The Mathematical Association of Victoria

Trial Exam 2017

SPECIALIST MATHEMATICS

Written Examination 1

STUDENT NAME _____

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of Book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 14 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the space provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (3 marks)

Consider the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\sqrt{2}\underset{\sim}{k}$ and $\underset{\sim}{b} = -\underset{\sim}{i} + m\underset{\sim}{j} - 2\sqrt{3}\underset{\sim}{k}$, where m is a real constant.

Find the value of m for which the scalar resolute of $\underset{\sim}{b}$ in the direction of $\underset{\sim}{a}$ is equal to $-\frac{\sqrt{26}}{13}$.

Question 2 (4 marks)

The number of minutes it takes for an LG V10 phone to fully charge from 0% is normally distributed with a mean of 65 minutes and a standard deviation of 3 minutes. The number of minutes it takes for an LG G5 phone to fully charge from 0% is normally distributed with a mean of 76 minutes and a standard deviation of 5 minutes.

Let the random variable X represent the charging time of the LG V10 phone and let the random variable Y represent the charging time of the LG G5.

a. The probability that the LG G5 phone fully charges from 0% in less time than the LG V10 phone is equal to $\Pr(Z \leq a)$ where Z has the standard normal distribution. Find the value of a. 2 marks

A phone seller decides to test the claim that the LG V10 phone takes an average time of 65 minutes to fully charge from 0% using the following hypothesis:

$H_0: \mu = 65$ minutes.

$H_1: \mu \neq 65$ minutes.

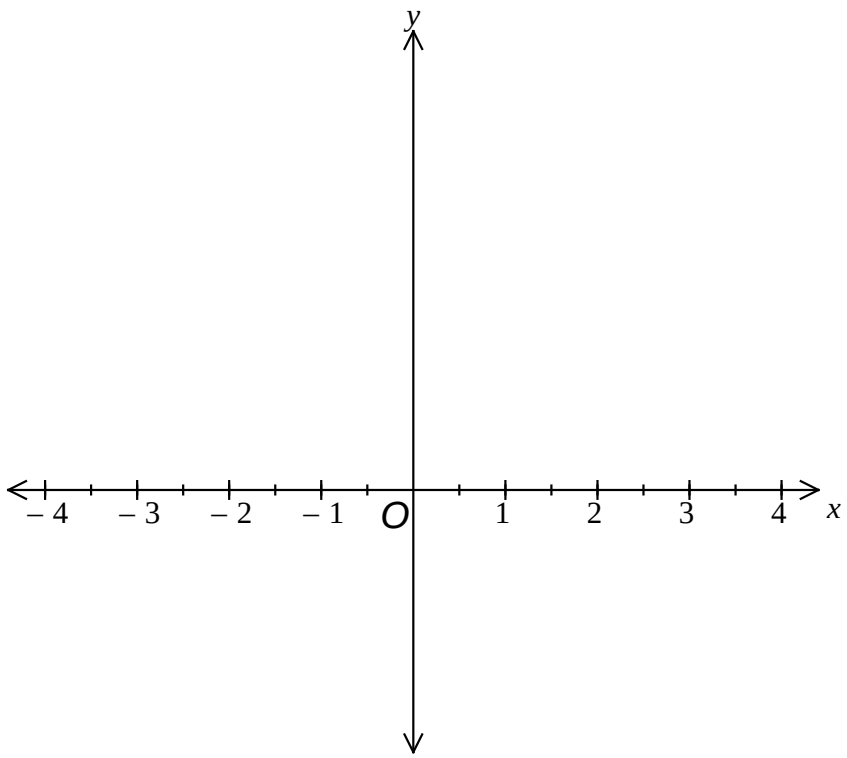
The seller collected a sample of 50 LG V10 phones and found that the average time to fully charge from 0% was 68 minutes.

b. The p-value for this test is given by the expression $2\Pr(Z \leq b)$, where Z has the standard normal distribution. Find the value of b. 2 marks

Question 3 (5 marks)

a. Find the equation of all asymptotes of $f(x) = \dfrac{x^3 - 3x + 2}{x - 2}$. 2 marks

b. Draw a graph of $y = f(x)$ over its maximal domain. Label all axes intercepts with their coordinates and all asymptotes with their equations. 3 marks



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Question 4 (3 marks)

Find in polar form all solutions of $z^4 = \sqrt{3}z^2 - 1$, $z \in C$.

Question 5 (7 marks)

A lift begins moving vertically upwards from rest with an acceleration given by $\frac{1}{3}\sqrt{25-v^2}$ ms^{-2}

where v ms^{-1} is its speed t seconds after it starts moving. This acceleration is maintained until the lift reaches its maximum speed.

a. The lift moves a distance of b metres before reaching its maximum speed. Find the value of b. 3 marks

b. The speed of the lift is increasing for $t \in [0,\ t_1)$. Find the largest value of t_1. 2 marks

Once the lift reaches its maximum speed the acceleration changes to $-\frac{1}{3}\sqrt{10v - v^2}$ ms^{-2} and the lift slows down.

c. There is a 20 kg parcel sitting on a set of bathroom scales inside the lift.
Find the reading of the scales when the lift is slowing down and has a speed of $2\,\text{ms}^{-1}$.
Give your answer in units of **kg wt**. 2 marks

Question 6 (3 marks)

Let C be a curve defined by the equation $y = f(x)$. The arc length of C from the points (a, b) to $(2, -3)$, where $a < 2$, is given by $\int_a^2 \frac{\sqrt{x^2 - 6x + 13}}{3 - x}\, dx$.

Find the two rules for the function $f(x)$.

Question 7 (3 marks)

The position of a particle at time t is given by

$$\underset{\sim}{\mathrm{r}}(t) = \arctan\left(2t^2\right)\underset{\sim}{\mathrm{i}} + \arccos\left(\frac{\sqrt{t}}{2}\right)\underset{\sim}{\mathrm{j}}, \quad t \in [0, 4].$$

Find the gradient of the path followed by the particle when $t = 3$.

Question 8 (4 marks)

The region enclosed by the graph of $y = \frac{2}{\sqrt{\pi}} \frac{1}{\sqrt{\left(3+\sqrt{x}\right)}}$ and the lines $y = 0$, $x = 1$ and $x = 4$ is rotated about the *x*-axis to form a solid of revolution of volume *V*.

a. Express *V* as a definite integral. 1 mark

b. By making an appropriate substitution in the above integral, calculate the value of *V*.

Give your answer in the form $2a\left(1 + b\ln\left(\frac{a}{5}\right)\right)$ where *a* and *b* are positive integers. 3 marks

Question 9 (4 marks)

Solve the differential equation $\frac{dy}{dx} = \frac{\cos^2(2y)}{2x^2+1}$, given that $y\left(-\frac{\sqrt{6}}{6}\right) = 0$. Express y as a function of x.

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Question 10 (4 marks)

Find the implied domain and range of the function $h(x) = -\frac{1}{2}\sin^{-1}\left(4x^2 + 2\sqrt{2}x\right)$.

END OF QUESTION AND ANSWER BOOK