(100-t)^3$

$$\frac{dx}{dt} = -\frac{1}{10} + \frac{15}{1000000}(100-t)^2$$

[M1]

L.H.S $= \dfrac{dx}{dt} + \dfrac{3x}{100-t}$

$$= -\frac{1}{10} + \frac{15}{1000000}(100-t)^2 + \frac{3}{(100-t)}\left(\frac{1}{10}(100-t) - \frac{5}{1000000}(100-t)^3\right)$$

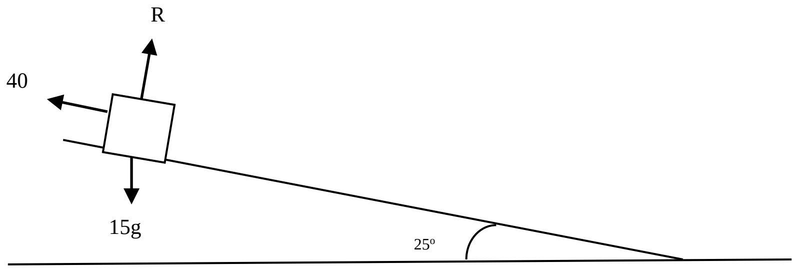
$$= -\frac{1}{10} + \frac{15}{1000000}(100-t)^2 + \frac{3}{10} - \frac{15}{1000000}(100-t)^2$$

[A1]

$$= \frac{2}{10} = 0.2 = \text{ R.H.S}$$

Question 5

a.



[A1]

b.

[M1]

$$15a = 15g\sin 25^o - 40$$

$$a = g\sin 25^o - 8/3$$

$$= 1.47\text{ms}^{-2}$$

[A1]

c.

$u = 0\text{ ms}^{-1},\ a = 1.47499\text{ ms}^{-2},\ \ s = 6\text{ m}$

[M1]

$$s = ut + \frac{1}{2}at^2$$

$$\therefore 6 = \frac{1}{2}\times 1.47499\times t^2$$

$$t = \sqrt{\frac{12}{1.47499}} = 2.85\text{ secs}$$

[A1]

Note: accept 2.85 if student has used the rounded answer from part b.

d.

$$15a = 30 - 50t + 15\times 1.47$$

[A1]

the acceleration is approximately $a = 3.47 - \dfrac{10t}{3}$

e.

[M1]

$$\text{speed } v = \int_0^{0.6}\left(3.47 - \frac{10t}{3}\right)dt$$

[A1]

$$= 1.48\text{ ms}^{-1}$$

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∫[0, 0.6] (3.47-(10/3)x)dx
                                        1.482
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f.

1^{st} part of journey:

$v = 3.47t - \frac{10}{6}t^2$

[M1]

$\therefore$ distance travelled in the first 0.6 secs $= \int_0^{0.6} \left(3.47t - \frac{10}{6}t^2\right) dt = 0.5046$

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∫[0, 0.6] (3.47x-(10/6)x^2)dx
                                        0.5046
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2^{nd} part of journey:

$u = 1.48 \text{ ms}^{-1}, \; a = 1.47 \text{ ms}^{-2}, \; s = 6 - 0.5046 = 5.4954 \text{ m}$ [M1]

$5.4954 = 1.48t + \frac{1}{2} \times 1.47t^2 \quad \therefore t = 1.9070$

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solve(5.4954=1.48x+0.5×1.47×x^2,x)
      {x=-3.920629486,x=1.907024043}
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Total time $= 0.6 + 1.9070 = 2.51$ secs [A1]

Question 6 (9 marks)

a. $X \sim \text{N}(40500, 4500^2)$

[M1]

For a sample size of 3 $\quad \bar{X} \sim \text{N}(40500, \frac{4500^2}{3})$

Therefore $\Pr(\bar{X} \geq 42000) = 0.282$ [A1]

b.

Let $Y = X_1 + X_2 + ... + X_{12}$ [M1]

$Y \sim N(12 \times 40500, 12 \times 4500^2)$

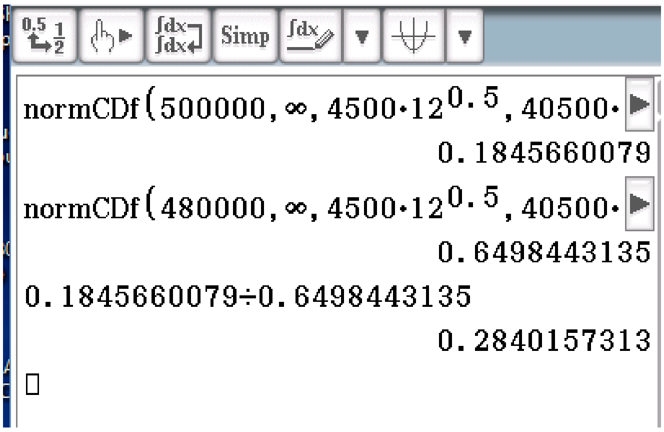
$\Pr(Y \geq 500000) = 0.185$ [A1]

c.

$\Pr(Y \geq 480000) = 0.6498$

$$\Pr(Y \geq 500000 / Y \geq 480000) = \frac{\Pr(Y \geq 500000)}{\Pr(Y \geq 480000)}$$ [M1]

$$\Pr(Y \geq 500000 / \geq 480000) = \frac{0.184566}{0.649844} = 0.284$$ [A1]



d. As we are interested in whether there has been a significant increase we use a one-sided test

$H_0 : \mu = 40500$

$H_1 : \mu > 40500$ [M1]

For a sample size of 3 $\quad \bar{X} \sim N(40500, \frac{4500^2}{3})$

$\Pr(\bar{X} \geq 46500) = 0.01046$

p-value $= 0.01046$ [A1]

$p < 0.05$ there is significant evidence to reject the Null Hypothesis

There is evidence to suggest that the advertising campaign has lead to a significant increase in the monthly turnover [A1]

$$\text{normCDf}\left(46500, \infty, \frac{4500}{3\text{^}0.5}, 40500\right)$$

0.01046066767